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The Propagation of Love Waves in an Irregular Fluid Saturated Porous Anisotropic Layer

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Abstract

The present paper discusses the dispersion equation for Love waves in a transversely-isotropic fluid saturated porous layer over a semi-infinite non-homogeneous elastic medium with an irregularity. In the absence of the irregularity, the dispersion equation reduces to standard dispersion equation for Love waves in a transversely-isotropic fluid saturated porous layer over a semi-infinite non-homogeneous elastic medium. It can be seen that the phase velocity is strongly influenced by the wave number and the depth of the irregularity.

Keywords: Love wave, Irregular boundary, Anisotropic layer, Dispersion Equation.

INTRODUCTION

Anisotropy is a general property of geological media. Transverse isotropy, the simplest form of anisotropy which characterizes media with a single symmetry axis, can be used to describe anisotropy in many real media of geophysical interest. This is for example the case for a stack of sedimentary layers, the layered lower crust, the upper mantle and the inner core.

The propagation of Love waves with and without the presence of irregularities has been studied by many researchers at the interface. However, most the work done on this subject does not concern porous media filled with fluid with irregular interface. Bhattacharya (1962) considered the irregularity in the thickness of the transversely isotropic crustal layer. Chattopadhyay (1975) studied the effect of irregularities and non-homogeneities in the crustal layer on the propagation of Love waves. Chattopadhyay et al. (2008) studied the effect of irregularity on the propagation of SH waves in an irregular monoclinic crustal layer. Gupta et al. (2010) discussed the effect of irregularity anisotropy on the propagation of shear waves. They derived the dispersion equation by applying the perturbation method, and the phase velocity curve was obtained for different irregularities by using the parameters of the porous medium which were suggested by Biot (1961).

As the earth's crust and mantle are not homogeneous, it is desirable to have information about Love wave propagation in an inhomogeneous medium. Konczak (1988) derived dispersion equation for Love waves in a transversely isotopic fluid saturated porous layer overlying an elastic non-homogeneous half-space. In the present paper, we have extended the work done by Konczak (1988) by introducing irregularity at the lower half-space. The irregularity is in the form of a rectangle. To solve the problem we have used the perturbation technique as indicated by Eringen and Samuels (1959). It is shown that the phase velocity of Love waves depends on the depth of the irregularity. To study the effect of irregularity in the medium, the variation of dimensionless phase velocity against the dimensionless wave number for transversely isotropic fluid saturated porous layer is shown graphically for different values of irregularity size.

FORMULATION OF THE PROBLEM

A transversely isotropic fluid saturated porous layer of thickness H, resting on a non-homogeneous elastic half space is considered. The Cartesian coordinate system (x, y, z) is chosen with z-axes taken vertically downward in the half space and x-axes is chosen parallel to the layer in the direction of propagation of the disturbance. We assume the irregularity in the form of a rectangle with length s and depth H'. The origin is placed at the middle point of the interface irregularity. The source of the disturbance is placed on positive z axes at a distance d (d> H') from the origin. Therefore, the upper layer describes the medium M_1: $-H' \leq z \leq 0$, and the non-homogeneous elastic half space describes the medium M_2: $0 \leq z < \infty$ The geometry of the problem is shown in figure: 1.

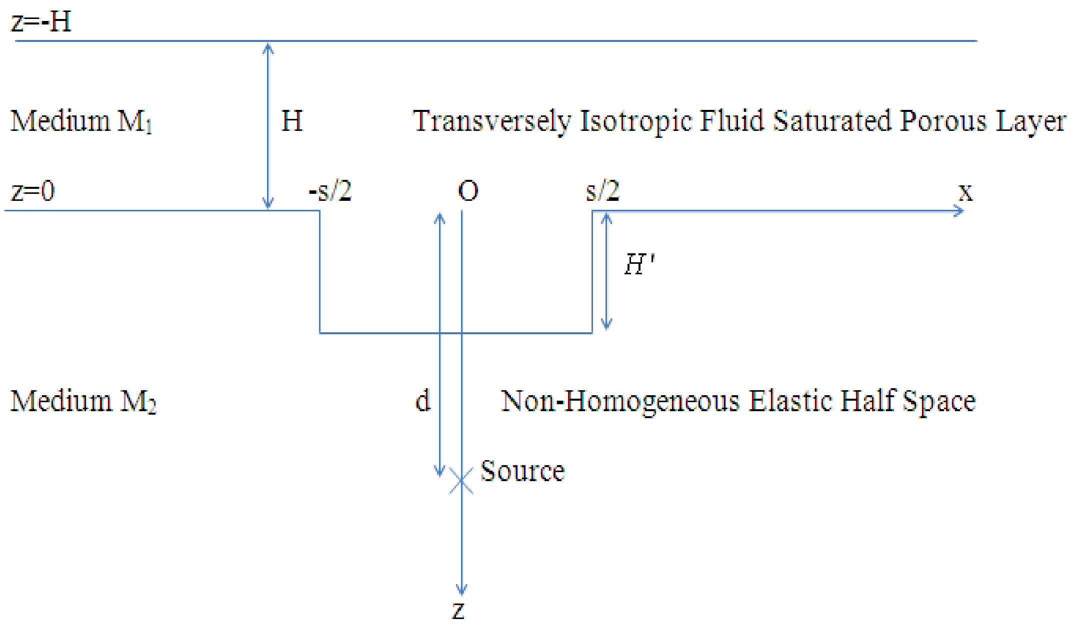

Figure 1: Geometry of the problem

The interface between the layer and half space is defined as

$$z = \varepsilon h(x) \tag{1}$$

where

$$h(x) = \begin{cases} 0; x \le -\frac{s}{2}, x \ge \frac{s}{2} \\ f(x); -\frac{s}{2} \le x \le \frac{s}{2} \end{cases} \tag{2}$$

where $\varepsilon = \frac{H'}{s}$ and $\varepsilon << 1$.

