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Coefficient Estimates for a New Subclass of Meromorphic Bi-Univalent Functions Defined by Al-Oboudi Differential Operator

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Abstract

In the present investigation, we define a new subclass of meromorphic bi-univalent function class $\Sigma^{'}$ of complex order $\gamma \in \mathbb{C} \setminus \{0\}$, using Al-Oboudi differential operator and obtain the estimates for the coefficients $|b_0|$ and $|b_1|$. Further we point out several known consequences of our result.

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1. Introduction

Let $\mathcal{A}$ denote the class of all normalized functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \tag{1.1}$$

which are analytic in the open unit disk,

$$\mathbb{U} = \{z : z \in \mathbb{C},\ |z| < 1\}.$$

A function $f \in \mathcal{A}$ is said to be in the class $S(\gamma)$ of univalent functions of complex order $\gamma \in \mathbb{C} \setminus \{0\}$ if and only if

$$\Re\left\{1+\frac{1}{\gamma}\left[\frac{zf'(z)}{f(z)}-1\right]\right\} > 0, \quad \frac{f(z)}{z} \neq 0, \quad (z \in \mathbb{U}). \tag{1.2}$$

Let $\mathcal{S}$ denote the class of all functions in the normalized analytic function class $\mathcal{A}$ which are univalent in $\mathbb{U}$.(see, for details [8, 15]). Then clearly, every function $f \in \mathcal{S}$ has an inverse f^{-1} defined by

$$f^{-1}(f(z)) = z, \quad (z \in \mathbb{U})$$

and

$$f(f^{-1}(w)) = w, \quad \left(|w| < r_0(f),\ r_0(f) \geq \frac{1}{4}\right)$$

where

$$f^{-1}(w) = w - a_2w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2a_3 + a_4)w^4 + \cdots. \tag{1.3}$$

A function $f \in \mathcal{A}$ given by (1.1) is said to be bi-univalent in $\mathbb{U}$ if both f and f^{-1} are univalent in $\mathbb{U}$. The class of these bi-univalent functions is denoted by Σ. Followed by Brannan and Taha [3] (see also [2, 20]), Srivastava et al. [19] and many other researchers (viz. [4, 5, 7, 10, 13, 16] introduced certain subclasses of bi-univalent function class Σ.

Let $\mathcal{S}'$ denote the class of meromorphic univalent functions g of the form:

$$g(z) = z + \sum_{n=0}^{\infty} \frac{b_n}{z^n}, \tag{1.4}$$

defined on the domain:

$$\mathbb{U}^* = \{z : z \in \mathbb{C},\ 1 < |z| < \infty\}.$$

Since $g \in \mathcal{S}'$ is univalent, it has an inverse say g^{-1}, which satisfies:

$$g^{-1}(g(z)) = z, \quad (z \in \mathbb{U}^*)$$

and

$$g(g^{-1}(w)) = w, \quad (M < |w| < \infty;\ M > 0).$$

Moreover, the inverse function $g^{-1} = h$ has a series expansion of the form:

$$g^{-1}(w) = h(w) = w + \sum_{n=0}^{\infty} \frac{c_n}{w^n}, \quad (M < |w| < \infty). \tag{1.5}$$

A function $g \in \mathcal{S}'$ is said to be meromorphic bi-univalent if both g and g^{-1} are meromorphic univalent in $\mathbb{U}^*$. Let Σ' denote the class of all meromorphic bi-univalent functions in $\mathbb{U}^*$ given by (1.4). A simple calculation shows that,

$$g^{-1}(w) = h(w) = w - b_0 - \frac{b_1}{w} - \frac{b_2 + b_0b_1}{w^2} - \cdots \tag{1.6}$$

Let ϕ is an analytic function with positive real part in the unit disk $\mathbb{U}$, satisfying $\phi(0) = 1$, $\phi^{'}(0) > 0$ and $\phi(\mathbb{U})$ is symmetric with respect to the real axis. Then such functions $\phi(z)$ has a series expansion of the form:

$$\phi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \cdots, \quad (B_1 > 0). \tag{1.7}$$

An analytic function ϕ is said to be subordinate to another analytic function ψ, written as

$$\phi(z) \prec \psi(z), \quad (z \in \mathbb{U})$$

if there exists a Schwarz function w, which is analytic in $\mathbb{U}$ with $w(0) = 0$ and $|w(z)| < 1$ for $z \in \mathbb{U}$ such that $\phi(z) = \psi(w(z))$. In particular, if the function ψ is univalent in $\mathbb{U}$, then we have the following equivalence:

$$\phi(z) \prec \psi(z) \iff \phi(0) = \psi(0), \quad \phi(\mathbb{U}) \subset \psi(\mathbb{U}).$$

