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Degree Splitting of More Heronian Mean Graphs

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Abstract

In this paper, We contribute some new results on Heronian Mean labeling of degree splitting graphs. We prove that degree splitting of Star $K_{1,3}$, Star $K_{1,4}$, Star $K_{1,5}$, Triangular Snake T_n , Quadrilateral Snake Q_n , Ladder L_n are Heronian Mean graphs.

Key words: Heronian Mean graphs, Degree Splitting graphs, Union of graphs, Star, Triangular Snake, Quadrilateral Snake, Ladder.

AMS Subject Classification: 05C78

1. INTRODUCTION:

The graph considered here will be simple, finite and undirected graph without loops or parallel edges. For all detailed survey of graph labeling, we refer to J.A. Gallian [1]. For all other standard terminology and notations we follow Harary[2].The concept of Mean labeling was introduced in [3]. Motivated by the above results and by the motivation of the authors we study the Heronian Mean labeling on Degree Splitting graphs. The concept of Heronian Mean labeling was introduced in [5]. The concept of Degree Splitting of Heronian Mean graph was introduced in [6]. We will provide brief summary of definitions and other information which are necessary for our present investigation.

A Complete Bipartite graph $\mathbf{K_{m,n}}$ is a bipartite graph with bipartition $\mathbf{(V_1, V_2)}$ such that every vertex of $\mathbf{V_1}$ is joined to all the vertices of $\mathbf{V_2}$,Where $|\mathbf{V_1}| = \mathbf{m}$ and $|\mathbf{V_2}| = \mathbf{n}$. **A Star** graph is the complete bipartite graph $\mathbf{K_{1,n}}$. A **Triangular Snake $\mathbf{T_n}$** is obtained from a path $\mathbf{u_1,u_2,\ldots\ldots.u_n}$ by joining $\mathbf{u_i}$ and $\mathbf{u_{i+1}}$ to a new vertex $\mathbf{v_i}$ for $1 \leq i \leq n-1$. That is every edge of a path is replaced by a triangle $\mathbf{C_3}$**.** A **Quadrilateral Snake $\mathbf{Q_n}$** is obtained from a path $\mathbf{u_1,u_2,\ldots\ldots.u_n}$ by joining $\mathbf{u_i}$ and $\mathbf{u_{i+1}}$ to two new vertices $\mathbf{v_i}$ and $\mathbf{w_i}$ respectively and then joining $\mathbf{v_i}$ and $\mathbf{w_i}$**.** That is every edge of a path is replaced by a cycle $\mathbf{C_4}$**.** The **Ladder $\mathbf{L_n}$** is the product graph $\mathbf{P_2 \times P_n}$**.**

Definition 1.1:

A graph **G=(V,E)** with p vertices and q edges is said to be a **Heronian Mean graph** if it is possible to label the vertices $\mathbf{x \in V}$ with distinct labels $\mathbf{f(x)}$ from **1,2,…,q+1** in such a way that when each edge $\mathbf{e = uv}$ is labeled with,

$$\mathbf{f(e=uv)} = \left\lceil \frac{\mathbf{f(u)} + \sqrt{\mathbf{f(u)f(v)}} + \mathbf{f(v)}}{\mathbf{3}} \right\rceil (\mathbf{OR}) \left\lfloor \frac{\mathbf{f(u)} + \sqrt{\mathbf{f(u)f(v)}} + \mathbf{f(v)}}{\mathbf{3}} \right\rfloor$$

then the edge labels are distinct. In this case **f** is called a **Heronian Mean labeling** of G.

Definition 1.2:

Let **G=(V,E)** be a graph with $V = S_1 \cup S_2 \cup \ldots.\cup S_t \cup T$, Where each S_i is a set of vertices having atleast two vertices and $T = V - \cup S_i$. The degree splitting graph of G is denoted by DS(G) and is obtained from G by adding vertices $w_1, w_2, \ldots.w_t$ and joining w_i to each vertex of S_i ($1 \leq i \leq t$). The graph G and its degree splitting graph DS(G) are given in figure:1.