BASIC EQUATIONS

The basic equations for medium considered are as follows:

Medium M$_1$

The equation of motion for the fluid saturated porous layer in the absence of body forces are given by: [Biot (1955)]

$$\sigma^{(1)}{}_{ij,j} = \rho_{11}\ddot{u} + \rho_{12}\ddot{U}_i - b_{ij}(\dot{U}_j - \dot{u}_j) \tag{3.1}$$

$$\sigma^{(1)}{}_{,i} = \rho_{12}\ddot{u}_i + \rho_{22}\ddot{U}_i + b_{ij}(\dot{U}_j - \dot{u}_j) \tag{3.2}$$

where comma denotes the differentiation with respect to position and dot represents that with respect to time, $\sigma^{(1)}{}_{ij}$ are the components of stress tensor in the solid skeleton, $\sigma^{(1)} = -fp$ is the reduced pressure of the fluid (p is the pressure in the fluid, and f is the porosity of the medium M_1), u_i are the components of the displacement vector of the solid skeleton and U_i are these of fluid.

The stress-strain relations for the transverse-isotropic fluid saturated porous layer [Biot (1955)]

$$\begin{bmatrix} \sigma^{(1)}{}_{11} \\ \sigma^{(1)}{}_{22} \\ \sigma^{(1)}{}_{33} \\ \sigma^{(1)}{}_{23} \\ \sigma^{(1)}{}_{31} \\ \sigma^{(1)}{}_{12} \\ \sigma^{(1)} \end{bmatrix} = \begin{bmatrix} 2C_1+C_2 & C_2 & C_3 & 0 & 0 & 0 & C_6 \\ C_2 & 2C_1+C_2 & C_3 & 0 & 0 & 0 & C_6 \\ C_3 & C_3 & 2C_4 & 0 & 0 & 0 & C_7 \\ 0 & 0 & 0 & 2C_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2C_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2C_1 & 0 \\ C_6 & C_6 & C_7 & 0 & 0 & 0 & C_8 \end{bmatrix} \begin{bmatrix} e_{11} \\ e_{22} \\ e_{33} \\ e_{23} \\ e_{31} \\ e_{12} \\ E \end{bmatrix}, \tag{4}$$

where

$$\begin{aligned} e_{ij} &= \frac{1}{2}(u_{i,j} + u_{j,i}), \\ E &= div U \equiv U_{j,j}, \\ e &= div u \equiv u_{k,k} \end{aligned} \tag{5}$$

and $C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8$ are the material constants.

On substituting from equations (4) and (5) in equations (3), we obtain the system of equations

$$(C_2\varepsilon + C_6E)_{,1} + [C_1(u_{1,1} + u_{2,2}) + (C_5 + C_3 - C_2)u_{3,3}]_{,1} + [C_1(u_{1,11} + u_{1,22}) + C_5u_{1,33}] = \rho_{11}\ddot{u}_1 + \rho_{12}\ddot{U}_1 - b_{11}(\dot{U}_1 - \dot{u}_1),$$

$$(C_2\varepsilon + C_6E)_{,2} + [C_1(u_{1,1} + u_{2,2}) + (C_5 + C_3 - C_2)u_{3,3}]_{,2} + [C_1(u_{2,11} + u_{2,22}) + C_5u_{2,33}] = \rho_{11}\ddot{u}_2 + \rho_{12}\ddot{U}_2 - b_{11}(\dot{U}_2 - \dot{u}_2),$$

$$\begin{aligned} &C_7E_{,3} + [(C_3 + C_5)(u_{1,1} + u_{2,2}) + C_4u_{3,3}]_{,3} + [C_5(u_{3,11} + u_{3,22}) + C_5u_{3,33}] \\ &= \rho_{11}\ddot{u}_3 + \rho_{12}\ddot{U}_3 - b_{33}(\dot{U}_3 - \dot{u}_3). \end{aligned} \tag{6}$$

$$(C_6\varepsilon + C_8 E)_{,1} + (C_7 - C_6)u_{3,31} = \rho_{12}\ddot{u}_1 + \rho_{22}\ddot{U}_1 + b_{11}(\dot{U}_1 - \dot{u}_1),$$
$$(C_6\varepsilon + C_8 E)_{,2} + (C_7 - C_6)u_{3,32} = \rho_{12}\ddot{u}_2 + \rho_{22}\ddot{U}_2 + b_{11}(\dot{U}_2 - \dot{u}_2),$$
$$(C_6\varepsilon + C_8 E)_{,3} + (C_7 - C_6)u_{3,33} = \rho_{12}\ddot{u}_3 + \rho_{22}\ddot{U}_3 + b_{33}(\dot{U}_3 - \dot{u}_3). \tag{7}$$