For $f \in \mathcal{A}$, Al-Oboudi [1] introduced the following operator:

$$D_\delta^0 f(z) = f(z)$$

$$D_\delta^1 f(z) = (1-\delta) f(z) + \delta z f'(z) = D_\delta f(z); \quad (\delta \geq 0) \tag{1.8}$$

$$D_\delta^n f(z) = D_\delta(D_\delta^{n-1} f(z)); \quad (n \in \mathbb{N}) \tag{1.9}$$

If f is given by (1.1), then from (1.8) and (1.9) we see that,

$$D_\delta^n f(z) = z + \sum_{k=2}^{\infty} [1 + (k-1)\delta]^n a_k z^k; \quad (n \in \mathbb{N}_0) \tag{1.10}$$

with $D_\delta^n f(0) = 0$ and when $\delta = 1$, we get Sălăgean's differential operator $D_1^n = D^n$ (see [17]). Similarly, for $g \in \mathcal{S}^{'}$ as given in (1.4), Al-Oboudi differential operator can be defined as:

$$D_\delta^0 g(z) = g(z),$$

$$D_\delta^1 g(z) = (1-\delta) g(z) + \delta z g'(z) = D_\delta g(z); \quad (\delta \geq 0) \tag{1.11}$$

$$D_\delta^n g(z) = D_\delta(D_\delta^{n-1} g(z)); \quad (n \in \mathbb{N}). \tag{1.12}$$

Then from (1.11) and (1.12) we have,

$$D_\delta^n g(z) = z + (1-\delta)^n b_0 + \sum_{k=1}^{\infty} [1 - (k+1)\delta]^n b_k z^{-k}; \quad (n \in \mathbb{N}_0) \tag{1.13}$$

The coefficient estimates of several subclasses of meromorphic bi-univalent function class $\Sigma^{'}$ were investigated by some researchers recently, such as Bulut [6], Janani and Murugusundaramoorthy [11], Panigrahi [14] etc. (see also [9, 12, 18]). In the present investigation, we define a new subclass of meromorphic bi-univalent function class $\Sigma^{'}$ of complex order $\gamma \in \mathbb{C} \setminus \{0\}$ associated with Al-Oboudi differential operator and estimated the coefficients $|b_0|$ and $|b_1|$ of the functions in this new subclass.

In order to prove our main result we require the following lemma.

Lemma 1.1. [15] If $p(z) \in \mathcal{P}$, the class of all functions analytic in $\mathbb{U}^*$, for which

$$\Re(p(z)) > 0,$$

then $|p_n| \leq 2$ for each $n \in \mathbb{N} := \{1, 2, 3, \ldots\}$, where

$$p(z) = 1 + p_1 z^{-1} + p_2 z^{-2} + p_3 z^{-3} + \cdots, \quad \left(z \in \mathbb{U}^*\right).$$

2. Coefficient Estimates for the Function Class $\Sigma'_{\delta,n,\gamma}(\lambda, \mu, \phi)$

Definition 2.1. For $0 \leq \lambda \leq 1$, $\mu \geq 0$, $\lambda \neq \mu$, $\delta \geq 0$, $n \in \mathbb{N}$ and $\gamma \in \mathbb{C} \setminus \{0\}$; a function $g(z) \in \mathcal{S}'$ given by (1.4) is said to be in the class $\Sigma'_{\delta,n,\gamma}(\lambda, \mu, \phi)$ if the following conditions are satisfied:

$$\left\{1 + \frac{1}{\gamma}\left[(1-\lambda)\left(\frac{D^n_\delta g(z)}{z}\right)^{\mu} + \left(\lambda D^n_\delta g(z)\right)'\left(\frac{D^n_\delta g(z)}{z}\right)^{\mu-1} - 1\right]\right\} \prec \phi(z)$$

and

$$\left\{1 + \frac{1}{\gamma}\left[(1-\lambda)\left(\frac{D^n_\delta h(w)}{w}\right)^{\mu} + \left(\lambda D^n_\delta h(w)\right)'\left(\frac{D^n_\delta h(w)}{w}\right)^{\mu-1} - 1\right]\right\} \prec \phi(w)$$

where the functions $z, w \in \mathbb{U}^*$ and $g^{-1}(w) = h(w)$ is given by (1.6).

