

Solving Fully Fuzzy Assignment Problem Using Branch and Bound Technique

Dr. S. Muruganandam¹ and K. Hema²

¹ *Professor and Head, Department of Mathematics, Imayam College of Engineering,
Kannanur, Thuraiyur , Trichy-621 206, Tamilnadu. India.*

² *Associate Professor, Department of Mathematics, M.A.M. College of Engineering
and Technology, Trichy-621 105, Tamilnadu, India.*

Abstract

This paper develops an approach to solve fuzzy assignment problem where profit is not deterministic number but a imprecise one. Here, the elements of the profit matrix of the assignment problem are triangular fuzzy numbers and optimal solution can be obtained using branch and bound method without converting the fuzzy numbers into crisp numbers. The efficiency of the proposed method is illustrated by a numerical example.

AMS Mathematics Subject Classification (2010): 03E72

1. INTRODUCTION

The main objective of Assignment problem which is also a special type of linear programming problem is to obtain the optimum assignment for number of tasks (jobs), that is equal to the number of resources (workers) at a minimum cost or maximum profit. This plays a vital role in assigning persons to jobs, classes to rooms, operators to machines etc. An optimal solution to assignment problems, can be achieved using various algorithm such as linear programming, Hungarian algorithm, Neural networks genetic algorithm, etc. In 1965, Zadeh [12] has introduced fuzzy sets which provided a new mathematical tool to deal with uncertainty of information.

Terry Ross.G and Richard M.Soland [10] developed branch and bound algorithm to solve the generalized assignment problem by solving a series of binary knapsack problem to determine the bounds. Anchal Choudhary et.al [1] proposed a new algorithm to solve trapezoidal fuzzy assignment problem using branch and bound techniques. He has converted trapezoidal fuzzy numbers into crisp numbers using a new algorithm ranking.M.A Lone et.al [5] proposed a solution to Intuitionistic fuzzy assignment problem using Branch and Bound method. He also considered the fuzzy costs as Intuitionistic triangular fuzzy numbers. Elias Munapo[4] introduced a transportation branch and bound algorithm for solving the generalized assignment problem in which the sub problems are solved by the available efficient transportation techniques rather than the usual simplex method. Anil D.Gotmare and P.G.Khot6[2] used branch and bound techniques to solve fuzzy assignment problem ,where the objective is to minimize the costs. They have also used linguistic variables to describe the qualitative expression and Yager's ranking method was used to convert the fuzzy numbers to crisp numbers. Srinivas.B and Ganesan[9] proposed Branch and bound method to solve fuzzy assignment problem with Triangular and Trapezoidal fuzzy numbers and Robust ranking indices is used to convert the fuzzy numbers to crisp numbers. Muruganandam et.al [7] has given a special procedure to solve a two objective fuzzy assignment problem through the Graded Mean Integration Representation.

Marius Posta et.al[6] proposed a simple and very effective algorithm to solve the generalized assignment problem. Further optimization problem is reformulated into sequence of decision problems and are solved by lagrangian branch and bound method and improved by variable fixing rules , these rules rely on lagrangian relative costs which was computed using dynamic programming algorithm. Deepak Gupta et.al[3] projected an optimal two stage flow shop scheduling problem with branch and bound method including transportation time ,the objective was to get optimal sequence of jobs in order to minimize the total elapsed time. Rajarajeswari.P et.al [8] solved a fully fuzzy linear programming problem using Robust ranking method. Ventepaka Yadaiah[11] discussed a new approach to get an optimal solution to an unbalanced assignment problem in which a lexi search algorithm was used to assign jobs to machines.

In this paper, Branch and Bound method is proposed to find the optimal solution to the fully fuzzy assignment problem. We organize this paper as follows:. In section 2, Some basic definition of fuzzy sets, fuzzy number and arithmetic operations of fuzzy number. In section 3 ,we construct the mathematical formulation for the problem. In section 4, we give the methodology for the problem. In section 5, we give a solution procedure for the proposed method to solve the problem. In section 6, a numerical

example is given to show the efficiency of the proposed method and finally we give a conclusion for the problem.

2. PRELIMINARIES

Definition 2.1: Fuzzy Set

A fuzzy set is characterized by a membership function mapping element of a domain, Space of the universe of discourse X to unit interval $[0,1]$. i.e $A = \{(x, \mu_A(x)); x \in X\}$. Here $\mu_A: X \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in a fuzzy set A . These membership grades are often represented by real numbers ranging from $[0,1]$.

