Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 9 (2017), pp. 4437–4450

http://www.ripublication.com/gjpam.htm

Soft linear pq-functions and soft β kernel in Vector Soft Topological Spaces

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Abstract

In this paper we prove some basic properties of the soft sets in a vector soft topological space(VSTS). Also we establish the soft linearity of pq-functions and soft β kernel of a soft linear pq-function in a VSTS.

AMS subject classification: 03E72, 46S40, 57N17, 06D72.
Keywords: Vector soft topology, open soft set, closed soft set, compact soft set, convex soft set, balanced soft set.

1. Introduction

Soft set theory emerged in 1999 as a general mathematical tool for modelling uncertainties. Molodstov [8] initiated this theory and pointed out its several applications in solving many practical problems. Operations on soft sets were introduced by Maji et al. [6] in 2003. Later much study was done in soft sets especially connecting soft sets and algebra. In 2011, Shabir and Naz [16] introduced soft topological spaces and Zorlutuna et al. [18], Cagman et al. [1], Hussain et al. [3] etc. contributed to soft topological spaces. Majumdar and Samanta[7] introduced soft mappings. In 2013, Sujoy Das, Pinaki Majumdar and S. K. Samanta [17] introduced the concepts of soft linear spaces and soft normed linear spaces. In 2011 Kharal and Ahmad [5] introduced soft pu-functions connecting any two families of soft sets. And in 2015, Moumita Chiney and S. K. Samanta [9] introduced vector soft topology, connecting soft set theory and topological vector spaces. With that motivation, we like to study some more concepts of soft sets, soft linearity of pq-functions and soft β kernel of a soft linear pq-function in a VSTS.

Section 2 deals with the preliminaries such as definition of soft sets, its basic operations, definition of soft topology and some properties. In section 3, we proved some theorems on soft sets in a VSTS. In Section 4 we present some properties of convex and balanced sets in a VSTS. Also we defined the concept of soft subspace topology and proved some results based on it. Section 5 contains a main results of the paper, the condition for linearity of a soft pq-function and the necessary and sufficient conditions for the continuity of a soft linear pq-function. In section 6 we defined soft β kernel of a soft linear pq-function and proved that soft β kernel of a soft linear map is a vector space. The definition of soft quotient topology and a necessary and sufficient condition for the continuity of a function on soft quotient topology is also proved in this section.

2. Preliminaries

Definition 2.1. [8] A pair (F, A) is called a *soft set* over a universal set X, where F is a mapping $F : A \to \wp(X)$, A is a set of parameters.

Notation [2]: The family of all soft sets over X is denoted by $SS(X, A)$.

Definition 2.2. [8] The soft set $(F, A) \in SS(X, A)$ where $F(\alpha) = \phi, \forall \alpha \in A$ is called *the null soft set* of $SS(X, A)$ and is denoted by ϕ_A.
The soft set $(F, A) \in SS(X, A)$ where $F(\alpha) = X, \forall \alpha \in A$ is called *the absolute soft set* of $SS(X, A)$ and is denoted by X_A.

Definition 2.3. [16] Let τ be a collection of soft sets over X. Then τ is said to be a *soft topology* if

1. ϕ_A, X_A belong to τ
2. the soft union of any number of soft sets in τ belongs to τ
3. the soft intersection of any two soft sets in τ belongs to τ

The triplet (X, τ, A) is called a *soft topological space*.

Definition 2.4. [16] Let (X, τ, A) be a soft topological space over X, then the members of τ are said to be *soft open sets* in X.
A soft set (F, A) over X is said to be *soft closed set* in X if its soft complement (F^c, A) belongs to τ.

Proposition 2.5. [3] Let (X, τ, A) be a soft topological space over X. Then for a fixed $\alpha \in A$, $\tau_\alpha = \{F(\alpha) : (F, A) \in \tau\}$ defines a topology on X.

Definition 2.6. [14] Let $SS(X, A)$ denote the set of all soft sets over X under the parameter set A. A soft set $(F, A) \in SS(X, A)$ is said to be *pseudo constant* soft set if $F(\alpha) = X$ or $\phi, \forall \alpha \in A$.
Let $CS(X, A)$ denote the set of all pseudo constant soft sets over X under the parameter set A.

Definition 2.7. [14] A soft topology τ on X is said to be an *enriched soft topology* if (1) of the Definition 2.7 is replaced by (1') $(F, A) \in \tau, \forall (F, A) \in CS(X, A)$.
Then the triplet (X, τ, A) is called an *enriched soft topological space* over X.

Proposition 2.8. [9] Let X be a non-empty set, A be the set of parameters and for each $\alpha \in A$, τ_α is a crisp topology on X. Then $\tau^* = \{(G, A) \in SS(X, A) : G(\alpha) \in \tau_\alpha, \forall \alpha \in A\}$ is an enriched soft topology on X.