Theorem 2.2. Let the function $g(z) \in \mathcal{S}'$ given by (1.4) be in the function class $\Sigma'_{\delta,n,\gamma}(\lambda, \mu, \phi)$. Then,

$$|b_0| \leq \left|\frac{\gamma B_1}{(1-\delta)^n (\mu - \lambda)}\right| \tag{2.1}$$

and

$$|b_1| \leq \left|\frac{\gamma}{(1-2\delta)^n}\sqrt{\left[\frac{(\mu-1)\gamma B_1^2}{2(\mu-\lambda)^2}\right]^2 + \left[\frac{B_2}{\mu - 2\lambda}\right]^2}\right| \tag{2.2}$$

where $0 \leq \lambda \leq 1$, $\mu \geq 0$, $\lambda \neq \mu$, $\delta \geq 0$, $n \in \mathbb{N}$, $\gamma \in \mathbb{C} \setminus \{0\}$ and $z, w \in \mathbb{U}^*$.

Proof. Since the function $g \in \Sigma'_{\delta,n,\gamma}(\lambda, \mu, \phi)$, we have

$$\left\{1 + \frac{1}{\gamma}\left[(1-\lambda)\left(\frac{D^n_\delta g(z)}{z}\right)^{\mu} + \left(\lambda D^n_\delta g(z)\right)'\left(\frac{D^n_\delta g(z)}{z}\right)^{\mu-1} - 1\right]\right\} = \phi\left(u\left(z\right)\right) \tag{2.3}$$

and

$$\left\{1+\frac{1}{\gamma}\left[(1-\lambda)\left(\frac{D_\delta^n h(w)}{w}\right)^{\mu}+\left(\lambda D_\delta^n h(w)\right)'\left(\frac{D_\delta^n h(w)}{w}\right)^{\mu-1}-1\right]\right\} \\ =\phi\left(v\left(w\right)\right) \tag{2.4}$$

Define the functions p_1 and p_2 in $\mathcal{P}$ by

$$p_1(z)=\frac{1+u(z)}{1-u(z)}=1+c_1z^{-1}+c_2z^{-2}+c_3z^{-3}+\cdots$$

and

$$p_2(w)=\frac{1+v(w)}{1-v(w)}=1+d_1w^{-1}+d_2w^{-2}+d_3w^{-3}+\cdots$$

It follows that

$$u(z)=\frac{p_1(z)-1}{p_1(z)+1}=\frac{1}{2}\left[\frac{c_1}{z}+\left(c_2-\frac{c_1^2}{2}\right)\frac{1}{z^2}+\cdots\right] \tag{2.5}$$

and

$$v(w)=\frac{p_2(w)-1}{p_2(w)+1}=\frac{1}{2}\left[\frac{d_1}{w}+\left(d_2-\frac{d_1^2}{2}\right)\frac{1}{w^2}+\cdots\right] \tag{2.6}$$

Also we have,

$$|c_n|\leq 2,\ \ |d_n|\leq 2; \quad (n\in\mathbb{N}) \tag{2.7}$$

Using (2.5) and (2.6) in (2.3) and (2.4) considering (1.7), we get

$$\left\{1+\frac{1}{\gamma}\left[(1-\lambda)\left(\frac{D_\delta^n g(z)}{z}\right)^{\mu}+\left(\lambda D_\delta^n g(z)\right)'\left(\frac{D_\delta^n g(z)}{z}\right)^{\mu-1}-1\right]\right\} \\ =1+\frac{1}{\gamma}\left(1-\delta\right)^n\left(\mu-\lambda\right)\frac{b_0}{z}+ \\ \frac{1}{\gamma}\left(\mu-2\lambda\right)\left[\left(1-\delta\right)^{2n}\frac{\left(\mu-1\right)}{2}b_0^2+\left(1-2\delta\right)^n b_1\right]\frac{1}{z^2}+\cdots \\ =1+B_1c_1\frac{1}{2z}+\left[\frac{1}{2}B_1\left(c_2-\frac{c_1^2}{2}\right)+\frac{1}{4}B_2c_1^2\right]\frac{1}{z^2}+\cdots \tag{2.8}$$

and

$$\left\{1+\frac{1}{\gamma}\left[(1-\lambda)\left(\frac{D_\delta^n h(w)}{w}\right)^{\mu}+\left(\lambda D_\delta^n h(w)\right)'\left(\frac{D_\delta^n h(w)}{w}\right)^{\mu-1}-1\right]\right\}$$
$$=1-\frac{1}{\gamma}(1-\delta)^n(\mu-\lambda)\frac{b_0}{w}+$$
$$\frac{1}{\gamma}(\mu-2\lambda)\left[(1-\delta)^{2n}\frac{(\mu-1)}{2}b_0^2-(1-2\delta)^n b_1\right]\frac{1}{w^2}+\cdots \tag{2.9}$$
$$=1+B_1d_1\frac{1}{2w}+\left[\frac{1}{2}B_1\left(d_2-\frac{d_1^2}{2}\right)+\frac{1}{4}B_2d_1^2\right]\frac{1}{w^2}+\cdots$$