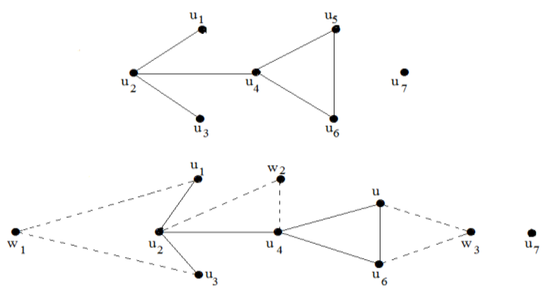

Figure:1

Remark 1.3:

Any graph G is a subgraph of DS(G). If G has atleast two vertices, then G contains atleast two vertices of the same degree. Hence $G = K_1$ is the only graph such that G=DS(G).

Remark 1.4:

If G is regular, then DS(G)= $G + K_1$.

Theorem 1.5: Star $K_{1,n}(n \leq 5)$ is a Heronian mean graph.

Theorem 1.6: Any Triangular Snake T_n is a Heronian mean graph.

Theorem 1.7: Any Quadrilateral Snake Q_n is a Heronian mean graph.

Theorem 1.8: Any Ladder L_n is a Heronian mean graph.

2. MAIN RESULTS:

Theorem: 2.1

$nDS(K_{1,3})$ is a Heronian mean graph.

Proof:

The graph $DS(K_{1,3})$ is shown in figure:2

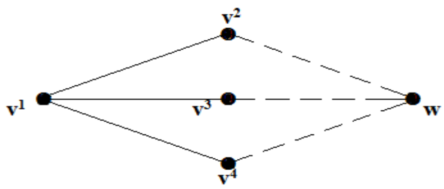

Figure:2

Let $G = nDS(K_{1,3})$. Let the vertex set of G be $V = V_1 \cup V_2 \cup \ldots . \cup V_n$,

Where $V = \{V_i^{\,1}, V_i^{\,2}, V_i^{\,3}, V_i^{\,4}, w_i / 1 \leq i \leq n\}$ } is the vertex set of i^{th} copy of $DS(K_{1,3})$.

Define a function $f: V(G) \rightarrow \{1,2, \ldots \ldots q+1\}$ by $f(V_i^{\,1})= 6i-4$,$1 \leq i \leq n$

$f(V_i^{\,2})= 6i-5$,$1 \leq i \leq n$

$f(V_i^{\,3})= 6i-1$,$1 \leq i \leq n$

$f(V_i^4) = 6i - 3$ $, 1 \leq i \leq n$

$f(w_i) = 6i$ $, 1 \leq i \leq n$

Then the edges are labeled with $f(V_i^1 V_i^2) = 6i - 5$ $, 1 \leq i \leq n$

$f(V_i^1 V_i^3) = 6i - 2$ $, 1 \leq i \leq n$

$f(V_i^1 V_i^4) = 6i - 4$ $, 1 \leq i \leq n$

$f(V_i^2 w_i) = 6i - 3$ $, 1 \leq i \leq n$

$f(V_i^3 w_i) = 6i$ $, 1 \leq i \leq n$

Hence G is a Heronian mean graph.

Example 2.2: Heronian mean labeling of $4DS(K_{1,3})$ is shown in figure 3.

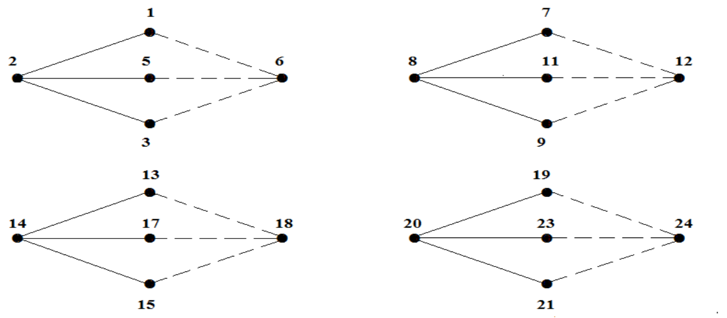

Figure:3

Theorem:2.3

$nDS(K_{1,4})$ is a Heronian mean graph.