Proposition 2.9. [13] Let (X, τ, A) be a soft topological space and if $\tau^* = \{(G, A) \in SS(X, A) : G(\alpha) \in \tau_\alpha, \forall \alpha \in A\}$, then τ^* is an enriched soft topology on X such that $\tau \subseteq \tau^*$ and $[\tau^*]_\alpha = \tau_\alpha, \forall \alpha \in A$. And τ^* is called *the enriched topology derived from* τ.

Definition 2.10. [13] Let X and Y be two non-empty sets and $f : X \to Y$ be a mapping. Then

1. the image of a soft set $(F, A) \in SS(X, A)$ under the mapping f is denoted by $f[(F, A)]$ and is defined by $f[(F, A)] = (f(F), A)$ where $[f(F)](\alpha) = f[F(\alpha)], \forall \alpha \in A$.

2. the inverse image of a soft set $(G, A) \in SS(Y, A)$ under the mapping f is denoted by $f^{-1}[(G, A)]$ and is defined by $f^{-1}[(G, A)] = (f^{-1}(G), A)$ where $[f^{-1}(G)](\alpha) = f^{-1}[G(\alpha)], \forall \alpha \in A$.

Definition 2.11. [13] Let (X, τ, A) and (Y, υ, A) be soft topological spaces. The mapping $f : (X, \tau, A) \to (Y, \upsilon, A)$ is said to be

1. *soft continuous* if $f^{-1}(F, A) \in \tau, \forall (F, A) \in \upsilon$.

2. *soft homeomorphism* if f is bijective and f and f^{-1} are soft continuous.

3. *soft open* if $(F, A) \in \tau \Rightarrow f(F, A) \in \upsilon$.

4. *soft closed* if (F, A) is soft closed in $(X, \tau, A) \Rightarrow f(F, A)$ is soft closed in (Y, υ, A).

Definition 2.12. [9] Let (F, A) and (G, A) be two soft sets over a vector space V, over K, the field of real or complex numbers. Then

1. $(F, A) + (G, A) = (F + G, A)$ where $(F + G)(\alpha) = F(\alpha) + G(\alpha), \forall \alpha \in A$

2. $k(F, A) = (kF, A)$ where $(kF)(\alpha) = \{kx : x \in F(\alpha)\}, \forall \alpha \in A$ and $\forall k \in K$.

3. $x + (F, A) = (x + F, A)$ where $(x + F)(\alpha) = \{x + y : y \in F(\alpha)\}, \forall\alpha \in A$ and $\forall x \in V$.

4. If (E, A) is any soft set over K, then $(E, A) \cdot (F, A) = (E \cdot F, A)$ where $(E \cdot F)(\alpha) = E(\alpha) \cdot F(\alpha), \forall\alpha \in A$.

Definition 2.13. [13] A soft set (E, A) over X is said to be a soft element if there exists $\alpha \in A$ such that $E(\alpha)$ is a singleton say $\{x\}$ and $E(\beta) = \phi, \forall\beta(\neq \alpha) \in A$. Such a soft element is denoted by E^x_α. A soft element E^x_α is said to be in the soft set (G, A) denoted by $E^x_\alpha \in (G, A)$ if $x \in G(\alpha)$.

Definition 2.14. [13] Let (X, τ, A) be a soft topological space over X. A soft set (F, A) is said to be *a soft neighbourhood* of the soft set (H, A) if there exists a soft open set (G, A) such that $(H, A) \sqsubseteq (G, A) \sqsubseteq (F, A)$.
If $(H, A) = E^x_\alpha$, then (F, A) is said to be soft neighbourhood of the soft element E^x_α. The neighbourhood system of a soft element E^x_α is denoted by $N_\tau(E^x_\alpha)$, which is the family of all its soft neighbourhoods.

Definition 2.15. [5] Let $SS(U, A)$ and $SS(V, B)$ be two families of soft sets. Let $q : U \to V$ and $p : A \to B$ be mappings. Then a mapping $f_{pq} : SS(U, A) \to SS(V, B)$ is defined as

1. Let (F, A) be a soft set in $SS(U, A)$. The image of (F, A) under f_{pq} written as $f_{pq}(F, A) = (f_{pq}(F), p(A))$ is a soft set in $SS(V, B)$ such that

$$f_{pq}(F)(y) = \begin{cases} \cup_{x\in p^{-1}(y)} q(F(x)) & \textit{if } p^{-1}(y) \neq \phi \\ \phi & \textit{otherwise} \end{cases}, \forall y \in B$$

2. Let (G, B) be a soft set in $SS(V, B)$. Then the inverse image of (G, B) under f_{pq} written as $f^{-1}_{pq}(G, B) = (f^{-1}_{pq}(G), p^{-1}(B))$ is a soft set in $SS(U, A)$ such that

$$f^{-1}_{pq}(G)(x) = \begin{cases} q^{-1}(G(p(x))) & \text{if } p(x) \in B \\ \phi & \text{otherwise} \end{cases}, \forall x \in A$$

The soft function f_{pq} is called surjective if p and q are surjective. The soft function f_{pq} is called injective if p and q are injective.