For Love-waves propagating in the x-direction with the displacement in the z-direction, we have

$$u_1 \equiv 0, \qquad u_2 \equiv u(x,z,t), \qquad u_3 \equiv 0,$$
$$U_1 \equiv 0, \qquad U_2 \equiv U(x,z,t), \qquad U_3 \equiv 0, \tag{8}$$

Eq. (6) and eq. (7) are reduced to the form

$$C_1 \frac{\partial^2 u_2}{\partial x^2} + C_5 \frac{\partial^2 u_2}{\partial z^2} = \frac{\partial^2}{\partial t^2}(\rho_{11}u_2 + \rho_{12}U_2) - b_{11}\frac{\partial}{\partial t}(U_2 - u_2),$$
$$0 = \frac{\partial^2}{\partial t^2}(\rho_{12}u_2 + \rho_{22}U_2) + b_{11}\frac{\partial}{\partial t}(U_2 - u_2). \tag{9}$$

By eliminating u_2 and U_2 from equation (9), we obtain

$$\left\{ C_1 \frac{\partial^2}{\partial x^2} + C_5 \frac{\partial^2}{\partial z^2} - \left[\begin{array}{l} \rho_{11}\partial_t^2 + b_{11}\partial_t \\ -\dfrac{(\rho_{12}\partial_t^2 - b_{11}\partial_t)^2}{\rho_{22}\partial_t^2 + b_{11}\partial_t} \end{array} \right] \right\}(u_2, U_2) = 0. \tag{10}$$

Medium M2

For the lower non-homogeneous half-space the basic equations of motion, without the body force, are [Ewing (1957)]

$$\sigma^{(2)}{}_{ij,j} = \rho^* \ddot{v}_i, \tag{11}$$

where $\sigma^{(2)}{}_{ij}$ are the components of stress tensor, v_i are the components of displacement vector, and ρ^* is the density.

The constitutive relation is given by

$$\sigma^{(2)}{}_{ij} = \lambda p_{kk}\delta_{ij} + 2\mu p_{ij}, \tag{12}$$

where λ and μ are Lame's elastic coefficients and are functions of x, y, z; δ_{ij} is the Kronecker delta and

$$p_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}), \quad p_{kk} = v_{k,k} = p. \tag{13}$$

With the help of equation (12) and (13), the equations of motion take the form of

$$\begin{aligned}&[(\lambda+2\mu)p],_i - 2\mu,_i\, p \\ &- \mu p,_i + \mu,_j\,(v_{i,j} + v_{j,i}) + \mu v_{i,ji} = \rho^* \ddot{v}_i .\end{aligned} \tag{14}$$

The equation (14), for Love waves $v_1 = v_3 = 0, v_2 = v_2(x,z,t),$ reduces to the form

$$\left\{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right\} v_2 + \frac{1}{\mu}\frac{d\mu}{dz}\frac{\partial v_2}{\partial z} = \frac{1}{\beta^2}\frac{\partial^2 v_2}{\partial t^2}. \tag{15}$$

The inhomogeneity and heterogeneity of the elastic half-space is characterized by

$$\mu = \mu_0 \exp(qz), \rho^* = \rho_0^* \exp(qz), \tag{16}$$

where μ_0, ρ_0^* *and* q are constants.

BOUNDARY CONDITIONS

The appropriate boundary conditions for the considered problem are as:

(i) At the free surface $z = -H$, the shear stress component vanishes, i.e.,

$$\sigma^{(1)}{}_{32}(x, z = -H, t) = 0. \tag{17.1}$$

(ii) The stresses are continuous at the interface $z = \varepsilon h(x)$:

$$C_5 \frac{\partial u_2}{\partial z} - C_1 \varepsilon h'(x)\frac{\partial u_2}{\partial x} = \mu\left(\frac{\partial v_2}{\partial z} - \varepsilon h'(x)\frac{\partial v_2}{\partial x}\right) \tag{17.2}$$

where $h'(x) = \dfrac{dh(x)}{dx}$.

(iii) At the interface $z = \varepsilon h(x)$, the displacements are continuous:

$$u_2(x, z = \varepsilon h(x), t) = v_2(x, z = \varepsilon h(x), t). \tag{17.3}$$

SOLUTION OF THE PROBLEM

For wave changing harmonically with time, we take

$$u_2(z,x,t) = u_2^0(z,x)\exp(i\omega t),$$

$$U_2(z,x,t) = U_2^0(z,x)\exp(i\omega t), i^2 = -1, \tag{18}$$

where ω is the angular frequency. On substituting from equation (18) into equation (10), we obtain

$$\left(C_1\frac{\partial^2}{\partial x^2} + C_5\frac{\partial^2}{\partial z^2} + \xi_1^2\right)(u_2^0, U_2^0) = 0, \tag{19.1, 2}$$

where

$$\xi_1^2 = \alpha_1 + i\alpha_2, \tag{20}$$

$$\alpha_1 = F\omega^2 / c_G^2, \alpha_2 = R\omega^2 / c_G^2,$$

$$F = F(\omega) = \frac{1+\Omega^2\gamma_{22}C'}{1+(\Omega\gamma_{22})^2}.\frac{\gamma_{22}}{C'},$$

$$R = R(\omega) = \frac{(C'-\gamma_{22})\Omega}{1+(\Omega\gamma_{22})^2}.\frac{\gamma_{22}}{C'}, \tag{21}$$

$$C' = \gamma_{11}\gamma_{22} - \gamma_{12}^2, \gamma_{kl} = \frac{\rho_{kl}}{\rho}(k,l = 1,2),$$

$$c_G^2 = (\rho_{11} - \rho_{12}^2/\rho_{22})^{-1}, \Omega = \frac{\rho\omega}{b_{11}}.$$

Ω is the dimensionless frequency and c_G is the velocity of shear wave in the porous layer.

