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An analysis of Fractional Explicit Method over Black Scholes method for pricing option

Aasiya Lateef[1] and C.K.Verma[2]

[1]*Research Scholar, Department of Mathematics and Computer Applications, Maulana Azad National Institute of Technology, Bhopal, India.*

[2]*Assistant Professor, Department of Mathematics and Computer Applications, Maulana Azad National Institute of Technology, Bhopal, India.*

ABSTRACT

Finance is one of the most rapid growing areas in the corporate business world. The world of corporate finance was managed by business persons and business students earlier but now it is controlled by mathematicians and computer scientists. Several methods have been developed to control the risk of financial market. Among all, Numerical methods play an important role in pricing the financial derivatives and especially in cases when there is no closed form analytical solution. Option pricing is one of the foremost research areas in this context. Also Fractional calculus is recently used for the finance and stock market analysis. The differential equation involving fractional order derivatives is a powerful tool for studying fractal geometry and fractal dynamics. In this paper, Fractional version of Explicit Finite Difference method has been used to price European call option. Historical data from banking sector of National Stock Exchange(NSE) have been used which classify into three different money market conditions i.e in-the-money(ITM), at-the-money(ATM) and Out-of-the-money(OTM) option. The results are compared with Benchmark Black -Scholes model with the help of the most well known hypothesis t-test.

Keywords—Fractional calculus; Fractional Explicit Method; Black-Schole model; European call options; Money market conditions (Moneyness).

MSC 2010 NO.: 26A33, 65M06, 65TXX

1. INTRODUCTION

Financial securities have become essential tools for corporations and investors over the past few decades. Options are the important financial derivatives that control the investment risks of investors in financial market. Options can be used to hedge assets and portfolios in order to control the risk due to the movement in stock prices in which its owner gets the right to trade in a fixed number of shares of a specified common stock at a fixed price known as strike price at any time on or before a given date known as *expiration date.* It can be traded in two ways European and American. European style allows to exercise on the expiry date while American style option pricing allows to exercise at any time prior to expiry date as given in Hull (2003).With the existence of Black-Scholes model partial differential equations (PDE) have become an important part of financial market. PDEs are adopted for both finding numerical and analytical solutions and developing new models for option pricing as given in Hull (2003), Chandra *et al.*(2013) and Wilmott *et al.*(1995).

The fractional order model is based on historical data of the system. So now a days, Fractional Calculus is also used in finance and stock market analysis. The financial variables such as stock market prices need more long-term memory to forecast future fluctuations in prices based on the past fluctuations. In financial markets, the main aim is making profit by trading through the right estimations as given in Song *et al.* (2013) and Baxter *et al.*(1996).

In this paper, the fractional version of the most common finite difference method i.e the explicit method is applied to price European call option. The Data has been taken from Banking sector of National Stock Exchange (NSE) which is divided into three different money market conditions(also known as Moneyness) which are, in-the-money(ITM), at-the-money(ATM) and Out-of-the-money(OTM) . The results are compared with the most well known Black Scholes Model by t-Test**.**

2. RELATED WORK

Recently Fractional calculus have played a key role in financial Theory. It is used in many fields of science and engineering. Leibniz(1695) was the first who presented the non-integer order of differential operator. After that so many renown personalities made important contribution to the literature in which Fourier(1822), Abel(1823), Liouville(1832) and Riemann(1847) defined and developed fractional integral and differentiation. Jafari *et al.* (2012), Jumarie (2010),Wyss (2000) proposed new approaches for derivation of fractional Black Scholes PDE.

Sousa (2012) derived a second order numerical method for the fractional advection diffusion equation which is explicit and also analyzed the convergence of the numerical method through the consistency and the stability. Meerschaert and Tadjeran

(2006) developed explicit and implicit Euler methods for the space fractional advection-dispersion equation. Rostami *et al.*(2012) introduced a novel numerical method for solving a fractional heat and wave like equations.

Herguner(2015) investigate the fractional Black Scholes equation and its application by different ways. Ghandehari *et al.*(2014) established the decomposition method for solution of the fractional Black-Scholes equation with boundary condition for a European option pricing problem.