Proposition 2.16. [5] Let $SS(U, A)$ and $SS(V, B)$ be families of soft sets. For a function $f_{pq} : SS(U, A) \to SS(V, B)$, the following statements are true:

1. $f_{pq}(\phi_A) = \phi_B$

2. $f_{pq}(U_A) \sqsubseteq U_B$

3. $f_{pq}((F, A)\sqcup(G, A)) = f_{pq}(F, A)\sqcup f_{pq}(G, A)$ where $(F, A), (G, A) \in SS(U, A)$. In general $f_{pq}(\sqcup_i(F_i, A)) = \sqcup_i f_{pq}(F_i, A)$ where $(F_i, A) \in SS(U, A)$.

4. If $(F, A) \sqsubseteq (G, A)$ then $f_{pq}(F, A) \sqsubseteq f_{pq}(G, A)$ where $(F, A), (G, A) \in SS(U, A)$.
5. If $(G, B) \sqsubseteq (H, B)$ then $f_{pq}^{-1}(G, B) \sqsubseteq f_{pq}^{-1}(H, B)$ where $(G, B), (H, B) \in SS(V, B)$.

Proposition 2.17. [18] Let $SS(U, A)$ and $SS(V, B)$ be families of soft sets. For a function $f_{pq} : SS(U, A) \to SS(V, B)$, the following statements are true

1. $f_{pq}^{-1}((G, B)^c) = (f_{pq}^{-1}(G, B))^c$
2. $f_{pq}(f_{pq}^{-1}(G, B)) \sqsubseteq (G, B) \forall (G, B) \in SS(V, B)$. If f_{pq} is surjective, the equality holds.
3. $(F, A) \sqsubseteq f_{pq}^{-1}(f_{pq}(F, A))$ for any soft set (F, A) in $SS(U, A)$. If f_{pq} is injective, the equality holds.

Definition 2.18. [18] Let (U_1, τ_1, A_1) and (U_2, τ_2, A_2) be soft topological spaces. Let $q : U_1 \to U_2$ and $p : A_1 \to A_2$ be mappings. Let $f_{pq} : SS(U_1, A_1) \to SS(U_2, A_2)$ be a soft function and $E_\alpha^x \in U_{1_{A_1}}$

1. f_{pq} is soft pq - continuous at $E_\alpha^x \in U_{1_{A_1}}$ if for each $(G, B) \in N_{\tau_2}(f_{pq}(E_\alpha^x))$, $\exists$ a $(H, A) \in N_{\tau_1}(E_\alpha^x)$ such that $f_{pq}(H, A) \sqsubseteq (G, B)$
2. f_{pq} is soft pq- continuous on $U_{1_{A_1}}$ if f_{pq} is soft pq continuous at each soft points in $U_{1_{A_1}}$.

Proposition 2.19. [18] Let (U, τ, A) and (V, ν, B) be soft topological spaces. Let $f_{pq} : SS(U, A) \to SS(V, B)$ be a function and $E_\alpha^x \in U_A$. Then the following statements are equivalent.

1. f_{pq} is soft pq-continuous at E_α^x
2. For each $(G, B) \in N_\nu(f_{pq}(E_\alpha^x))$, $\exists$ a $(H, A) \in N_\tau(E_\alpha^x)$ such that $(H, A) \sqsubseteq f_{pq}^{-1}(G, B)$.
3. For each $(G, B) \in N_\nu(f_{pq}(E_\alpha^x))$, $f_{pq}^{-1}(G, B) \in N_\tau(E_\alpha^x)$.

3. Properties of soft sets in a Vector Soft Topological Space

Definition 3.1. [9] Let K be the field of real or complex numbers, A be the set of parameters and ν_α be the usual topology on $K, \forall \alpha \in A$. Then the soft topology ν derived from ν_α is called *the soft usual topology* on K.

Definition 3.2. [9] Let V be a vector space over a scalar field K, endowed with the soft usual topology, ν, A be the parameter set and τ be a soft topology on V. Then τ is said to be *a vector soft topology* on V if the mapping:

1. $f : (V \times V, A, \tau \times \tau) \to (V, A, \tau)$ defined by $f(x, y) = x + y$
   and

2. $g : (K \times V, A, \nu \times \tau) \to (V, A, \tau)$ defined by $g(k, x) = kx$
   are soft continuous, $\forall x, y \in V$ and $k \in K$.

Proposition 3.3. [9] Let τ be a vector soft topology on a vector space V over the field K, A be the parameter set and ν be the soft usual topology on K. Then τ_α is a vector topology on V, $\forall \alpha \in A$.

Proposition 3.4. [9] Let V be a vector space over a scalar field K, endowed with the soft usual topology, ν, A be the parameter set and $\forall \alpha \in A$, τ_α is a vector topology on V, then τ^* is a vector soft topology on V, where τ^* is defined as in Proposition 2.2.

Proposition 3.5. L et τ be a vector soft topology on a vector space V over the field K, A be the parameter set. Then for any $(F, A) \in SS(V, A)$ and $x \in V$, $[x + (F, A)]^- = x + (F, A)^-$ and $[\lambda(F, A)]^- = (\lambda F, A)^-, \forall \lambda \in K$.