For elastic heterogeneous half-space, waves changing harmonically with time, we obtain

$$v_2(z,x,t) = v_2^0(z,x)\exp(i\omega t), \tag{22}$$

Hence equation (15) with the aid of (16) takes the form of

$$\frac{\partial^2 v_2^0}{\partial x^2}+\frac{\partial^2 v_2^0}{\partial z^2}+q\frac{\partial v_2^0}{\partial z}+\frac{\omega^2}{\beta^2}v_2^0=0, \tag{23}$$

Define the Fourier Transform $\bar{u}_2^0(z,\eta) of\, u_2^0(z,\eta)$ as

$$\bar{u}_2^0(z,\eta)=\int_{-\infty}^{\infty}u_2^0(z,x)e^{i\eta x}dx, \tag{24}$$

And inverse Fourier Inverse Transform is given by

$$u_2^0(z,x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{u}_2^0(z,\eta)e^{-i\eta x}d\eta, \text{etc.} \tag{25}$$

The Fourier Transform of equations (19.1, 2) and (23) then are

$$\frac{d^2\bar{u}_2^0}{dz^2}+\chi_1^2\bar{u}_2^0=0, \tag{26}$$

$$\frac{d^2\bar{U}_2^0}{dz^2}+\chi_1^2\bar{U}_2^0=0, \tag{27}$$

$$\frac{d^2\bar{v}_2^0}{dz^2}+q\frac{d\bar{v}_2^0}{dz}-\chi_2^2\bar{v}_2^0=0. \tag{28}$$

where $\chi_1^2=\frac{C_1}{C_5}\left(\frac{\xi_1^2}{C_1}-\eta^2\right), \chi_2^2=\left(\eta^2-\frac{\omega^2}{\beta^2}\right)$

The solution of equations (26) and (27) are

$$\bar{u}_2^0=A\cos\chi_1 z+B\sin\chi_1 z, \tag{29}$$

$$\bar{U}_2^0=\bar{A}\cos\chi_1 z+\bar{B}\sin\chi_1 z, \tag{30}$$

The appropriate solution of equation (28) is

$$\bar{v}_2^0=D.\exp(-\zeta z), 0\le z<\infty, \tag{31}$$

where

$$\zeta = \frac{1}{2}\left(q + \sqrt{q^2 + 4\chi_2^2}\right) \tag{32}$$

Where $A, B, \overline{A}, \overline{B}, D$ are functions of η.

Thus, by inverse Fourier Transform, we obtain

$$u_2^0(z,x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}(A\cos\chi_1 z + B\sin\chi_1 z)e^{-i\eta x}d\eta, \tag{33}$$

$$U_2^0(z,x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}(\overline{A}\cos\chi_1 z + \overline{B}\sin\chi_1 z)e^{-i\eta x}d\eta, \tag{34}$$

$$v_2^0(z,x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}(De^{-\zeta z} + \frac{2}{\zeta}e^{\zeta z}e^{-\zeta d})e^{-i\eta x}d\eta, \tag{35}$$

where the second term in the integrand of $v_2^0(z,x)$ is introduced due to the source in the lower medium.

The relations between the constants $\overline{A}, \overline{B}$ and A, B are provided by equation (9).

We set the following approximations due to small value of ε

$$A \cong A_0 + A_1\varepsilon, B \cong B_0 + B_1\varepsilon, D \cong D_0 + D_1\varepsilon. \tag{36}$$

Since the boundary is not uniform, the terms A, B, D in equation (36) are also functions of ε. Expanding these terms in ascending powers of ε and keeping in view that ε is small and so retaining the terms up to the first order of ε, A, B, D can be approximated as in equation (36). In physical situations, when the depth H' of the irregular boundary is too small with respect to the length of the boundary s, the above assumptions are justified. Further for small ε

$$e^{\pm\alpha\varepsilon h} \cong 1 \pm \alpha\varepsilon h, \cos\chi_1\varepsilon h \cong 1, \sin\chi_1\varepsilon h \cong \chi_1\varepsilon h,$$

where α is any quantity.

Now, by using boundary condition (17.1), we obtain

$$\begin{aligned}&(A_0 + A_1\varepsilon)\sin\chi_1 H \\ &+ (B_0 + B_1\varepsilon)\cos\chi_1 H = 0.\end{aligned} \tag{37}$$

Using boundary condition (17.3), we have

$$\varepsilon\int_{-\infty}^{\infty}[(B_0\chi_1+\zeta D_0-2e^{-\zeta d})+\varepsilon(B_1\chi_1+\zeta D_1)]h(x)e^{-i\eta x}d\eta$$
$$=\int_{-\infty}^{\infty}[((\zeta D_0-A_0)+\frac{2}{\zeta}e^{-\zeta d})+\varepsilon(\zeta D_1-A_1)]e^{-i\eta x}d\eta.$$