The organisation of this paper is as follows: In section 3 there is a brief background on Black-Scholes model. Section 4 is based on definitions related to the study. In section 5 working of Fractional Explicit Method is given. Section 6 is based on description of the data preparation process, Analysis of the results of pricing European call option on different moneyness conditions and its comparison with Black Scholes method by t-test. In section 7 conclusion and discussion of results are given.

3. BLACK SCHOLES MODEL

The Black-Scholes model for calculating the premium of an option was introduced in 1973 in a paper entitled, "The Pricing of Options and Corporate Liabilities" published in the *Journal of Political Economy* . It is used to calculate the theoretical price of European put and call options. The model assumes that the price of heavily traded assets follow a geometric Brownian motion with constant drift and volatility. The formula, developed by three economists – Fischer Black, Myron Scholes and Robert Merton – is perhaps the world's most well-known options pricing model. Their dynamic hedging strategy led to a stochastic partial differential equation, now called the Black–Scholes equation as follows:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0,$$

- $V(S,T)$ is the price of a derivative as a function of time and stock price.
- S, be the price of the stock.
- σ, is the volatility of the stock's returns.
- r, is the annualized risk-free interest rate.

It estimates the price of the option over time with the following assumptions:

- The options are European and can only be exercised at expiration.
- No dividends are paid out during the life of the option.
- Efficient markets i.e., market movements cannot be predicted.

- No commissions.
- The risk-free rate and volatility of the underlying are known and constant.
- Follows a lognormal distribution, that is, returns on the underlying are normally distributed.

The Black Schole model is the most useful part of modern financial theory, which has been using continuously for estimating the fair price of option since 1973 as given in Black *et al.*(1973), Baxter *et al.*(1996), Baz *et al.*(2004), Wilmott *et al.*(1995) and Lateef *et al.*(2015).

4. BASIC DEFINITIONS AND CONCEPTS

In order to understand the concept of this study, we need to go through the following definitions:

A. Differ-Integrals

The differ-integral is a combined differentiation and integration operator that studies the possibility of taking real number powers, real number fractional powers or complex number powers of the differentiation operator which is obtained by the inverse of differentiation as given in Hergüner (June 2015). The n-th order derivative of a function f(x) is:

$$\frac{d^n f(x)}{dx^n} = f^{(n)}(x) = D_x^n f(x)$$

Similarly, when the integral considered as inverse of derivative, the notations for n-the integral of a function f(x)

$$\frac{d^{-n} f(x)}{dx^{-n}} = f^{(-n)}(x) = D_x^{-n} f(x)$$

It is more convenient to use q when the power n is real or complex number. Therefore, combined derivative and integral definitions for arbitrary q gives:

$$\frac{d^q f(x)}{dx^q} = f^{(q)}(x) = D_x^q f(x)$$

B. Grünwald-Letnikov definition

Grünwald and Letnikov defined fractional differintegral in 1868 as limit of a sum which is generalized form of definition of differentiation and successive integration for arbitrary q numbers as given in Hergüner (June 2015) and Mohammed (2010). Grünwald -Letnikov q^{th} order differintegral for a continuous function f(x) is given

$$ {}_aD_x^q f(x) = \frac{d^q f}{d(x-a)^q} = \lim_{N\to\infty}\left\{\frac{\left(\frac{x-a}{N}\right)^{-q}}{\Gamma(-q)}\sum_{j=0}^{N-1}\frac{\Gamma(j-q)}{\Gamma(j+1)} f\left(x - j\left(\frac{x-a}{N}\right)\right)\right\} $$

Here, the expression a is lower limit for a < x and N represents the number of segments which the interval $(x - a)$ divided into. It can also b expressed as,

$$ \frac{d^q f}{d(x-a)^q} = \lim_{N\to\infty}\left\{\frac{(\delta_N x)^{-q}}{\Gamma(-q)}\sum_{j=0}^{N-1}\frac{\Gamma(j-q)}{\Gamma(j+1)} f(x - j(\delta_N x))\right\} $$

Where $\delta_N x = \frac{(x-a)}{N}$, for N = 1,2,3.... and $(-1)^j \binom{n}{j} = \binom{j+n-1}{j} = \frac{\Gamma(j-n)}{\Gamma(-n)\Gamma(j+1)}$.