Proof:

The graph $DS(K_{1,4})$ is shown in figure:4

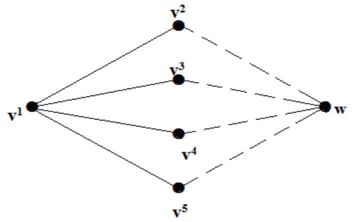

Figure:4

Let $G = nDS(K_{1,4})$. Let the vertex set of G be $V = V_1 \cup V_2 \cup \dots \cup V_n$, Where $V = \{V_i^{1}, V_i^{2}, V_i^{3}, V_i^{4}, V_i^{5}, w_i / 1 \le i \le n\}$ is the vertex set of i^{th} copy of $DS(K_{1,4})$.

Define a function $f: V(G) \to \{1,2,\ldots\ldots q+1\}$ by $f(V_i^{1}) = 8i-6$,$1 \le i \le n$

$f(V_i^{2}) = 8i-7, 1 \le i \le n$

$f(V_i^{3}) = 8i-1$,$1 \le i \le n$

$f(V_i^{4}) = 8i-3$,$1 \le i \le n$

$f(V_i^{5}) = 8i-4$,$1 \le i \le n$

$f(w_i) = 8i$,$1 \le i \le n$

Then the edges are labeled with $f(V_i^{1}V_i^{2}) = 8i-7$,$1 \le i \le n$

$f(V_i^{1}V_i^{3}) = 8i-3$,$1 \le i \le n$

$f(V_i^{1}V_i^{4}) = 8i-5$,$1 \le i \le n$

$f(V_i^{1}V_i^{5}) = 8i-6$,$1 \le i \le n$

$f(V_i^{2}w_i) = 8i-4$,$1 \le i \le n$

$f(V_i^{3}w_i) = 8i$,$1 \le i \le n$

$f(V_i^{4}w_i) = 8i-1$,$1 \le i \le n$

$f(V_i^{5}w_i) = 8i-2$,$1 \le i \le n$

Hence G is a Heronian mean graph.

Example 2.4: Heronian mean labeling of $4DS(K_{1,4})$ is shown in figure 5.

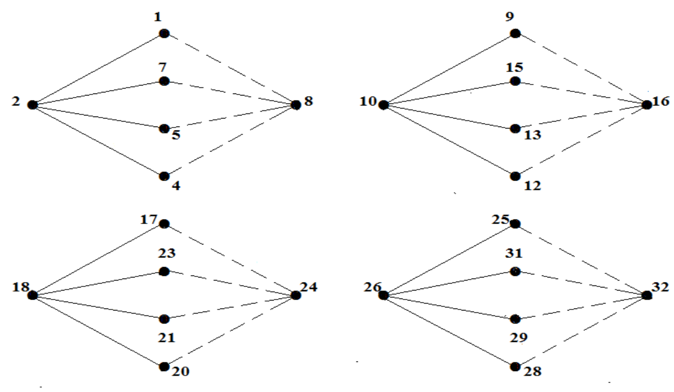

Figure:5

Theorem:2.5

$nDS(K_{1,5})$ is a Heronian mean graph.

Proof:

The graph $DS(K_{1,5})$ is shown in figure:6

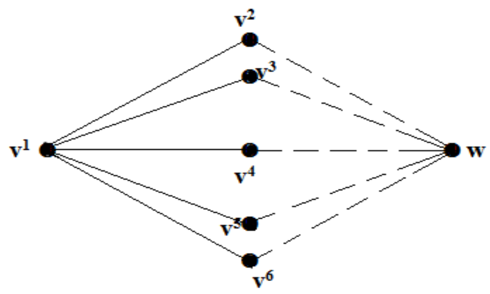

Figure:6

Let $G = nDS(K_{1,5})$. Let the vertex set of G be $V = V_1 \cup V_2 \cup \dots . \cup V_n$,

Where $V = \{V_i^1, V_i^2, V_i^3, V_i^4, V_i^5, V_i^6, w_i / 1 \le i \le n\}$ is the vertex set of i^{th} copy of $DS(K_{1,5})$.