Definition 2.2: Fuzzy Number: The fuzzy set A defined on the set of real numbers is said to be a fuzzy number if its membership function $\mu_A: X \rightarrow [0,1]$ has the following properties.

1. A is normal. It means that there exists an $x \in X$, such that $\mu_A(x) = 1$.
2. A is convex it means that for every $x_1, x_2 \in X$.
3. $\mu_A(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$, $\lambda \in [0,1]$.
4. $3-\mu_A$ is upper semi continuous.
5. $\text{Sup}(A)$ is bounded in X .

2.3: Arithmetic operations of triangular fuzzy numbers:

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers. Then

1. $(a_1, a_2, a_3) \oplus (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
2. $k (a_1, a_2, a_3) = (ka_1, ka_2, ka_3)$ for $k \geq 0$
3. $k (a_1, a_2, a_3) = (ka_3, ka_2, ka_1)$ for $k < 0$
4. $(a_1, a_2, a_3) \otimes (b_1, b_2, b_3) = \begin{cases} (a_1 b_1, a_2 b_2, a_3 b_3) & \text{for } a_1 \geq 0 \\ (a_1 b_1, a_1 b_1, a_1 b_1) & \text{for } a_1 < 0, a_3 \geq 0 \\ (a_1 b_3, a_2 b_2, a_3 b_1) & \text{for } a_3 < 0 \end{cases}$

3. MATHEMATICAL FORMULATION

3.1 The general assignment problem

Suppose there are 'n' people and 'n' jobs. Each job must be done by exactly one person; also each person can do, at most, one job. The problem is to assign jobs to the people so as to minimize the total cost of completing all of the jobs. The general assignment problem can be mathematically stated as follows:

$$\text{Minimize } Z = \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij}$$

Subject to
$$\sum_{j=1}^n x_{ij} = 1 \text{ for } i = 1, 2, \dots, n$$

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$$x_{ij} = 0 \text{ or } 1$$

3.2 Fuzzy assignment problem

Minimize
$$Z = \sum_{i=1}^n \sum_{j=1}^n \tilde{C}_{ij} x_{ij}$$

Subject to
$$\sum_{j=1}^n x_{ij} = 1 \text{ for } i = 1, 2, \dots, n$$

$$\sum_{i=1}^n x_{ij} = 1 \text{ for } j = 1, 2, \dots, n$$

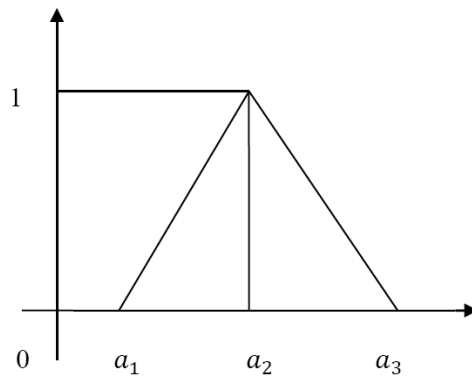
$$x_{ij} = 0 \text{ or } 1$$

4. METHODOLOGY

4.1 Triangular fuzzy number

A fuzzy number \tilde{A} is a triangular fuzzy number denoted by (a_1, a_2, a_3) and its

membership function $\mu_{\tilde{A}}(x)$ is given below:
$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$



Triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$

5. THE PROPOSED METHOD – BRANCH AND BOUND TECHNIQUES:

Solution Procedure:

The terminologies of branch and bound technique applied to fuzzy assignment problem are given below.

1. Let k be the level number in the branch tree ϵ be the assignment in the current node of a branching tree. Assume that the root node is 0.
2. P_ϵ^k be an assignment at level k of the branching tree. A is the set of assigned cells up to the node P_ϵ^k from the root node (set of i,j values with respect to the assigned cells up to the node P_ϵ^k from the root node), V_ϵ be the upper bound of the partial assignment up to P_ϵ^k such that,

$$V_\epsilon = \sum_{i,j \in X} C_{i,j} + \sum_{i \in X} \sum_{j \in Y} \max C_{i,j}$$

Where $C_{i,j}$ is the cell entry of the profit matrix with respect to the i^{th} row and j^{th} column. X be the set of rows which are not deleted up to the node P_ϵ^k from the node in the branching node.