Proof. $(F, A)^-$ is the intersection of all soft closed sets containing (F, A).
Let (G, A) be a soft closed set containing (F, A).
Then $x + (G, A) = (x + G, A)$, where $(x + G)(\alpha) = \{x + y : y \in G(\alpha)\}$.
$x + (G, A)$ is also soft closed, since the addition map is continuous.
Also $x + (F, A) \sqsubseteq x + (G, A)$.
$[x + (F, A)]^- = \sqcap\{x + (G, A) : (G, A)$ is soft closed and $(G, A) \sqsupseteq (F, A)\}$
$= x + \sqcap\{(G, A) : (G, A)$ is soft closed and $(G, A) \sqsupseteq (F, A)\}$
$= x + (F, A)^-$ Now $\lambda(G, A) = (\lambda G, A)$ where $(\lambda G)(\alpha) = \{\lambda y : y \in G(\alpha)\}$.
Since scalar multiplication is continuous, $\lambda(G, A)$ is closed if (G, A) is closed.
And $\lambda(F, A) \sqsubseteq \lambda(G, A)$.
$[\lambda(F, A)]^- = \sqcap\{\lambda(G, A) : (G, A)$ is soft closed and $(G, A) \sqsupseteq (F, A)\}$
$= \lambda \sqcap \{(G, A) : (G, A)$ is soft closed and $(G, A) \sqsupseteq (F, A)\}$
$= \lambda(F, A)^-$ Thus the closure of $\lambda(F, A)$ is $(\lambda F, A)^-$. ■

Proposition 3.6. Let (V, τ, A) be a vector soft topological space(VSTS). Then for any $(F, A), (G, A) \in SS(V, A)$, $(F, A)^- + (G, A)^- \sqsubseteq (F + G, A)^-$.

Proof. For a fixed $\alpha \in A$, consider $F^-(\alpha) + G^-(\alpha)$. Let $x_\alpha \in F^-(\alpha)$ and $y_\alpha \in G^-(\alpha)$. Then $(x_\alpha, y_\alpha) \in F^-(\alpha) \times G^-(\alpha)$, and $F^-(\alpha) \times G^-(\alpha)$ is a closed set since it is the product of two closed sets. Then by the continuity of the addition map, $x_\alpha + y_\alpha \in$ any closed set containing $F(\alpha) + G(\alpha)$. Therefore $x_\alpha + y_\alpha \in (F + G)^-(\alpha)$ Since this is true for all $\alpha \in A$, $(F, A)^- + (G, A)^- \sqsubseteq (F + G, A)^-$. ■

Proposition 3.7. In the VSTS (V, τ^*, A), the sum of any soft set and a soft open set is soft open.

Proof. Let (F, A) be any soft set in (V, τ^*, A) and (U, A) be a soft open set. Fix $\alpha \in A$ $F(\alpha) + U(\alpha) = \cup_x\{x + U(\alpha) : x \in F(\alpha)\}$. Since $U(\alpha)$ is open, $x + U(\alpha)$ is open and

by the property of a topological space, $\cup_x\{x + U(\alpha) : x \in F(\alpha)\}$ is open in τ_α.
i.e. $(F + U)(\alpha)$ is open in τ_α.
Since this is true for all $\alpha \in A$, $(F + U, A)$ is soft open in τ^*. ■

Definition 3.8. [18] A family Ψ of soft sets is a *cover* of a soft set (F, A) if $(F, A) \sqsubseteq \sqcup\{(F_i, A) : (F_i, A) \in \Psi, i \in I\}$. It is a soft open cover, if each member of Ψ is a soft open set. A subcover of Ψ is a subfamily of Ψ which is also a cover.

Definition 3.9. [18] A soft topological space (U, τ, A) is *compact* if each soft open cover of U_A has a finite subcover.

Proposition 3.10. Let (V, τ, A) be a VSTS. If (C, A) and (D, A) are two soft compact sets in V, then $(C, A) + (D, A)$ is also a soft compact set.

Proof. If (C, A) and (D, A) are two soft compact sets, $C(\alpha)$ and $D(\alpha)$ are compact sets for all $\alpha \in A$. This implies $C(\alpha) \times D(\alpha)$ is compact, for all $\alpha \in A$. Then $C(\alpha) + D(\alpha)$ is compact, for all $\alpha \in A$, by the continuity of addition in vector soft topology. Hence $(C, A) + (D, A)$ is compact. ■

Proposition 3.11. Let (V, τ, A) be a VSTS. If (C, A) is a soft compact set in V, then $(\lambda C, A)$ is also a soft compact set, $\forall \lambda \in K$.