As ε is very- very small, we have

$$\varepsilon\int_{-\infty}^{\infty}[(B_0\chi_1+\zeta D_0-2e^{-\zeta d})]h(x)e^{-i\eta x}d\eta$$
$$=\int_{-\infty}^{\infty}[(D_0-A_0)+\frac{2}{\zeta}e^{-\zeta d})+\varepsilon(D_1-A_1)]e^{-i\eta x}d\eta. \tag{38}$$

Now we define Fourier Transform of $h(x)$ as

$$\bar{h}(\lambda)=\int_{-\infty}^{\infty}h(x)e^{i\lambda x}dx. \tag{39}$$

And the inverse Fourier Transform is

$$h(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{h}(\lambda)e^{-i\lambda x}d\lambda. \tag{40}$$

Therefore,

$$h'(x)=\frac{-i}{2\pi}\int_{-\infty}^{\infty}\lambda\,\bar{h}(\lambda)e^{-i\lambda x}d\lambda. \tag{41}$$

Now by using equation (40), equation (38) takes the form of

$$\frac{\varepsilon}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}[(B_0\chi_1+\zeta D_0-2e^{-\zeta d})]\bar{h}(\lambda)e^{-i(\eta+\lambda)x}d\eta\right]d\lambda=\int_{-\infty}^{\infty}[((D_0-A_0)+\frac{2}{\zeta}e^{-\zeta d})+\varepsilon(D_1-A_1)]e^{-i\eta x}d\eta. \tag{42}$$

Putting $\eta+\lambda=k$ for the inner integral in the left hand side of equation (42), so that λ may be treated as a constant such that $d\eta=dk$, replacing η by k in the right hand side of equation (42), and finally after taking Fourier transform as defined just above, we have

$$((D_0 - A_0) + \frac{2}{\zeta} e^{-\zeta d}) + \varepsilon (D_1 - A_1) = \varepsilon R_1(k), \tag{43}$$

where

$$R_1(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [(B_0 \chi_1 + \zeta D_0 - 2e^{-\zeta d})]^{\eta = k-\lambda} \bar{h}(\lambda) d\lambda. \tag{44}$$

Now by using boundary condition (17.2), we have

$$\begin{aligned} &\varepsilon \int_{-\infty}^{\infty} \begin{matrix} ([\chi_1^2 C_5 A_0 - \mu_0 (q-\zeta)(\zeta D_0 - 2e^{-\zeta d})] h(x) \\ + i\eta [-C_1 A_0 + \mu_0 (D_0 + \frac{2}{\zeta} e^{-\zeta d}] h'(x)) e^{-i\eta x} d\eta \end{matrix} \\ &= \int_{-\infty}^{\infty} \begin{matrix} [(B_0 \chi_1 C_5 + \zeta \mu_0 D_0 - 2\mu_0 e^{-\zeta d}) \\ + \varepsilon (\mu_0 \zeta D_1 + B_1 \chi_1 C_5)] e^{-i\eta x} d\eta \end{matrix}. \end{aligned} \tag{45}$$

Similarly, using equations (39), (40) & (41) in equation (45), proceeding as in equation (43), we obtain

$$\begin{aligned} &(B_0 \chi_1 C_5 + \zeta \mu_0 D_0 - 2\mu_0 e^{-\zeta d}) \\ &+ \varepsilon (\mu_0 \zeta D_1 + B_1 \chi_1 C_5) = \varepsilon R_2(k) \end{aligned} \tag{46}$$

where

$$R_2(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\chi_1^2 C_5 A_0 - \mu_0 (q-\zeta)(\zeta D_0 - 2e^{-\zeta d}) + \lambda k (C_1 A_0 - \mu_0 (D_0 + \frac{2}{\zeta} e^{-\zeta d}) \right]^{\eta = k-\lambda} \bar{h}(\lambda) d\lambda.$$

(47)

Equating the absolute term (terms not containing ε) and the coefficients of ε from equations (37), (43), and (46), we obtain

$$\begin{aligned} &A_0 \sin \chi_1 H + B_0 \cos \chi_1 H = 0, \\ &A_0 - D_0 = \frac{2}{\zeta} e^{-\zeta d}, \\ &C_5 \chi_1 B_0 + \mu_0 \zeta D_0 = 2\mu_0 e^{-\zeta d}, \\ &A_1 \sin \chi_1 H + B_1 \cos \chi_1 H = 0, \\ &D_1 - A_1 = R_1(k), \\ &C_5 \chi_1 B_1 + \mu_0 \zeta D_1 = R_2(k). \end{aligned} \tag{48}$$

Solving the above six equations given as in (48), we deduce that

$$A_0 = \frac{4\mu_0 e^{-\zeta d}}{E(k)},$$

$$B_0 = \frac{-4\mu_0 e^{-\zeta d}\tan\chi_1 H}{E(k)},$$

$$D_0 = \frac{2e^{-\zeta d}(\mu_0\zeta + C_5\chi_1\tan\chi_1 H)}{\zeta E(k)}, \tag{49}$$

$$A_1 = \frac{R_2 - \mu_0\zeta R_1}{E(k)},$$

$$B_1 = \frac{-(R_2 - \mu_0\zeta R_1)\tan\chi_1 H}{E(k)},$$

$$D_1 = \frac{R_2 - R_1 C_5\chi_1\tan\chi_1 H}{E(k)},$$

where

$$E(k) = \mu_0\zeta - C_5\chi_1\tan(\chi_1 H).$$

The displacement in the anisotropic layer is

$$u_2^0 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{4\mu_0 e^{-\zeta d}}{E(k)}\left[1 + \frac{\varepsilon(R_2 - \mu_0\zeta R_1)e^{\zeta d}}{4\mu_0}\right] (\cos\chi_1 z - \tan(\chi_1 H)\sin\chi_1 z)e^{-ikx}dk. \tag{50}$$