Branching Methodology:

1. At level k, the column marked as k of the assignment problem ,will be assigned with the best row of the assignment problem.
2. If there are tie on the upper bound, then the terminal node at the upper most level is to be considered for further branching.
3. If the maximum upper bound happens to be at any one of the terminal nodes at the (n-1)th level, the optimality is reached. Then the assignments on the path from the root node to that node along with the missing pair of row-column combination will form the optimum solution.

6. NUMERICAL EXAMPLE

Let us consider a fuzzy assignment problem with rows representing four Machines and column representing four Jobs

Machines/Job	Job1	Job2	Job3	Job4
Machine1	(1,3,6)	(8,12,16)	(20,22,24)	(34,38,40)
Machine 2	(8,12,16)	(2,3,5)	(23,26,28)	(27,30,32)
Machine 3	(13,15,17)	(34,38,40)	(2,3,5)	(1,3,6)
Machine 4	(27,30,32)	(8,12,16)	(19,22,24)	(19,22,24)

Find an optimal assignment of machines to jobs that will maximize the total profit.(profit in rupees).

Solution:

Consider the fully fuzzy assignment problem and we solve it by branch and bound techniques to get the optimal solution.

Branching: The four different sub problems under the root nodes and upper bounds are shown in fig.1.

Compute the upper bound P_{11}^1

$$V_{\epsilon} = \sum_{i,j \in X} C_{i,j} + \sum_{i \in X} \sum_{j \in Y} \max C_{ij}$$

Where $\epsilon = (1,1)$ $A = (1,1)$ $X = (2,3,4)$ $Y = (2,3,4)$

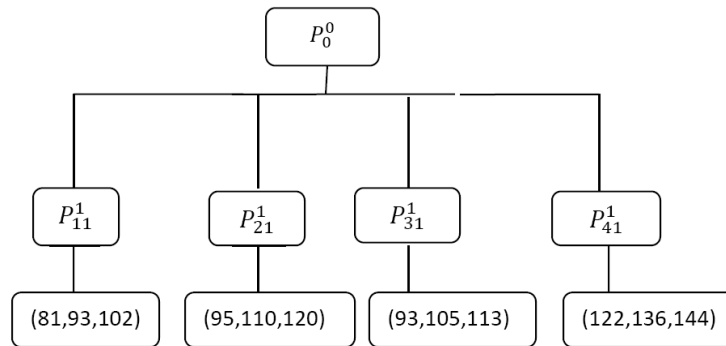
$$V_{11} = C_{11} + \sum_{i \in (2,3,4)} \sum_{j \in (2,3,4)} \max C_{ij}$$

$$P_{11}^1 = (1,3,6) + \{ (27,30,32) + (34,38,40) + (19,22,24) \} = (81,93,102)$$

$$P_{21}^1 = (8,12,16) + \{ (34,38,40) + (34,38,40) + (19,22,24) \} = (95,110,120).$$

$$P_{31}^1 = (13,15,17) + \{ (34,38,40) + (27,30,32) + (19,22,24) \} = (93,105,113)$$

$$P_{41}^1 = (27,30,32) + \{ (34,38,40) + (27,30,32) + (34,38,40) \} = (122,136,144)$$



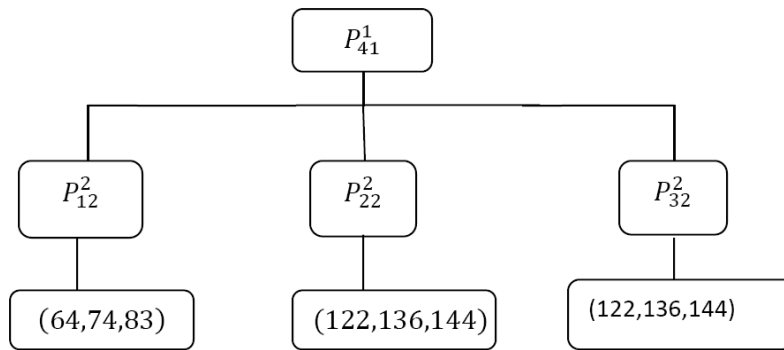
Further branching: Further branching is done from the terminal node which has the greatest upper bound. At this stage, the nodes $P_{11}^1, P_{21}^1, P_{31}^1, P_{41}^1$ are the terminal nodes. The node P_{41}^1 has the greatest upper bound. Eliminate fourth row and first column. Hence further branching from this node is shown as follows.