C . Riemann- Liouville definition of Differintegral

Riemann and Liouville defined q-th order fractional differintegral in 1832 from an integral definition for a continuous function f(x) $\forall q < 0$,

$$ {}_aD_x^q f(x) = \frac{d^q f}{d(x-a)^q} = \frac{1}{\Gamma(-q)}\int_a^x (x-\xi)^{-q-1} f(\xi)\, d\xi, $$

for $q - n < 0$ and $\forall\, q < 0$:

$$ {}_aD_x^q f(x) = \frac{d^q f}{d(x-a)^q} = \frac{d^n f}{d(x-a)^n}\left(\frac{1}{\Gamma(n-q)}\int_a^x (x-\xi)^{-(q-n)-1} f(\xi)\, d\xi\right) $$

where the expression a is lower limit for a < x and n is an integer number as given in Jumarie (2010) and Song *et al.*(2013).

D. Caputo Definition of Fractional Derivative

Caputo defined fractional derivative of qth order differintegral of a function f(x) in 1960s, using Laplace transform at a point x for 0 < q < 1: This definition is widely used, especially for viscoelasicity problems.

$$ \frac{d^q f}{dx^q} = \frac{1}{\Gamma(1-q)}\int_a^x (x-\acute{x})^{-q}\left(\frac{df(\acute{x})}{d\acute{x}}\right) d\acute{x} $$

Note that since it is valid for 0 < q < 1, this definition is given only for derivative.

E. The Mittag-Leffler Function

The Mittag-Leffler function is defined for α > 0 plays an important role for representing solutions of fractional order partial differintegral equations and fractional integral equations.

$$E_\alpha(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(\alpha k + 1)}.$$

F. *t-test*

A t-test is an analysis of two populations means through the use of statistical examination.An Hypothesis test that is used to determine questions regarding to mean in situations where data is collected from two random data samples.

The two sample t-test is often used to evaluating the means of two variables or distinct groups providing the information that whether the two samples differs with each other or not. The variate t is define by the relation,

$$t = \frac{\bar{x} - \bar{y}}{s} \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

where

$$s = \sqrt{\frac{\sum (x-\bar{x})^2 + (y-\bar{y})^2}{n_1 + n_2 - 2}}$$

$$\bar{x} = \frac{\sum x}{n_1} \quad ; \quad \bar{y} = \frac{\sum y}{n_2}$$

$n_1 =$ Number of observations of Sample A; $n_2 =$ Number of observations of Sample B; Degree of freedom $(dof) = n_1 + n_2 - 2$

5. FRACTIONAL EXPLICIT METHOD

Explicit method is the most popular one within finite difference methods. The basic idea behind each finite difference method is to replace partial derivatives in the PDE by finite difference approximations and solving the resulting system of equations as given in Lateef *et al.*(2015) and Thomson (1995). All finite difference method involves similar four step process:

- Discretize the appropriate differential equation.
- Specify a grid of stock price and time .

- Calculate the payoff of the option at specfic *boundaries* of the grid of underlying prices.
- Iteratively determine the option price at all other grid points, including the point for the current time and underlying price (i.e. the option price today).