Now equating the coefficients in (2.8) and (2.9), we get

$$\frac{1}{\gamma}(1-\delta)^n(\mu-\lambda)b_0=\frac{1}{2}B_1c_1 \tag{2.10}$$

$$\frac{1}{\gamma}(\mu-2\lambda)\left[(1-\delta)^{2n}(\mu-1)\frac{b_0^2}{2}+(1-2\delta)^n b_1\right]$$
$$=\frac{1}{2}B_1\left(c_2-\frac{c_1^2}{2}\right)+\frac{1}{4}B_2c_1^2 \tag{2.11}$$

$$-\frac{1}{\gamma}(1-\delta)^n(\mu-\lambda)b_0=\frac{1}{2}B_1d_1 \tag{2.12}$$

$$\frac{1}{\gamma}(\mu-2\lambda)\left[(1-\delta)^{2n}(\mu-1)\frac{b_0^2}{2}-(1-2\delta)^n b_1\right]$$
$$=\frac{1}{2}B_1\left(d_2-\frac{d_1^2}{2}\right)+\frac{1}{4}B_2d_1^2 \tag{2.13}$$

Using (2.10) and (2.12), we get

$$c_1=-d_1 \tag{2.14}$$

and also,

$$8(1-\delta)^{2n}(\mu-\lambda)^2b_0^2=\gamma^2B_1^2\left(c_1^2+d_1^2\right)$$

Or

$$b_0^2=\frac{\gamma^2B_1^2\left(c_1^2+d_1^2\right)}{8(1-\delta)^{2n}(\mu-\lambda)^2} \tag{2.15}$$

Using (2.7) in (2.15), we get

$$|b_0|\le\left|\frac{\gamma B_1}{(1-\delta)^n(\mu-\lambda)}\right|$$

which is our desired result (2.1). Next, for coefficient bounds on $|b_1|$, adding (2.11) and (2.13), we get

$$(\mu - 2\lambda)(1-\delta)^{2n}(\mu - 1)\, b_0^2 = \gamma \left[\frac{B_1}{2}(c_2 + d_2) - \frac{1}{2}(B_1 - B_2)\, c_1^2 \right] \tag{2.16}$$

Subtracting (2.13) from (2.11), we get

$$2(\mu - 2\lambda)(1 - 2\delta)^n\, b_1 = \gamma \left[\frac{B_1}{2}(c_2 - d_2) \right] \tag{2.17}$$

Subtracting the square of (2.16) from the square of (2.17), we get

$$\begin{aligned}(1-2\delta)^{2n}(\mu - 2\lambda)^2\, b_1^2 &= (1-\delta)^{4n}(\mu - 2\lambda)^2(\mu - 1)^2\,\frac{b_0^4}{4} \\ -\gamma^2 &\left[\frac{B_1^2}{4} c_2 d_2 + (B_2 - B_1)\, B_1 (c_2 + d_2)\,\frac{c_1^2}{8} + (B_1 - B_2)^2\,\frac{c_1^4}{16} \right]\end{aligned} \tag{2.18}$$

Using (2.7) and (2.15) in (2.18), we get

$$|b_1| \leq \left| \frac{\gamma}{(1-2\delta)^n} \sqrt{\left[\frac{(\mu - 1)\gamma B_1^2}{2(\mu - \lambda)^2} \right]^2 + \left[\frac{B_2}{\mu - 2\lambda} \right]^2} \right|$$

which is our desired result (2.2). This completes the proof of Theorem 2.2. ■

If we take $n = 0$ in Definition 2.1, we get the function class $\Sigma_{\gamma}^{'}(\lambda, \mu, \phi)$ defined by Janani and Murugusundaramoorthy [11]. Hence we have the following corollary.