$$V_{22} = C_{41} + C_{i2} + \sum_{i \in (3,4)} \sum_{j \in (3,4)} \max C_{ij}$$

$$P_{12}^2 = (27,30,32) + (8,12,16) + \{ (27,30,32) + (2,3,5) \} = (64,77,85)$$

$$P_{22}^2 = (27,30,32) + (2,3,5) + \{ (34,38,40) + (1,3,6) \} = (64,74,83)$$

$$P_{32}^2 = (27,30,32) + (34,38,40) + \{ (34,38,40) + (27,30,32) \} = (122,136,144)$$

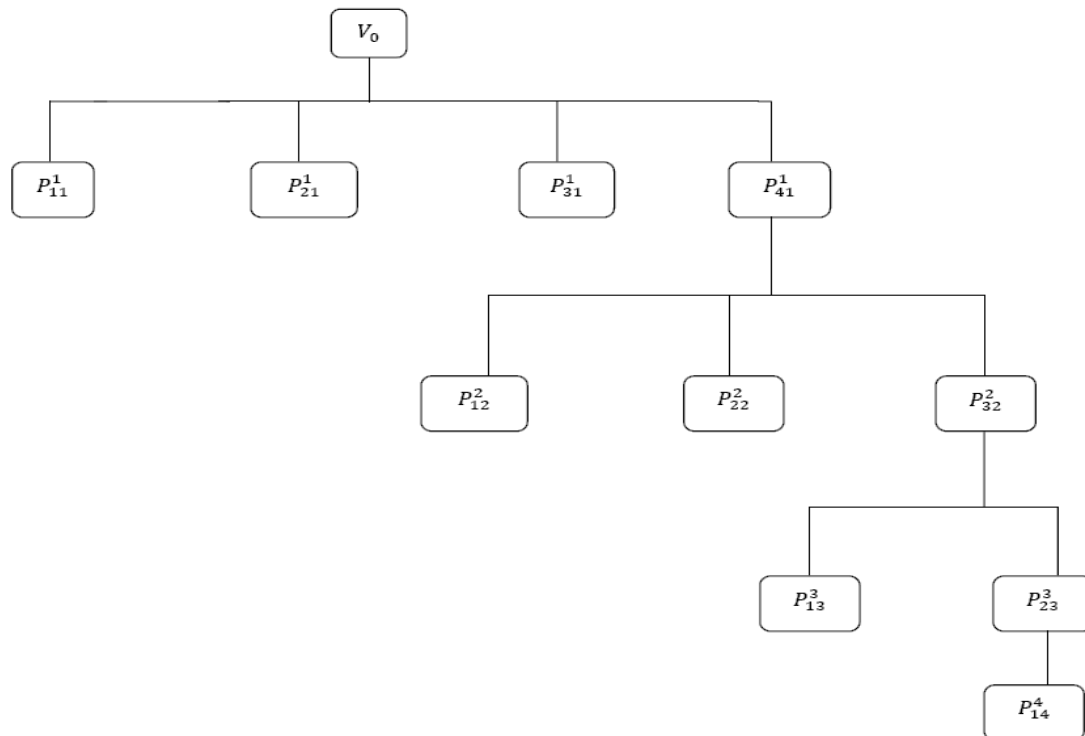


Further Branching: At this stage the nodes are $P_{41}^1, P_{12}^2, P_{22}^2, P_{32}^2$ are the terminal nodes. Among these nodes P_{32}^2 is the upper bound. Then these terminal nodes at the upper-most is considering for further branching. Eliminate third row and second column.

$$V_{33} = C_{41} + C_{32} + C_{i3} + \sum_{i \in (4)} \sum_{j \in (4)} \max C_{ij}$$

$$P_{13}^3 = (27,30,32) + (34,38,40) + (20,22,24) + \{ (27,30,32) \} = (108,120,128)$$

$$P_{23}^3 = (27,30,32) + (34,38,40) + (23,26,28) + \{ (34,38,40) \} = (118,132,140)$$



The optimal assignment: Person4 \rightarrow Job1, Person 3 \rightarrow Job2, Person2 \rightarrow Job3, Person1 \rightarrow Job4.

The optimal cost = $(34,38,40) + (23,26,28) + (34,38,40) + (27,30,32) = (118,132,140)$

6. CONCLUSION

In this paper, profit has been considered as fuzzy numbers which are realistic and general in nature. Optimal solution to fully fuzzy assignment problem is obtained using Branch and Bound techniques. Numerical example has been shown that the total profit obtained is optimal. Branch and Bound method is a systematic procedure, which is easy to apply in all type of assignment problems be it to maximize or minimize the objective function.

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