A finite difference scheme is said to be *explicit* when it can be computed forward in time using quantities from previous time steps. Originally it is applied on Black-Scholes Partial differential equation. But here our approach is to find price of option by fractional explicit method as given in Davis(2005) and Thomsan (1995). For this let us consider the qth Order time fractional Black Scholes Equation:

$$\frac{\partial^q V}{\partial t^q} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0, \qquad \ldots(1)$$

where q is arbitrary real or complex number, V (S, t) is the price of European option as a function of stock price S and time t, r is the risk-free interest rate, σ is the volatility of the stock. The boundary conditions for European call option can be given as:

Final condition:

$$V(S, T) = \max(S - E, 0),\ S > 0$$

Boundary condition as:

$$V(S_{min}, t) = 0,$$

$$V(S_{max}, t) = S - Ee^{-r(T-t)},$$

Where K is a strike price and S_{min} and S_{max} represents the minimum and maximum values of stock price. Now first we will divide the domain S and t into N and M .

$$\Delta t = \frac{T - t_0}{M} \qquad \text{for } t_0 \le t \le T$$

$$\Delta S = \frac{S_{max} - S_{min}}{N} \qquad \text{for } S_{min} \le S \le S_{max}$$

On making some arrangements S and t are obtained as

$$t_i = t_0 + i\Delta t \qquad \text{for } t_0 \le t \le T$$

$$S_K = S_{min} + k\Delta S \qquad \text{for } S_{min} \le S \le S_{max}$$

For the notation of points (S_K, t_i), we denote the approximation of option price as

$$V(S_K, t_i) \approx \mu_{k,i},$$

Also the final and boundary conditions for European call option in terms of $\mu_{k,i}$, M and N are given as

$$\mu_{k,M} \approx \max(S_K - E, 0),\ S > 0$$

$$\mu_{0,i} = 0, \tag{2}$$

$$\mu_{N,i} = S_N - \mathrm{E}e^{-r(t_M - t_i)}$$

Next we have to discretize the qth order time fractional Black Scholes Equation, for which we have to replace all the partial derivatives as follows:

$$\frac{\partial V(S_K, t_i)}{\partial S_k} \approx \frac{\mu_{k+1,i} - \mu_{k-1,i}}{2\Delta S}$$

$$\frac{\partial^2 V(S_K, t_i)}{\partial {S_k}^2} \approx \frac{\mu_{k+1,i} - 2\mu_{k,i} + \mu_{k-1,i}}{(\Delta S)^2}$$

The approximation for q-th order time fractional derivative of V(S_K, t_i) can be stated as the sum differences with the coefficients g_j as given in Herguner (2015).

$$\frac{\partial^q V}{\partial t^q} = \frac{1}{(\Delta t)^q} \sum_{j=0}^{i} g_j \mu_{k,i-j} \tag{3}$$

where g_j is the function of gamma functions of q and j,

$$g_j = \frac{\lceil(j-q)}{\lceil(-q)\lceil(j+1)}$$

On substituting all the derivatives in (1) and making some arrangements and simplification we will get,

$$\sum_{j=1}^{i} g_j \mu_{k,i-j} = \propto_k \mu_{k-1,i} + \beta_k \mu_{k,i} + \gamma_k \mu_{k+1,i} \tag{4}$$

for i = M, M-1,.....,1 and k = 1,.....N-1 and where the terms are $\propto_k$ β_k and γ_k

$$\propto_k = -\frac{1}{2}(\Delta t)^q \left\{\sigma^2 \left(\frac{S_k}{\Delta S}\right)^2 - r\frac{S_k}{\Delta S}\right\}$$

$$\beta_k = (\Delta t)^q \left\{\sigma^2 \left(\frac{S_k}{\Delta S}\right)^2 + r\right\} - 1$$

$$\gamma_k = -\frac{1}{2}(\Delta t)^q \left\{r\frac{S_k}{\Delta S} + \sigma^2 \left(\frac{S_k}{\Delta S}\right)^2\right\}.$$

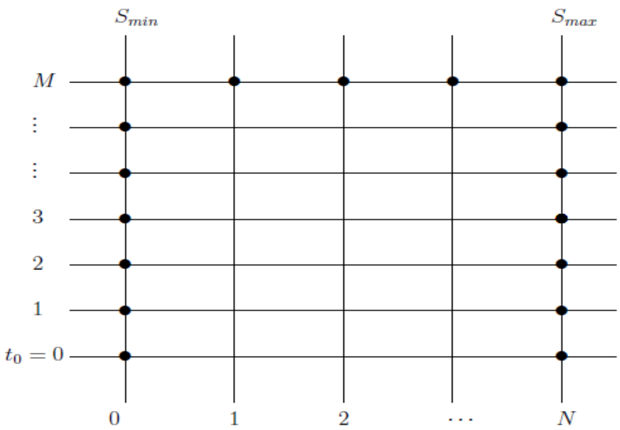

Figure 1: Grid for Fractional Explicit Method (Ecem Hergüner, June 2015)

black dots in the figure shows the data for time T, and stock price S_{min} and S_{max}. We have to find the data when t_0= 0 for S = S_0 by using the fractional explicit method.