Now from equation (1), we have

$$\bar{h}(\lambda) = \frac{2s}{\lambda}\sin\frac{\lambda s}{2}. \tag{51}$$

Using equations (44), (47) and (51), we obtain

$$R_2 - \mu_0\zeta R_1 = \frac{2s\mu_0}{\pi}\int_{-\infty}^{\infty}\begin{pmatrix}\phi(k-\lambda)\\ +\phi(k+\lambda)\end{pmatrix}\frac{1}{\lambda}\sin\frac{\lambda s}{2}d\lambda, \tag{52}$$

where

$$\phi(k-\lambda) = \left[(A_2 + A_3)\frac{e^{-\zeta d}}{E(k)}\right]^{\eta = k-\lambda},$$

and

$$A_2 = 2C_5\chi_1^2 - 2C_5 q\chi_1 \tan\chi_1 H,$$
$$A_3 = 2\chi_1\zeta \tan\chi_1 H + 2\lambda k(C_1 - \mu_0). \tag{53}$$

Here, the argument of $\phi(k-\lambda)$ is because of $\eta + \lambda = k$.

Using asymptotic formula of Willis (1948) and Tranter (1966) and neglecting the terms containing 2/s and highest powers of 2/s for large s, we obtain

$$\int_{-\infty}^{\infty}[\phi(k-\lambda)+\phi(k+\lambda)]\frac{1}{\lambda}\sin\frac{\lambda s}{2}d\lambda \cong \frac{\pi}{2}.2\phi(k) = \pi\phi(k). \tag{54}$$

Now using equation (52) and equation (54), we obtain

$$R_2 - \mu_0\zeta R_1 = 2s\mu_0\phi(k) = 2\mu_0\frac{H'}{\varepsilon}\phi(k). \tag{55}$$

Therefore the displacement in the anisotropic layer is

$$u_2^0 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{4\mu_0 e^{-\zeta d}}{E(k)[1-\frac{H'}{2}\phi(k)e^{\zeta d}]}.$$
$$(\cos\chi_1 z - \tan(\chi_1 H)\sin\chi_1 z)e^{-ikx}dk. \tag{56}$$

The value of this integral depends entirely on the contribution of the poles of the integrand. The poles are located at the roots of the equation

$$E(k)[1-\frac{H'}{2}\phi(k)e^{\zeta d}] = 0 \tag{57}$$

Equation (57) may be written as

$$B_2 - H' B_3 = 0 \tag{58}$$

where

$$B_2 = \mu_0\zeta - C_5\chi_1\tan(\chi_1 H),$$
$$B_3 = C_5\chi_1^2 - C_5\chi_1 q\tan\chi_1 H + \chi_1\zeta\tan\chi_1 H.$$

If c is the common wave velocity of wave propagating along the surface, then we can set in equation (58) $\omega = ck$ (ω is the circular frequency and k is the wave number), $\chi_1 = P_1 k, q = Qk$ and $\zeta = P_2 k$

where

$$P_1 = \sqrt{\left(\frac{1}{C_5}\left(\frac{c^2}{c_G^2}.F(\omega) - C_1\right) + i\frac{1}{C_5}.\frac{c^2}{c_G^2}.R(\omega)\right)} \text{ and } P_2 = \frac{1}{2}\left(Q + \sqrt{Q^2 + 4\left(1 - \frac{c^2}{\beta^2}\right)}\right).$$

Solving equation (58), we obtain

$$\tan P_1 kH = \frac{\mu_0 P_2 - H' P_1^2 k C_5}{P_1 (C_5 (1 - QkH') + P_2 H' k)}. \tag{59}$$

Since the quantity P_1^2 is complex, so we have

$$P_1 = k_1 + ik_2, \tag{60}$$

where

$$k_{1,2} = \left\{\frac{1}{2}\left(\left[\begin{array}{l}\left\{\frac{1}{C_5}\left(\frac{c^2}{c_G^2}.F(\omega) - C_1\right)\right\}^2 \\ + \left(\frac{1}{C_5}.\frac{c^2}{c_G^2}.R(\omega)\right)^2\end{array}\right]^{\frac{1}{2}} \pm \left\{\frac{1}{C_5}\left(\frac{c^2}{c_G^2}.F(\omega) - C_1\right)\right\}\right)\right\}^{\frac{1}{2}}. \tag{61}$$

Thus by using equation (60), the dispersion equation (59) for Love waves, simplifies to

$$\tan(k_1 + ik_2)kH = A_r + iA_i \tag{62}$$

where

$$A_r = \frac{\left(\mu_0 P_2 - H'\left(\frac{c^2}{c_G^2}.F(\omega) - C_1\right)k\right)k_1 + H'k\frac{c^2}{c_G^2}.R(\omega)k_2}{\left(k_1^2 + k_2^2\right)\left(C_5(1 - QkH') + P_2 H' k\right)}, \tag{63}$$

$$A_i = -\frac{\left(\mu_0 P_2 - H'\left(\frac{c^2}{c_G^2}.F(\omega) - C_1\right)k\right)k_2 + H'k\frac{c^2}{c_G^2}.R(\omega)k_1}{\left(k_1^2 + k_2^2\right)\left(C_5(1-QkH') + P_2 H'k\right)}. \tag{64}$$