Proof. If (C, A) is a soft compact set, $C(\alpha)$ is compact set for all $\alpha \in A$. This implies $\lambda C(\alpha)$ is compact, for all $\alpha \in A$. Then $(\lambda C)(\alpha)$ is compact, for all $\alpha \in A$, by the continuity of scalar multiplication in vector soft topology. Hence $(\lambda C, A)$ is compact. ■

Note:
By the propositions 3.6 and 3.7, the set of all soft compact sets in a VSTS forms a vector space with addition of soft sets and scalar multiplication of soft sets.

4. Convex and balanced soft sets in a VSTS and soft subspace topology

Definition 4.1. [9] A soft set (F, A) over a vector space V is said to be

1. *convex* if $k(F, A) + (1 - k)(F, A) \sqsubseteq (F, A), \forall k \in [0, 1]$.
2. *balanced* if $k(F, A) \sqsubseteq (F, A)$ for all scalar k with $|k| \leq 1$.
3. *absolutely convex* if it is balanced and convex.

Remark 4.2. [9]

1. (F, A) is convex (balanced) soft set if and only if for all $\alpha \in A$, the ordinary set $F(\alpha)$ is convex (balanced).

2. If (F, A) and (G, A) are two convex (balanced) soft sets in a vector space V over the scalar field K, then $k_1(F, A) + k_2(G, A)$ is convex (balanced) soft set in V for all scalars $k_1, k_2 \in K$.

3. If $\{(F_i, A)\}_{i\in I}$ is a family of convex (balanced) soft sets in a vector space V, then $(F, A) = \sqcap_{i\in I}(F_i, A)$ is a convex (balanced) soft set in V.

Proposition 4.3. The closure of a balanced soft set is balanced in any VSTS.

Proof. Let (F, A) be a balanced soft set in (V, τ, A), a VSTS.
Then by definition, $k(F, A) \sqsubseteq (F, A), \forall |k| \leq 1$
$k(F, A)^- = (kF, A)^- \sqsubseteq (F, A)^-, \forall k \leq 1$
Hence $(F, A)^-$ is a balanced soft set. ■

Proposition 4.4. The interior of a balanced soft set is balanced in any VSTS.

Proof. Let (F, A) be a balanced set.
Then by definition $(kF, A) \sqsubseteq (F, A), \forall k \in K$. And for any soft set (F, A), $k(F, A) = (kF, A)$
Hence $k(F, A)^o = (kF, A)^o \sqsubseteq (F, A)^o$
So $(F, A)^o$ is a balanced soft set. ■

Proposition 4.5. The closure of a convex soft set is a convex soft set in any VSTS.

Proof. Let (F, A) be a convex soft set.
Then by definition $k(F, A) + (1 - k)(F, A) \sqsubseteq (F, A), \forall k \in [0, 1]$
Now $k(F, A)^- + (1 - k)(F, A)^- = (kF, A)^- + ((1 - k)F, A)^-$
$\sqsubseteq (kF + (1 - k)F, A)^-$
$\sqsubseteq (F, A)^-$.
Thus $(F, A)^-$ is a convex soft set. ■

Definition 4.6. Let (V, τ, A) be a vector soft topology and W be a subspace of V. Then for $(F, A) \in \tau, \exists, (F|_W, A) \in \tau|_W$ where $F|_W(\alpha) = F(\alpha) \cap W, \forall \alpha \in A$.
Then clearly $\tau|_W$ is a soft topology on W.
If $\tau|_W$ is a vector soft topology on W, then $(W, \tau|_W, A)$ is called a *soft subspace topology*.

Proposition 4.7. Let (V, τ, A) be a VSTS, then the closure of a soft subspace in V is a soft subspace in V.

Proof. Let (FA) be a soft vector space in a vector space V
i.e. $F(\alpha)$ is a vecor space for all $\alpha \in A$.
Let b and c be any two scalars.
$b(F, A)^- + c(F, A)^- = (bF, A)^- + (cF, A)^-$
$\sqsubseteq (bF + cF, A)^-$
$= (F, A)^-$, since $(bF + cF, A) = (F, A)$, by the definition of soft vector space.
Thus $(F, A)^-$ is a soft vector space in V. ■

Proposition 4.8. Let (L, τ) be a topological vector space and A be any parameter set. Then the soft enriched topology τ^* derived from τ, is a vector soft topology on L, where for each $\alpha \in A$, $\tau_\alpha = \tau$.

Proof. Since vector addition and scalar multiplication are both continuous in a topological vector space, proof follows directly from the definitions of vector soft topology. ■

Proposition 4.9. If (L, τ) be a topological vector space and M is a subspace of L, then $\overline{M}$ is the closure of M in (L, τ). Then $(\overline{M}, \tau^*|_{\overline{M}}, A)$ is a soft subspace topology of (L, τ^*, A).

Proof. Since (L, τ) is a topological vector space and M is a subspace of L, by the property of topological vector space we have $\overline{M} + \overline{M} \subseteq \overline{M}$ and $k\overline{M} \subseteq \overline{M}, \forall k \in K$. Thus $\overline{M}$ is again a subspace of L. Then by the definition of soft subspace topology and the proposition 5.2, $(\overline{M}, \tau^*|_{\overline{M}}, A)$ is a soft subspace topology of (L, τ^*, A). ■

5. Soft $pq-$ functions in a VSTS

Definition 5.1. A soft zero element E^0_α is the soft element given by $E(\alpha) = \{0\}$ and $E(\beta) = \phi, \forall\, \beta\, (\neq \alpha) \in A$.