Define a function $f: V(G) \to \{1,2, \dots \dots q+1\}$ by $f(V_i^1) = 10i - 8$,$1 \le i \le n$

$f(V_i^2) = 10i - 9, 1 \le i \le n$

$f(V_i^3) = 10i - 1$,$1 \le i \le n$

$f(V_i^4) = 10i - 2$,$1 \le i \le n$

$f(V_i^5) = 10i - 4$,$1 \le i \le n$

$f(V_i^6) = 10i - 6$,$1 \le i \le n$

$f(w_i) = 10i$,$1 \le i \le n$

Then the edges are labeled with $f(V_i^1 V_i^2) = 8i - 7$,$1 \le i \le n$

$f(V_i^1 V_i^3) = 8i - 3$,$1 \le i \le n$

$f(V_i^1 V_i^4) = 8i - 5$,$1 \le i \le n$

$f(V_i^1 V_i^5) = 8i - 6$,$1 \le i \le n$

$f(V_i^1 V_i^6) = 8i - 6$,$1 \le i \le n$

$f(V_i^2 w_i) = 8i - 4$,$1 \le i \le n$

$f(V_i^3 w_i) = 8i$,$1 \le i \le n$

$$f(V_i^{\,4}w_i)=8i-1\ ,1\le i\le n$$

$$f(V_i^{\,5}w_i)=8i-2\ ,1\le i\le n$$

$$f(V_i^{\,6}w_i)=8i-2\ ,1\le i\le n$$

Hence G is a Heronian mean graph.

Example 2.6: Heronian mean labeling of $4DS(K_{1,5})$ is shown in figure 7.

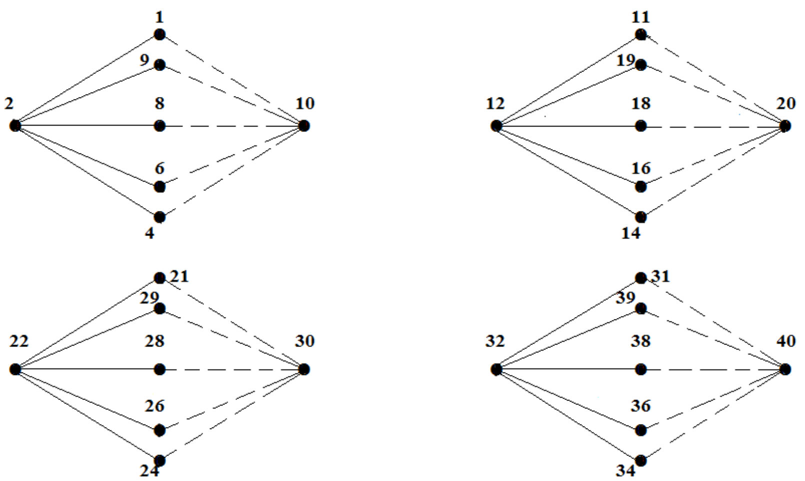

Figure:7

Theorem: 2.7

$nDS(T_2)$ is a Heronian mean graph.

Proof:

The graph $DS(T_2)$ is shown in figure:8

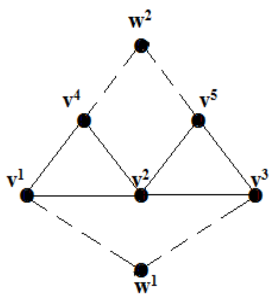

Figure:8

Let $G=nDS(T_2)$. Let the vertex set of G be $V=V_1\cup V_2\cup\ldots\cup V_n$, Where $V = \{V_i^{\,1}, V_i^{\,2}, V_i^{\,3}, V_i^{\,4}, V_i^{\,5}, w_i^{\,1}, w_i^{\,2} / 1 \le i \le n\}$ is the vertex set of i^{th} copy of $DS(T_2)$.

Define a function$f: V(G) \to \{1,2, \ldots\ldots q+1\}$ by $f(V_i^{\,1})= 10i-8$,$1 \le i \le n$

$f(V_i^{\,2})= 10i-4$,$1 \le i \le n$

$f(V_i^{\,3})= 10i-5$,$1 \le i \le n$

$f(V_i^{\,4})= 10i-3$,$1 \le i \le n$

$f(V_i^{\,5})= 10i-1$,$1 \le i \le n$

$f(w_i^{\,1})= 10i-9$,$1 \le i \le n$

$f(w_i^{\,2})= 10i$,$1 \le i \le n$

Then we get distinct edge labels from $\{$**1,2,…,q**$\}$ Hence G is a Heronian mean graph.

Example 2.8: Heronian mean labeling of $4DS(T_2)$ is shown in figure 9.