Corollary 2.3. [11] Let the function $g \in \mathcal{S}^{'}$ given by (1.4) be in the class $\Sigma_{\gamma}^{'}(\lambda, \mu, \phi)$. Then,

$$|b_0| \leq \left| \frac{\gamma B_1}{\mu - \lambda} \right|$$

and

$$|b_1| \leq \left| \gamma \sqrt{\left[\frac{(\mu - 1)\gamma B_1^2}{2(\mu - \lambda)^2} \right]^2 + \left[\frac{B_2}{\mu - 2\lambda} \right]^2} \right|$$

where $0 \leq \lambda \leq 1$, $\mu \geq 0$, $\lambda \neq \mu$, $\delta \geq 0$, $n \in \mathbb{N}$, $\gamma \in \mathbb{C} \setminus \{0\}$ and $z, w \in \mathbb{U}^*$.

Analogous to (1.7), by setting $\phi(z)$ as given below:

$$\phi(z) = \left(\frac{1+z}{1-z} \right)^{\alpha} = 1 + 2\alpha z + 2\alpha^2 z^2 + \cdots$$

where $0 < \alpha \le 1$, then we have $B_1 = 2\alpha$ and $B_2 = 2\alpha^2$ and hence for $\gamma = 1$, we state the following corollary.

Corollary 2.4. Let the function $g \in \mathcal{S}'$ given by (1.4) be in the class $\Sigma'_{\delta,n,1}\left(\lambda, \mu, \left(\frac{1+z}{1-z}\right)^{\alpha}\right) \equiv \Sigma'_{\delta,n}(\lambda, \mu, \alpha)$. Then,

$$|b_0| \le \frac{2\alpha}{\left|(1-\delta)^n (\mu-\lambda)\right|}$$

and

$$|b_1| \le \left| \frac{2\alpha^2}{(1-2\delta)^n} \sqrt{\frac{(\mu-1)^2}{(\mu-\lambda)^4} + \frac{1}{(\mu-2\lambda)^2}} \right|$$

where $0 \le \lambda \le 1$, $\mu \ge 0$, $\lambda \ne \mu$, $\delta \ge 0$, $n \in \mathbb{N}$ and $z, w \in \mathbb{U}^*$.

On the other hand, if we take

$$\phi(z) = \frac{1 + (1-2\beta)z}{1-z} = 1 + 2(1-\beta)z + 2(1-\beta)z^2 + \cdots$$

where $0 \le \beta < 1$, then we have $B_1 = B_2 = 2(1-\beta)$ and hence for $\gamma = 1$, we state the following corollary.

Corollary 2.5. Let the function $g \in \mathcal{S}'$ given by (1.4) be in the class $\Sigma'_{\delta,n,1}\left(\lambda, \mu, \frac{1+(1-2\beta)z}{1-z}\right) \equiv \Sigma'_{\delta,n}(\lambda, \mu, \beta)$. Then,

$$|b_0| \le \frac{2(1-\beta)}{\left|(1-\delta)^n (\mu-\lambda)\right|}$$

and

$$|b_1| \le \left| \frac{2(1-\beta)}{(1-2\delta)^n} \sqrt{\frac{(\mu-1)^2 (1-\beta)^2)}{(\mu-\lambda)^4} + \frac{1}{(\mu-2\lambda)^2}} \right|$$

where $0 \le \lambda \le 1$, $\mu \ge 0$, $\lambda \ne \mu$, $\delta \ge 0$, $n \in \mathbb{N}$ and $z, w \in \mathbb{U}^*$.

Example 2.6. If we put $n = 0$, $\mu = 1$ with $0 \le \lambda < 1$ in Definition 2.1, we get the function class $\Sigma'_{\gamma}(\lambda, 1, \phi) \equiv \mathcal{F}^{\gamma}_{\Sigma'}(\lambda, \phi)$. Hence by putting $\mu = 1$ in Corollary 2.3, we can obtain the coefficient estimates of $|b_0|$ and $|b_1|$ for this function class.

Example 2.7. If we put $n = 0$, $\lambda = 1$ with $0 \le \mu < 1$ in Definition 2.1, we get the function class $\Sigma'_{\gamma}(1, \mu, \phi) \equiv \mathcal{B}^{\gamma}_{\Sigma'}(\mu, \phi)$. Hence by putting $\lambda = 1$ in Corollary 2.3, we can obtain the coefficient estimates of $|b_0|$ and $|b_1|$ for this function class.

Example 2.8. If we put $n = 0$, $\lambda = 1$ and $\mu = 0$ in Definition 2.1, we get the function class $\Sigma'_{\gamma}(1, 0, \phi) \equiv \mathcal{S}^{\gamma}_{\Sigma'}(\phi)$. Hence by putting $\lambda = 1$ and $\mu = 0$ in Corollary 2.3, we can obtain the coefficient estimates of $|b_0|$ and $|b_1|$ for this function class.

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