Result:
$E^0_\alpha + E^x_\alpha = E^x_\alpha, \forall\, x \in X$.

Proposition 5.2. Let (X, τ, A) be a VSTS and τ^* be the enriched topology derived from τ. Let $(M, A) \in N_{\tau^*}(E^0_\alpha)$. Then $(M, A) + E^x_\alpha \in N_{\tau^*}(E^x_\alpha)$.

Proof. Since $(M, A) \in N_{\tau^*}(E^0_\alpha)$, there exists $(H, A) \in \tau^*$ such that $E^0_\alpha \in (H, A) \sqsubseteq (M, A)$. Then $\{0\} \subseteq H(\alpha)$. So $\{x\} \subseteq \{x\} + H(\alpha)$. Hence $E^x_\alpha \in (H, A) + E^x_\alpha$. Since (H, A) is soft open, $(H, A) + E^x_\alpha$ is soft open since $H(\beta)$ is open for each $\beta \in A$ and $H(\alpha)+x$ is open by the continuity of addition. Also $(H, A) \sqsubseteq (M, A) \Rightarrow (H, A)+E^x_\alpha \sqsubseteq (M, A) + E^x_\alpha$. Thus $(M, A) + E^x_\alpha \in N_{\tau^*}(E^x_\alpha)$. ■

Theorem 5.3. Let (V_1, τ_1^*, A_1) and (V_2, τ_2^*, A_2) be two enriched vector soft topological spaces. Let $q : V_1 \to V_2$ and $p : A_1 \to A_2$ be two mappings in which q is linear and $T_{pq} : (V_1, \tau_1^*, A_1) \to (V_2, \tau_2^*, A_2)$. Then $T_{pq}(E^0_\alpha) = E^0_{p(\alpha)}$.

Proof. Since q is a linear map $q(0) = 0$.
By definition of $pq-$soft mapping,

$$T_{pq}(E_\alpha^0)(y) = \begin{cases} \cup_{x\in p^{-1}(y)} q(E(x)) & \text{if} p^{-1}(y) \neq \phi \\ \phi & \text{otherwise} \end{cases}$$
$$= \begin{cases} q(\{0\}) & \text{if } p(\alpha) = y \\ \phi & \text{otherwise} \end{cases}$$
$$= \begin{cases} \{0\} & \text{if } p(\alpha) = y \\ \phi & \text{otherwise} \end{cases}$$

Thus $T_{pq}(E_\alpha^0) = E_{p(\alpha)}^0$. ■

Corollary 5.4. T_{pq} is soft $pq-$continuous at E_α^0 if for each neighbourhood (M, A) of $T_{pq}(E_\alpha^0) = E_{p(\alpha)}^0$, $\exists$ a neighbourhood (L, A) of E_α^0 such that $T_{pq}(L, A) \sqsubseteq (M, A)$.

Theorem 5.5. Let (V_1, τ_1, A_1) and (V_2, τ_2, A_2) be two VSTS. Let $q : V_1 \to V_2$ and $p : A_1 \to A_2$ be two mappings in which q is linear and $T_{pq} : (V_1, \tau_1^*, A_1) \to (V_2, \tau_2^*, A_2)$. Then T_{pq} is linear in the sense that
$T_{pq}[\alpha\ (L, A) + \beta\ (M, A)] = \alpha\ T_{pq}(L, A) + \beta\ T_{pq}(M, A)$.

Proof. Since p is one-one

$$T_{pq}(L)(y) = \begin{cases} q(L(x)) & \text{when } x = p^{-1}(y) \\ \phi & \text{when } p^{-1}(y) = \phi \end{cases}$$

So

$$T_{pq}[\alpha(L, A) + \beta(M, A)](y) = T_{pq}[\alpha(L, A) + \beta(M, A)](y)$$
$$= \begin{cases} q(\alpha L(x) + \beta M(x)) & \text{when } x = p^{-1}(y) \\ \phi & \text{when } p^{-1}(y) = \phi \end{cases}$$
$$= \begin{cases} \alpha q(L(x)) + \beta q(M(x)) & \text{when } x = p^{-1}(y) \\ \phi & \text{otherwise} \end{cases}, \text{ since } \quad q \text{ is linear}$$

Now

$$\alpha T_{pq}(L)(y) + \beta T_{pq}(M)(y)$$
$$= \alpha \begin{cases} q(L(x)) \text{ when } x = p^{-1}(y) \\ \phi \text{ when } p^{-1} = \phi \end{cases} + \beta \begin{cases} q(M(x)) \text{ when } x = p^{-1}(y) \\ \phi \text{ when } p^{-1} = \phi \end{cases}$$
$$= \begin{cases} \alpha q(L(x)) + \beta q(M(x)) \text{ when } x = p^{-1}(y) \\ \phi \text{ otherwise} \end{cases}$$