In order to solve the system we need matrices for i = M, M−1,....,1 and k = 1,.....,N−1. Then we will get N−1 simultaneous equations.

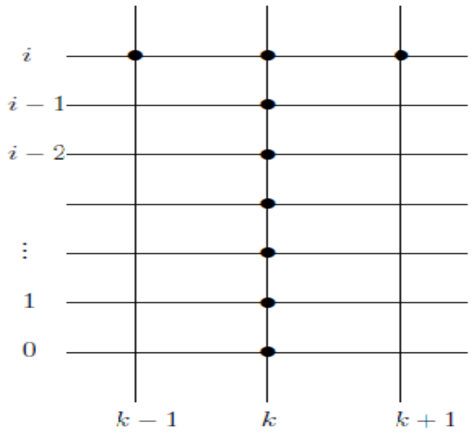

Figure 2 Molecule of fractional explicit method (Ecem Hergüner, June 2015)

The above figure shows the principle of the fractional explicit method for Equation (4).The notation i is for time and k is for stock price as mentioned earlier. The summation of the values of black dots on the i-th row with the coefficients $\propto_k$, β_k and γ_k equal to the summation of the values of black dots on the (i-1), (i -2),... ,0^{th} rows and kth column with the coefficients g_i Herguner (2015). These equations can be written in matrix form as:

$$G \times W = B$$

where G and B matrices are known and W is unknown.

Then we can find the matrix W as $W = G^{-1} \times B$ and find the values of all the grid points including our point of interest i.e option price.

6. DATA PREPARATION

In this study European call option pricing data (1 March to 1 April 2016) of banking sector is collected from NSE website (2016). We restrict our data to the values where number of contracts are near to 100 or more than 100.We have taken four banks Axis Bank, ICICI Bank, SBI Bank and YES Bank. All the observations are classify into three moneyness conditions namely in-the-money (ITM), at-the-money (ATM) and Out-of-the money (OTM). Moneyness is defined as the ratio of stock price S to the strike price K i.e (S/K) as given in Verma *et al.*(2014) and Augustin *et al.*(2013).The results generated for the two methods using MATLAB based on the above three conditions for call option according to the range given below:

1. In-the-money(ITM) $(1.05 \le \frac{S}{K} \le 1.20)$
2. At-the-money(ATM) $(0.95 \le \frac{S}{K} \le 1.05)$
3. Out -the-money(OTM) $(0.80 \le \frac{S}{K} \le 0.95)$

The results obtained for the three conditions are given in following tables:

(Note: Tables are continued on next pages)

Table 1: *Option Price under in-the-money(**ITM**)*
by Fractional Explicit and Black Scholes method of different Banks

Bank Symbol	Strike Price	Stock Price	Fractional Explicit Method	Black Scholes Method
ICICI	210	220.5	15.3921	14.9747
ICICI	210	221.65	15.8239	14.6071
ICICI	210	221.35	15.5451	14.1055
ICICI	200	220	18.3516	21.9885
ICICI	200	218	16.5795	20.0087
ICICI	200	220.5	18.6671	22.2073
ICICI	200	216.85	15.4769	18.6801
ICICI	200	216.35	14.9894	18.0816
ICICI	200	215.5	14.2008	17.1612
ICICI	200	213.85	12.7235	15.5184