As k_2 is small, so we have

$$\tan(k_1 + ik_2)kH \approx \frac{\tan k_1 kH + ik_2 kh}{1 - ik_2 kh \tan k_1 kH}. \tag{65}$$

So making use of equations (63), (64), (65) and separating real and imaginary parts of equation (62), we obtain two real equations

$$\begin{aligned} \tan k_1 kH &= \frac{A_r}{1 - A_i k_2 kH}, \\ k_2 kH(1 + A_r k_2 kH.\tan k_1 kH) &= A_i. \end{aligned} \tag{66}$$

Since, the real part of equation (62) gives the dispersion equation for Love waves. Therefore, dispersion equation for SH waves is

$$\tan k_1 kH = \frac{A_r}{1 - A_i k_2 kH} \approx A_r(1 + A_i k_2 kH). \tag{67}$$

Putting $H' = 0$ in equation (59), we obtain the standard dispersion equation of Love waves in a transversely isotopic fluid saturated porous layer over a non homogeneous elastic half space which concludes with the results already obtained by Konczak (1988)

$$\tan(k_1 + ik_2)H = \frac{\mu_0 \zeta}{C_5 \chi_1} = \frac{\mu_0}{2C_5}.\frac{\left(q + \sqrt{q^2 + 4\chi_2^2}\right)}{\chi_1}. \tag{68}$$

NUMERICAL RESULTS AND DISCUSSIONS

In this section we intend to study the effect of irregularity present in the transversely isotropic fluid saturated porous layer and to compare the results numerically between the phase velocity and the wave number. We will use the values of elastic constants given by Ding et al. (2006) for medium M_1 and Konczak (1988) for medium M_2. And by using Mat Lab, we obtain the following graph

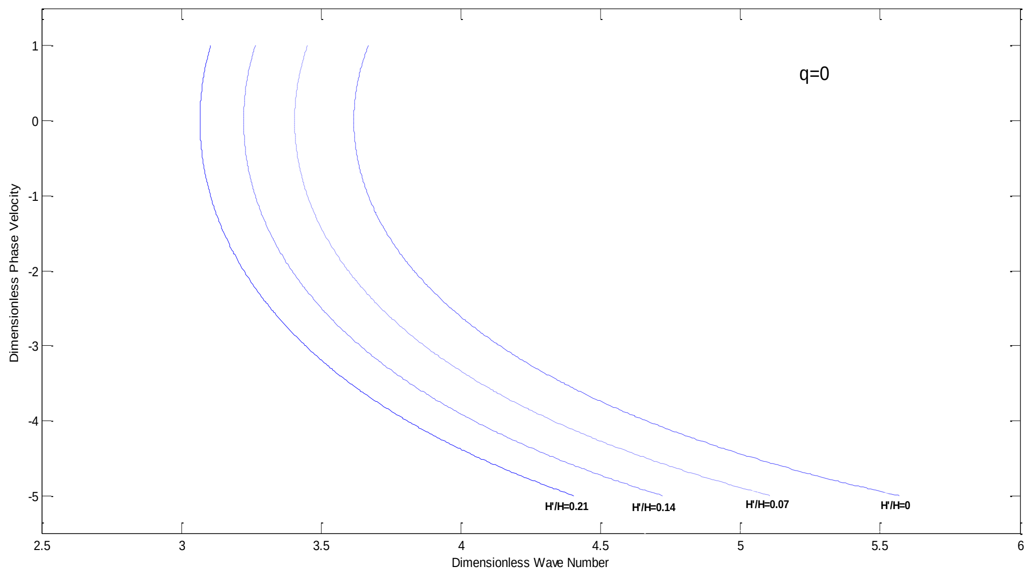

Figure 2: Variation of the dimensionless phase velocity (c/c_G) against the dimensionless wave number (kH) in a transversely isotropic fluid saturated porous layer over a homogeneous elastic half space for different values of H'/H (0, 0.07, 0.14, 0.21).