Thus $T_{pq}[\alpha(L, A) + \beta(M, A)] = \alpha T_{pq}(L, A) + \beta T_{pq}(M, A)$. ■

Theorem 5.6. Let (V_1, τ_1^*, A_1) and (V_2, τ_2^*, A_2) be two enriched vector soft topological spaces. Let $T_{pq} : (V_1, \tau_1^*, A_1) \to (V_2, \tau_2^*, A_2)$ be linear. Then T_{pq} is soft $pq-$continuous at E_α^x if and only if for each $(M, A) \in N_{\tau_2^*}(E^0_{p(\alpha)})$, there exists $(L, A) \in N_{\tau_1^*}(E^0_\alpha)$ such that $T_{pq}[(L, A) + E_\alpha^x] \sqsubseteq (M, A) + T_{pq}(E_\alpha^x)$.

Proof. Addition is a homeomorphism in a VSTS. Hence by Proposition 6.1 (F, A) is a soft neighbourhood of a soft point E_α^x if and only if $-E_\alpha^x + (F, A)$ is a soft neighbourhood of E^0_α. Thus any neighbourhood of E_α^x is obtained from a neighbourhood of E^0_α and vice versa.
Hence $(L, A) + E_\alpha^x$ is a soft neighbourhood of the soft point E_α^x for any $(L, A) \in N_{\tau_1^*}(E^0_\alpha)$.
Now $T_{pq}[(L, A) + E_\alpha^x] = T_{pq}(L, A) + T_{pq}[E_\alpha^x]$, by the linearity of T_{pq}.
This shows that any neighbourhood of $T_{pq}[E_\alpha^x]$ can be obtained from the image of the neighbourhood (L, A) of E^0_α. Also if (L, A) is a neighbourhood of E^0_α, $T_{pq}(L, A)$ is a neighbourhood of $T_{pq}(E^0_\alpha) = E^0_{p(\alpha)}$.
Let $(M, A) \in N_{\tau_2^*}(E^0_{p(\alpha)})$. Then $(M, A) + T_{pq}(E_\alpha^x) \in N_{\tau_2^*}(E^0_{p(\alpha)})$.
T_{pq} is soft $pq-$continuous at E_α^x if and only if there exists a neighbourhood of E_α^x say (D, A) such that

$$T_{pq}(D, A) \sqsubseteq (M, A) + T_{pq} E_\alpha^x$$

And corresponding to (D, A) we may find $(L, A) \in N_{\tau_1^*}(E^0_\alpha)$ such that

$$(L, A) + E_\alpha^x = (D, A).$$

$$\Rightarrow T_{pq}[(L, A) + E_\alpha^x] \sqsubseteq (M, A) + T_{pq}(E_\alpha^x)$$

■

6. Soft β kernel of a soft pq-linear map

Definition 6.1. [14] Let (X, τ, A) be a soft topological space. If for the soft elements E_α^x, E_β^y with $E_\alpha^x \neq E_\beta^y$, there exists,

1. $(F, A) \in \tau$ such that $E_\alpha^x \in (F, A)$ and $E_\beta^y \notin (F, A)$ or $E_\alpha^x \notin (F, A)$ and $E_\beta^y \in (F, A)$, then (X, τ, A) is called a soft T_0-space.
2. $(F, A), (G, A) \in \tau$ such that $E_\alpha^x \in (F, A)$ and $E_\beta^y \notin (F, A)$ and $E_\alpha^x \notin (G, A)$ and $E_\beta^y \in (G, A)$, then (X, τ, A) is called a soft T_1-space.
3. $(F, A), (G, A) \in \tau$ such that $E_\alpha^x \in (F, A)$, $E_\beta^y \in (G, A)$ and $(F, A) \cap (G, A) = \phi_A$, then (X, τ, A) is called a soft T_2-space.

Proposition 6.2. [14] A soft topological space (X, τ, A) is soft T_1 space if and only if all soft elements E_α^x is soft closed.

Definition 6.3. Let $T_{pq} : (V_1, \tau_1, A_1) \to (V_2, \tau_2, A_2)$ be a soft linear map. Then the soft- β kernel of T_{pq} denoted by $K_\beta(T_{pq})$ is the pre-image of the soft zero-element E^0_β for some $\beta \in A_2$.

Note:
$K_\beta(T_{pq}) = \{(F, A_1) \in SS(V_1, A_1) | T_{pq}(F, A_1) = E^0_\beta\}$
Since T_{pq} is soft linear, $T_{pq}(F)(y) = \begin{cases} q(F(x)) \text{ if} x = p^{-1}(y) \\ \phi \text{ if} p^{-1}(y) = \phi \end{cases}$
and $T_{pq}(F, A_1) = E^0_\beta \Rightarrow$
$K_\beta(T_{pq}) = \{(F, A_1) \in SS(V_1, A_1) | F(p^{-1}(\beta)) \subseteq Kerq$ and $F(\alpha) = \phi \; \forall \; \alpha \; (\neq p^{-1}(\beta)) \; \in A_1\}$.