ICICI	200	221.65	19.3753	22.7284
ICICI	200	221.35	19.0617	22.3281
SBIN	170	181.15	17.1678	15.3073
SBIN	170	182.75	17.9614	16.344
SBIN	170	188.4	20.8945	20.8548
SBIN	170	183.35	18.1699	16.4139
SBIN	170	183.4	18.1515	16.2473
SBIN	170	180.5	16.5587	13.7194
SBIN	170	180.1	16.2951	13.1773
SBIN	170	181.75	17.1102	14.2511
SBIN	170	185.3	18.9219	17.0287

The result of t-test for in-the-money condition is 0.49236 which is less than the tabulated value of t = 1.684 at 5% level of significance and degree of freedom 40. So there is no significant difference between two methods.

Table 2: *Option Price under at-the-money(**ATM**)*
by Fractional Explicit and Black Scholes method of different banks

Bank Symbol	Strike Price	Stock Price	Fractional Explicit Method	Black Scholes Method
AXISBANK	400	392	3.0525	6.3337
AXISBANK	400	407.25	10.0701	14.5767
AXISBANK	400	417.15	18.9098	21.8987
AXISBANK	400	416.05	17.6727	20.7024
AXISBANK	400	411.75	13.5344	16.9577
AXISBANK	400	416.85	17.9354	20.8025
AXISBANK	400	413	14.2254	17.525
AXISBANK	400	412.8	13.8238	16.8495
AXISBANK	400	415.45	15.9827	18.7333
AXISBANK	400	418.85	18.7991	21.4182
AXISBANK	410	392	2.956	4.0782
AXISBANK	410	407.25	8.861	9.8319
AXISBANK	410	417.15	16.1909	15.3929
AXISBANK	410	416.05	15.1086	14.3084
AXISBANK	410	411.75	11.6211	11.265
AXISBANK	410	416.85	15.2215	14.1151
AXISBANK	410	413	12.1006	11.416
AXISBANK	410	412.8	11.7212	10.7515

AXISBANK	410	415.45	13.4502	12.0228
AXISBANK	410	418.85	15.7108	13.9627
AXISBANK	420	407.25	7.7358	7.0896
AXISBANK	420	417.15	13.6619	11.146
AXISBANK	420	416.05	12.7121	10.1984
AXISBANK	420	411.75	9.8213	7.8222
AXISBANK	420	416.85	12.6634	9.7885
AXISBANK	420	413	10.0854	7.6786
AXISBANK	420	412.8	9.7181	7.0688
AXISBANK	420	415.45	11.0308	7.8165
AXISBANK	420	418.85	12.7515	9.0329
AXISBANK	430	417.15	10.9343	8.149
AXISBANK	430	416.05	10.1204	7.3397
AXISBANK	430	411.75	7.8094	5.4982
AXISBANK	430	416.85	9.9374	6.8343
AXISBANK	430	413	7.8905	5.2114
AXISBANK	430	412.8	7.5466	4.6848
AXISBANK	430	415.45	8.4693	5.095
AXISBANK	430	418.85	9.6788	5.8107
AXISBANK	440	418.85	6.5561	3.8161
ICICIBANK	200	204.95	5.5145	9.6898
ICICIBANK	210	204.95	4.9184	6.3357
ICICIBANK	210	220	15.1928	15.0826
ICICIBANK	210	218	13.7442	13.4037
ICICIBANK	210	220.5	15.3921	14.9747
ICICIBANK	210	216.85	12.8063	12.0971
ICICIBANK	210	216.35	12.3886	11.4905
ICICIBANK	210	215.5	11.7295	10.6447
ICICIBANK	210	213.85	10.5184	9.288
ICICIBANK	220	220	12.1713	11.0956
ICICIBANK	220	218	11.0167	9.7161
ICICIBANK	220	220.5	12.2466	10.7309
ICICIBANK	220	216.85	10.2206	8.4907
ICICIBANK	220	216.35	9.8628	7.9143
ICICIBANK	220	215.5	9.3208	7.1741
ICICIBANK	220	213.85	8.3576	6.0985
ICICIBANK	220	221.65	12.3749	9.6061
ICICIBANK	220	221.35	12.1205	9.0441
ICICIBANK	230	220	9.1884	8.5935
ICICIBANK	230	220.5	9.1437	8.096