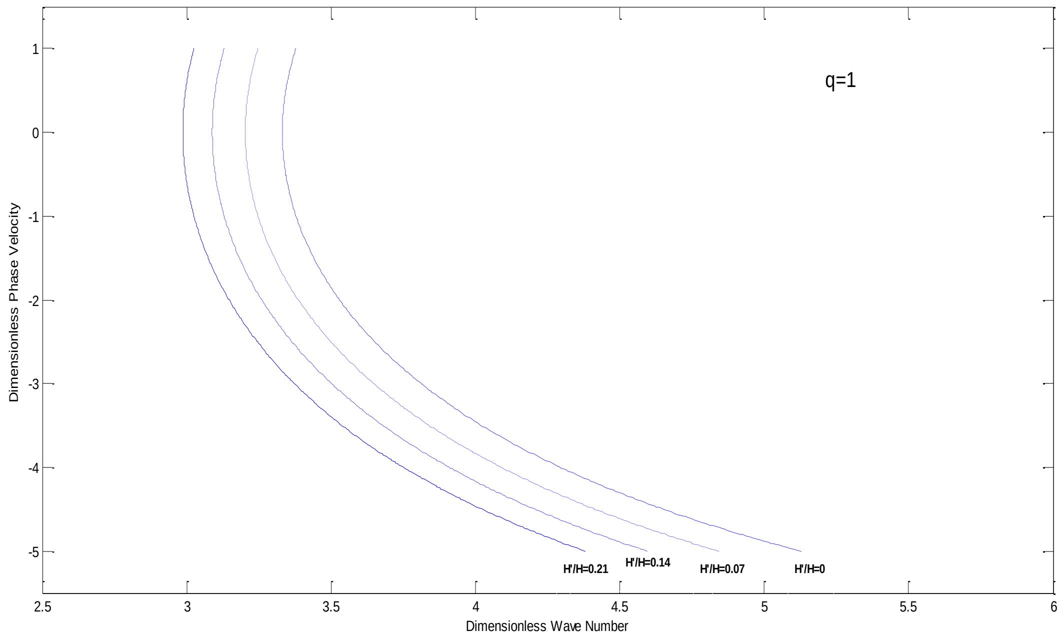

Figure 3: Variation of the dimensionless phase velocity (c/c_G) against the dimensionless wave number (kH) in a transversely isotropic fluid saturated porous layer over a non- homogeneous elastic half space for different values of H'/H (0, 0.07, 0.14, 0.21).

The dimensionless phase velocity (c/c_G) is plotted against the dimensionless wave number (kH) in Figures 2 and 3. It is clear from Figure 2 and 3 that the phase velocity decreases with increase in wave number and also increase in the value of H'/H. It is interesting to note that $\left(\frac{c}{c_G}\right)_{\frac{H'}{H}=0} \geq \left(\frac{c}{c_G}\right)_{\frac{H'}{H}\neq 0}$ in both the cases. It may also be interpreted from the two graphs that due to the effect of irregularity at the interface, the phase velocity of Love waves in the transversely isotropic fluid saturated porous layer over non-homogeneous elastic half space decreases faster than the phase velocity of Love waves in the transversely isotropic fluid saturated porous layer over homogeneous elastic half space for a fixed value of H'/H.

CONCLUSIONS

Propagation of Love waves in a transversely isotropic fluid saturated porous layer with irregular boundary over a non-homogeneous isotropic half space has been studied. The perturbation method is applied to find the displacement field in the layer. The result obtained is used to get dispersion relation in an irregular transversely isotropic fluid saturated porous layer. The dispersion relation for the layer with and without irregularity has been derived as a special case of the present problem. The effect of dimensionless wave number on dispersion curve is shown graphically for both homogeneous and non-homogeneous half spaces. Variation of phase velocity for different ratio of irregularity depth to the layer width is studied and shown graphically. From above discussion, we conclude that:

I. In general the phase velocity of Love waves in transversely isotropic fluid saturated porous layer over a homogeneous or a non-homogeneous half space with irregularity decreases with the increase in wave number.

II. Phase velocity is a function of wave number as well as layer width and depth of irregularity.

III. Increase in the depth of the irregularity decrease the magnitude of the phase velocity.

Hence, we conclude that the transversely isotropic fluid saturated porous layer with irregularity has a significant effect on the propagation of Love waves and the phase velocity in a layer with irregularity is affected by not only the shape of irregularity, but also by wave number, the ratio of the depth of the irregularity to layer width and layer structure.

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REFERENCES

[1] Bhattacharya J (1962). On the dispersion curve for Love waves due to irregularity in the thickness of the transversely isotropic crustal layer, Beitr. Geophysical, 71, pp 324–333.

[2] Biot MA (1955). Theory of Elasticity and Consolidation for a Porous Anisotropic Solid, Journal of Applied Physics, 26(2), pp182-85.

[3] Biot MA (1961). Mechanics of Incremental Deformation, John Wiley and Sons Inc., New York.

[4] Chattopadhyay A (1975). On the dispersion equation for Love wave due to irregularity in the thickness of the non-homogeneous crustal layer, Acta Geophysica Polonica, 23, pp 307–317.

[5] Chattopadhyay A, Gupta S, Sharma VK and Pato Kumari (2008). Propagation of SH waves in an irregular monoclinic crustal layer, Archive of Applied Mechanics, 78, pp 989–999.

[6] Eringen AC and Samuels CJ (1959). Impact and moving loads on a slightly curved elastic half-space, Journal of Applied Mechanics, 26, pp 491–498.

[7] Ewing WM, Jardetzky WS, and Press F (1957). Elastic Waves in Layered Media, McGraw-Hill, New York.

[8] Gupta S, Chattopadhyay A and Majhi DK (2010). Effect of irregularity on the propagation of torsional surface waves in an initially stressed anisotropic poro-elastic layer, Applied Mathematics and Mechanics (English Edition), 31(4), pp 481–492.

[9] Ding H and Hangzhou WC (2006). Elasticity of transversely isotropic materials, Springer P.R., China.

[10] Konczak Z (1988). The propagation of Love waves in fluid saturated porous anisotropic layer, Acta Mechnica, 79, pp 155-168.

[11] Tranter CJ (1966). Integral Transform in Mathematical Physics, Methuen and Co. Ltd., 63-67.

[12] Willis H. F. (1948). A formula for expanding an integral as a series, *Phil. Mag*., 39, 455-459.