Proposition 6.4. Let $T_{pq} : (V_1, \tau_1, A_1) \to (V_2, \tau_2, A_2)$ be a soft linear map between the VSTS (V_1, τ_1, A_1) and (V_2, τ_2, A_2) and $K_\beta(T_{pq})$ be the soft $\beta-$ kernel of the mapping. Then $K_\beta(T_{pq})$ is a vector space under addition of soft sets and scalar multiplication of a soft set.

Proof. Since Ker q is a subspace of V_1, if (F, A_1) and $(G, A_1) \in K_\beta(T_{pq})$, $F(p^{-1}(\beta)) + G(p^{-1}(\beta)) \subseteq Kerq$ and $F(\alpha) + G(\alpha) = \phi \forall \alpha \neq p^{-1}(\beta)$. Thus $(F, A_1) + (G, A_1) \in K_\beta(T_{pq})$. Similarly $\lambda(F, A_1) \in K_\beta(T_{pq}), \forall (F, A_1) \in K_\beta(T_{pq})$.
$E^0_{p^{-1}(\beta)}$ acts as the zero vector for addition and for any $(F, A_1) \in K_\beta(T_{pq})$, $-(F, A_1) \in K_\beta(T_{pq})$, which is the additive inverse of (F, A_1). ■

Remark 6.5. The soft union of all soft sets in $K_\beta(T_{pq})$ is the soft set (K_β, A_1) given by
$K_\beta(x) = \begin{cases} Kerq \; if x = p^{-1}(\beta) \\ \phi \text{ otherwise} \end{cases}$.

Remark 6.6. If q is one-one, T_{pq} is one-one and then

$$K_\beta(T_{pq}) = T^{-1}_{pq}(E^0_\beta) = E^0_{p^{-1}(\beta)}$$

Proposition 6.7. If (V_2, τ_2, A_2) is a soft Hausdorff space and T_{pq} is soft-continuous, then each soft set in $K_\beta(T_{pq})$ is soft closed and if A_1 is finite (K_β, A_1) is soft closed.

Proof. If (V_2, τ_2, A_2) is a soft Hausdorff space, E^0_β is soft closed and if T_{pq} is soft-continuous,then the inverse image of the closed set E^0_β is soft closed set. That is each soft set in $K_\beta(T_{pq})$ is soft closed.
Then if A_1 is finite (K_β, A_1) is soft closed being finite union of closed sets. ■

Definition 6.8. Let (V, τ, A) be a VSTS. Let W be a subspace of V. Then V/W is the quotient space and $Q : V \to V/W$ given by $Q(v) = v + W$ is the quotient map. The soft quotient topology, τ_Q on V/W is defined such that a soft set, (E, A) in V/W is soft open if and only if the inverse of (E, A) under the quotient map is soft open. $(V/W, \tau_Q, A)$ is called the vector soft topological quotient space.

Proposition 6.9. A soft $pq-$ function T_{pq} on V/W is continuous (open) if and only if the composition $T_{pq} \circ Q$ is soft continuous (open).

Proof. By the definition of the soft quotient topology the map $Q : (V, \tau, A) \to (V/W, \tau_Q, A)$ is soft open and soft continuous. Now consider the map $T_{pq} : (V/W, \tau_Q, A) \to (X, \upsilon, B)$. If T_{pq} is soft continuous (open), then clearly the composition $T_{pq} \circ Q$ is soft continuous (open). Assume that the composition $T_{pq} \circ Q$ is soft continuous (open). Let (Y, B) be soft open in (X, υ, B). $T_{pq}^{-1}(Y, B)$ is soft open if and only if $Q^{-1}[T_{pq}^{-1}(Y, B)]$ is soft open, by the definition of τ_Q.And $Q^{-1}[T_{pq}^{-1}(Y, B)] = [T_{pq} \circ Q]^{-1}(Y, B)$. But since the composition is continuous $[T_{pq} \circ Q]^{-1}(Y, B)$ is soft open and hence $T_{pq}^{-1}(Y, B)$ is soft open, showing that T_{pq} is soft pq-continuous. Let (F, A) be soft open in $(V/W, \tau_Q, A)$. Since Q is soft continuous $Q^{-1}(F, A) = (G, A)$ is soft open in (V, τ, A). Since the composition $T_{pq} \circ Q$ is soft open $[T_{pq} \circ Q](G, A) = T_{pq}[Q(G, A)] = T_{pq}(F, A)$ is soft open, showing that T_{pq} is soft pq-open. ■

7. Conclusion

The study of soft sets and soft topology has wide applications in classical and non-classical logic. The notion of soft mappings have been applied to medical diagnosis in medical expert systems [5]. We hope that our study connecting vector spaces, soft topology and soft mappings can be applied to many problems in several fields of uncertainty.

Acknowledgement

The author is indebted to the University Grants Commission as the work is under the Faculty Development Programme of UGC (XII plan).

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