ICICIBANK	230	221.65	8.9698	6.6144
ICICIBANK	230	221.35	8.734	6.0642
SBIN	170	162.05	7.0921	4.4585
SBIN	180	181.15	12.3112	10.8897
SBIN	180	182.75	12.828	11.5278
SBIN	180	188.4	14.8459	14.8414
SBIN	180	183.35	12.884	11.2822
SBIN	180	183.4	12.8181	10.9978
SBIN	180	180.5	11.668	8.9945
SBIN	180	180.1	11.4411	8.4418
SBIN	180	181.75	11.9625	9.0172
SBIN	180	185.3	13.1685	10.8288
SBIN	190	181.15	7.6322	8.1818
SBIN	190	182.75	7.8801	8.5554
SBIN	190	188.4	9.0277	10.9517
SBIN	190	183.35	7.7706	8.1356
SBIN	190	183.4	7.6577	7.7985
SBIN	190	180.5	6.9073	6.2245
SBIN	190	181.75	6.9362	5.9723
SBIN	190	185.3	7.5496	7.0763
YESBANK	800	790.6	19.407	25.211
YESBANK	800	791.4	18.3262	24.5301
YESBANK	800	800.6	19.1101	28.1766
YESBANK	800	802	19.4294	27.775
YESBANK	800	810.95	25.4298	31.7589
YESBANK	820	790.6	19.5175	21.2167
YESBANK	820	791.4	18.4652	20.384
YESBANK	820	800.6	19.1466	23.0082
YESBANK	820	802	18.7341	22.3392
YESBANK	820	810.95	23.4297	25.1228
YESBANK	840	800.6	20.4809	23.0991
YESBANK	840	802	19.7583	22.1213
YESBANK	840	810.95	22.5186	24.0337
YESBANK	860	831.85	14.4823	4.1825
YESBANK	860	832.2	14.1743	2.9155

The result of t-test for at-the-money condition is 0.71090 which is less than the tabulated value of t = 1.654 at 5% level of significance and degree of freedom 180. So we cannot discriminate between two methods.

Table 3: *Option Price under Out-of-the-money(**OTM**) by Fractional Explicit and Black Scholes method of different banks*

Bank Symbol	Strike Price	Stock Price	Fractional Explicit Method	Black Scholes Method
AXISBANK	420	392	2.8615	3.0076
AXISBANK	430	392	2.471	2.2444
AXISBANK	430	407.25	6.3297	5.1908
AXISBANK	440	392	1.8624	1.7493
AXISBANK	440	407.25	4.7382	3.9363
AXISBANK	440	417.15	8.111	6.1186
AXISBANK	440	416.05	7.429	5.4309
AXISBANK	440	411.75	5.6695	3.9925
AXISBANK	440	416.85	7.1283	4.9009
AXISBANK	440	413	5.59	3.6492
AXISBANK	440	412.8	5.2746	3.2056
AXISBANK	440	415.45	5.8314	3.4139
ICICIBANK	220	204.95	4.276	4.8508
ICICIBANK	230	204.95	3.4386	3.9233
ICICIBANK	230	218	8.298	7.4391
ICICIBANK	230	216.85	7.6286	6.3153
ICICIBANK	230	216.35	7.3254	5.7864
ICICIBANK	230	215.5	6.8916	5.1451
ICICIBANK	230	213.85	6.1603	4.2798
ICICIBANK	240	204.95	2.3482	3.0833
ICICIBANK	240	220	6.1357	6.6333
ICICIBANK	240	218	5.4911	5.6673
ICICIBANK	240	220.5	5.9837	6.0929
ICICIBANK	240	216.85	4.9465	4.6704
ICICIBANK	240	216.35	4.6984	4.2048
ICICIBANK	240	215.5	4.3711	3.6642
ICICIBANK	240	213.85	3.8641	2.9763
ICICIBANK	240	221.65	5.5338	4.5424
ICICIBANK	240	221.35	5.3171	4.0513
SBIN	180	162.05	5.2074	3.2838