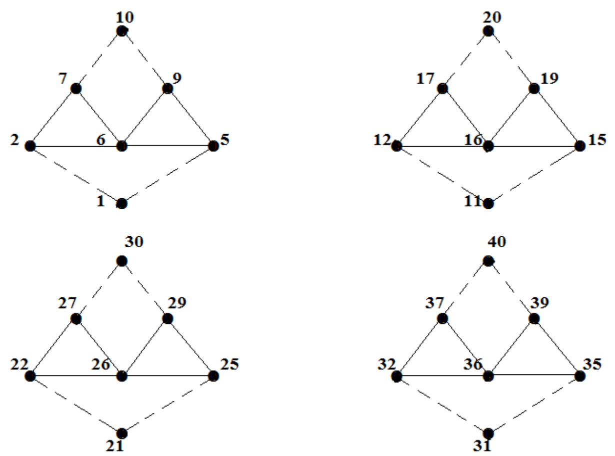

Figure:9

Theorem: 2.9

$nDS(Q_2)$ is a Heronian mean graph.

Proof:

The graph $DS(Q_2)$ is shown in figure:10

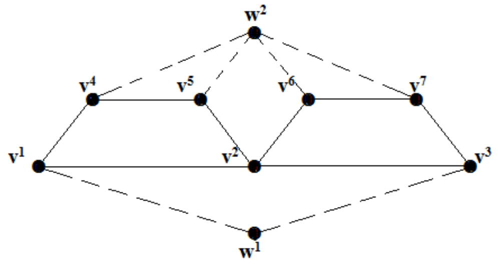

Figure:10

Let $G = nDS(Q_2)$. Let the vertex set of G be $V = V_1 \cup V_2 \cup \dots .\cup V_n$,

Where $V = \{V_i^{\ 1}, V_i^{\ 2}, V_i^{\ 3}, V_i^{\ 4}, V_i^{\ 5}, V_i^{\ 6}, V_i^{\ 7}, w_i^{\ 1}, w_i^{\ 2} / 1 \leq i \leq n\}$ is the vertex set of i^{th} copy of

$DS(Q_2)$.

Define a function $f: V(G) \rightarrow \{1,2, \dots \dots q+1\}$ by $f(V_i^{\ 1})= 14i - 12$,$1 \leq i \leq n$

$f(V_i^{\ 2})= 14i - 10$,$1 \leq i \leq n$

$f(V_i^{\ 3})= 14i - 9$,$1 \leq i \leq n$

$f(V_i^{\ 4})= 14i - 7$,$1 \leq i \leq n$

$f(V_i^{\ 5})= 14i - 5$,$1 \leq i \leq n$

$f(V_i^{\ 6})= 14i - 3$,$1 \leq i \leq n$

$f(V_i^{\ 7})= 14i - 1$,$1 \leq i \leq n$

$f(w_i^{\ 1})= 14i - 13$,$1 \leq i \leq n$

$f(w_i^{\ 2})= 14i$,$1 \leq i \leq n$

Then we get distinct edge labels from {**1,2,…,q**} Hence G is a Heronian mean graph.

Example 2.10: Heronian mean labeling of $4DS(Q_2)$ is shown in figure 11.

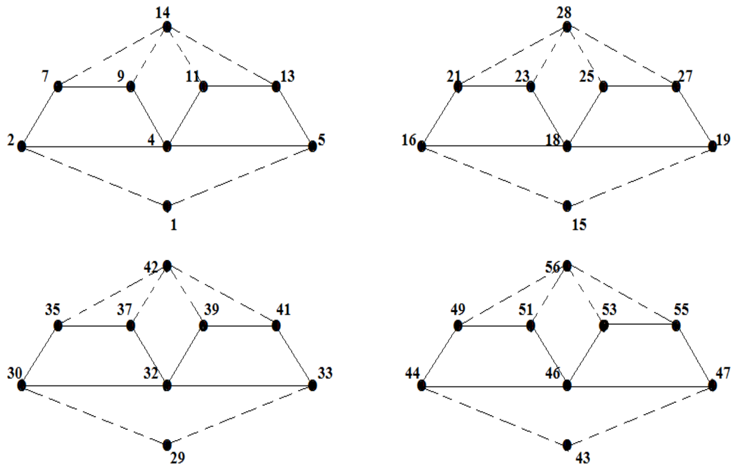

Figure:11

Theorem: 2.11

$nDS(L_3)$ is a Heronian mean graph.

Proof:

The graph $DS(L_3)$ is shown in figure:12

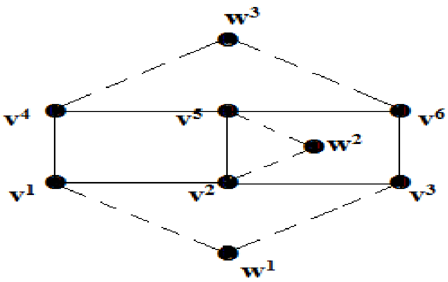

Figure:12

Let $G = nDS(L_3 = P_3 \times P_2)$. Let the vertex set of G be $V = V_1 \cup V_2 \cup \ldots.\cup V_n$, Where $V = \{V_i^{\,1}, V_i^{\,2}, V_i^{\,3}, V_i^{\,4}, V_i^{\,5}, V_i^{\,6}, w_i^{\,1}, w_i^{\,2}, w_i^{\,3} / 1 \leq i \leq n\}$ is the vertex set of i^{th} copy of $DS(L_3)$.

Define a function $f: V(G) \rightarrow \{1,2, \ldots\ldots q+1\}$ by $f(V_i^{\,1}) = 13i - 11$ $, 1 \leq i \leq n$

$$f(V_i^{\,2}) = 13i - 8 \ ,1 \leq i \leq n$$

$$f(V_i^{\,3}) = 13i - 9 \ ,1 \leq i \leq n$$

$$f(V_i^{\,4}) = 13i - 4 \ ,1 \leq i \leq n$$

$$f(V_i^{5}) = 13i - 3 \;, 1 \leq i \leq n$$

$$f(V_i^{6}) = 13i - 1 \;, 1 \leq i \leq n$$

$$f(w_i^{1}) = 13i - 12 \;, 1 \leq i \leq n$$

$$f(w_i^{2}) = 13i - 15 \;, 1 \leq i \leq n$$

$$f(w_i^{3}) = 13i \;, 1 \leq i \leq n$$

Then we get distinct edge labels from $\{\mathbf{1,2,\ldots,q}\}$ Hence G is a Heronian mean graph.

Example 2.12: Heronian mean labeling of $4DS(L_3)$ is shown in figure 13.

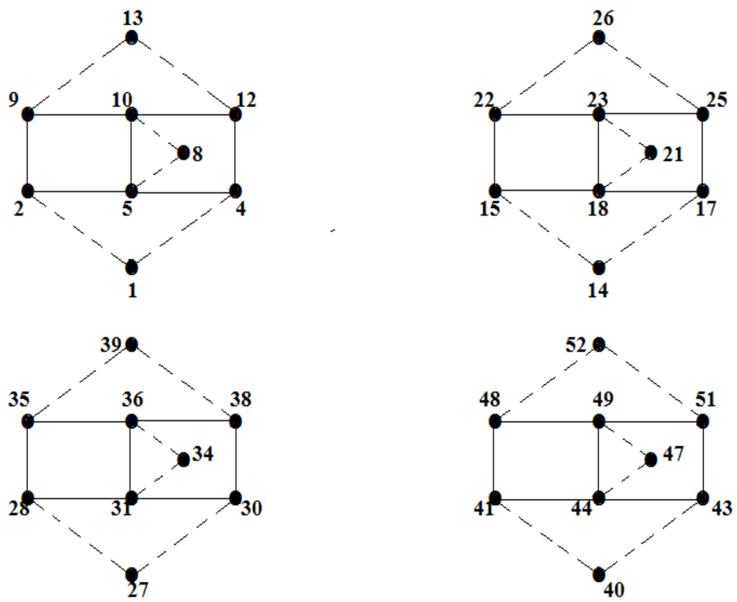

Figure:13

3. CONCLUSION

In this paper, we studied the degree splitting behavior of some standard Heronian mean graphs. The authors are of the opinion that the study of Heronian mean labeling of degree splitting graphs will lead to newer and different results.

4. ACKNOWLEDGEMENT

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