SBIN	190	162.05	3.2979	2.5962
SBIN	190	180.5	6.9073	6.2245
SBIN	190	180.1	6.7056	5.7086
SBIN	200	162.05	1.224	1.764
SBIN	200	181.15	2.8289	5.7632
SBIN	200	182.75	2.8165	5.9733
SBIN	200	188.4	3.1081	7.694
SBIN	200	183.35	2.5543	5.5139
SBIN	200	183.4	2.4005	5.1898
SBIN	200	180.5	2.0525	3.9997
SBIN	200	180.1	1.8821	3.5705
SBIN	200	181.75	1.8301	3.6669
SBIN	200	185.3	1.8606	4.3133
SBIN	210	181.15	2.7723	4.6523
SBIN	210	182.75	2.7586	4.7569
SBIN	210	188.4	3.0369	6.0303
SBIN	210	183.35	2.5006	4.2702
SBIN	210	183.4	2.3494	3.955
SBIN	210	180.5	2.0094	2.999
SBIN	210	180.1	1.8422	2.6227
SBIN	210	181.75	1.7904	2.6274
SBIN	210	185.3	1.8189	3.0094
YESBANK	800	719.4	6.9942	6.5618
YESBANK	800	755.25	15.4628	14.6992
YESBANK	800	757.65	15.2418	14.6766
YESBANK	800	759.05	14.7352	14.3078
YESBANK	800	759.55	13.999	13.6239
YESBANK	820	719.4	7.3857	6.4788
YESBANK	820	755.25	15.6195	13.371
YESBANK	820	757.65	15.3985	13.1946
YESBANK	820	759.05	14.8986	12.7221
YESBANK	820	759.55	14.172	11.9857
YESBANK	840	719.4	8.7317	9.0674

YESBANK	840	755.25	16.8897	15.9936
YESBANK	840	757.65	16.6596	15.6231
YESBANK	840	759.05	16.1497	14.9602
YESBANK	840	759.55	15.4076	14.0405
YESBANK	840	790.6	20.9076	22.2426
YESBANK	840	791.4	19.8662	22.1729
YESBANK	860	790.6	5.3948	3.6454
YESBANK	860	791.4	4.98	3.2124
YESBANK	860	800.6	5.1612	3.8963
YESBANK	860	802	5.2006	3.4541
YESBANK	860	810.95	8.1738	4.1407
YESBANK	860	804.4	5.1367	2.4671
YESBANK	860	799.55	2.907	1.4015
YESBANK	860	814.25	8.0168	2.2421
YESBANK	880	790.6	2.857	1.8276
YESBANK	880	791.4	2.6272	1.5777
YESBANK	880	800.6	2.7164	1.8977
YESBANK	880	802	2.7342	1.5961
YESBANK	880	810.95	4.3179	1.8803
YESBANK	880	804.4	2.6866	0.9829
YESBANK	880	799.55	1.5037	0.4756
YESBANK	880	814.25	4.167	0.7481
YESBANK	880	831.84	7.4856	1.4112

The result of t-test for Out-of-the-money condition is 0.65579 which is less than the tabulated value of t = 1.654 at 5% level of significance and degree of freedom 170. So the difference between the two methods is not significant.

7. CONCLUSION

Fractional Explicit method is the main outlook of this study. We employed Fractional Explicit method for pricing European call option and analyse its results and performance with the Benchmark Black Scholes model. For this, we have taken historical data from National Stock Exchange and found that the former gives approximately equal results as the latter one. For the comparison we have applied the well known t-test and observe that there is no significant difference between the results of Fractional Explicit method and Black–Scholes Method. In future fractional explicit method may be applied to other fractional PDEs and can be used to find solutions of other problems. Also the benefit of Fractional order model is that it is based on historical information of the system. So one can use long term memory benefit of fractional order model to make robust financial model.

Conflict of Interests:

The authors declare that there is no conflict of interests regarding the publication of this paper

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