

On $g\beta^*I$ -closed sets in Ideal Topological Spaces

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Abstract

In this paper, a new class of set called $g\beta^*I$ -closed set is introduced. Some of the basic properties of these sets in the ideal topological spaces are studied and also its relationship with the already existing generalized closed set namely β^* -closed set is brought about.

Keywords: $A_{**}(I, \tau)$, semi-pre local function, semi-pre $*$ -closed.

I. INTRODUCTION

Local function in topological space using ideals was introduced by Kuratowski [10]. Since then the properties of generalized closed sets in ideal topological spaces denoted by (X, τ, I) has been the topics for study for many researchers. Dontchev et al[3] were the first to introduce Ig -closed sets. In 2007, Navaneethakrishnan and Paulraj Joseph [16] further investigated and characterized g -closed sets in ideal topological spaces. Khan and Noiri [9] introduced semi-local functions in ideal topological space and using that a new class of sets called gI -closed sets in ideal topological spaces, which is a new generalization of Ig -closed sets were studied in 2010 by them. In this paper another generalization of Ig -closed sets namely $g\beta^*I$ -closed set is defined using semi-pre local function. In 2012, J. Antony Rex Rodgio and et al[2] introduced β^* -closed sets in topological spaces and studied their properties. The aim of this paper is to investigate the properties of these sets in the ideal topological spaces. In this paper, it will be shown that in ideal topological spaces, a set which is both $g\beta^*I$ -closed set and semi-pre dense set is a β^* -closed set.

II. PRELIMINARIES

An ideal I on a nonempty set X is a collection of subsets of X which satisfies the following properties: (i) $A \in I$ and $B \subseteq A$ implies $B \in I$.

(ii) $A \in I$ and $B \in I$ implies $A \cup B \in I$

An ideal topological space is a topological space (X, τ) with an ideal I on X , and is denoted by (X, τ, I) . Let Y be a subset of X . Then $I_Y = \{I_0 \cap Y \mid I_0 \in I\}$ is an ideal on Y and the ideal subspace is denoted by $(Y, \tau/Y, I_Y)$.

Given a topological space (X, τ) with an ideal I on X and if $P(X)$ is the set of all subsets of X , a set operator $(\cdot)^* : P(X) \rightarrow P(X)$, called a *local function* [10] of A with respect to τ and I is defined as follows: for $A \subset X$, $A^*(I, \tau) = \{x \in X \mid U \cap A \notin I \text{ for every}$

$U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau \mid x \in U\}$ when there is no chance for confusion $A^*(I, \tau)$ is denoted by A^* . For every ideal topological space (X, τ, I) , there exists a topology τ^* finer than τ , generated by the base $\beta(I, \tau) = \{U \setminus I \mid U \in \tau \text{ and } I \in I\}$. In general $\beta(I, \tau)$ is not a topology [7]. We will make use of the basic facts about the local functions [10] without mentioning it explicitly. A Kuratowski closure operator $cl^*(\cdot)$ for a topology $\tau^*(I, \tau)$, called the **-topology*, finer than τ is defined by $cl^*(A) = A \cup A^*(I, \tau)$ [15]. An ideal space I is said to be *codense* [4] if $\tau \cap I = \{\emptyset\}$. If $A \subset X$, $cl(A)$ and $int(A)$ will, respectively, denote the closure and interior of A in (X, τ) and $cl^*(A)$ and $int^*(A)$ will respectively denote the closure and interior of A in (X, τ^*) . A subset A of a space (X, τ) is α -open [15] (resp. *semi-open* [11], *pre-open* [12], β -open or *semi-pre-open* [1]) set if $A \subset int(cl(int(A)))$ (resp. $A \subset cl(int(A))$, $A \subset int(cl(A))$, $A \subset cl(int(cl(A)))$). The complement of an α -open (resp. *semi-open*, *pre-open*, β -open or *semi-pre-open*) set is α -closed (resp. *semi-closed*, *pre-closed*, β -closed or *semi-pre-closed*). The *semi closure* (resp. *semi-pre-closure*) of a subset A of X , denoted by $sclA$ (resp. $spclA$) is defined to be the intersection of all semi closed sets (resp. *semi-pre-closed* sets) containing A . A subset A of an ideal space (X, τ, I) is **-closed* [7] (resp. **-dense in itself* [5]) if $A^* \subset A$ (resp. $A \subseteq A^*$).

Definition-2.1: A subset A of a topological space (X, τ) is said to be

- i) ω -closed set [6] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open.
- ii) β^* -closed set [2] if $spcl(A) \subseteq intU$ whenever $A \subseteq U$ and U is ω -open.

Definition-2.2[9]: Let (X, τ, I) be an ideal topological space and A a subset of X . Then

$A_*(I, \tau) = \{x \in X / U \cap A \notin I \text{ for every } U \in SO(X, x)\}$ is called the semi-local function of A with respect to I and τ , where $SO(X, x) = \{U \in SO(X, x) / x \in U\}$, we will write simply A_* for $A_*(I, \tau)$.

Definition-2.3[9]: A subset A of an ideal space (X, τ, I) is said to be gI -closed set if $A_* \subseteq U$ whenever $A \subseteq U$ and U is open in X . The complement of gI -closed set is said to be gI -open.

Definition-2.4[9]: A subset A of an ideal space (X, τ, I) is said to be semi- $*$ -closed if $A_* \subseteq A$.

Definition-2.5[2]: A space (X, τ) is called a T_ω -space if every ω -closed set in it is closed.

Theorem -2.6[2]: Let (X, τ) be a topological space and A a subset of X . U is ω -open if and only if whenever $A \subseteq U$ and A is semi closed implies $A \subseteq \text{int}U$.

Lemma-2.7[1]: Let A and Y be subsets of a topological space X . If Y is open in X and A is semi-pre open in X , then $A \cap Y$ is semi-pre open in Y .

Definition-2.8[1]: Let (X, τ, I) be an ideal topological space and A be a subset of X . Then

$A_{**}(I, \tau) = \{x \in X / U \cap A \notin I \text{ for every } U \in SPO(X, x)\}$ is called the semi-prelocal function of A with respect to I and τ , where $SPO(X, x) = \{U \in SPO(X, x) / x \in U\}$ when there is no ambiguity, we will write simply A_{**} for $A_{**}(I, \tau)$.

III. $g\beta^*I$ -closed sets

Definition-3.1: A subset A of an ideal space (X, τ, I) is said to be $g\beta^*I$ -closed set if $A_{**} \subseteq \text{int}U$ whenever $A \subseteq U$ and U is ω -open in X . The complement of $g\beta^*I$ -closed set is said to be $g\beta^*I$ -open.

Lemma-3.2: Let (X, τ, I) be an ideal space and A, B subsets of X . Then, for the semi-pre local function, the following properties hold:

1. If $A \subset B$, then $A_{**} \subset B_{**}$.
2. $A_{**} = spcl(A_{**}) \subset spcl(A)$ and A_{**} is semi-pre closed in X .
3. $(A_{**})_{**} \subset A_{**}$
4. $(A \cup B)_{**} = A_{**} \cup B_{**}$

Remark- 3.3: Every $*$ -closed set is ${}_{g\beta^*}I$ -closed but not conversely.

Example - 3.4: Let $X = \{a, b, c, d\}$ with the topology $\tau = \{\phi, X, \{a, b\}\}$ and the ideal $I = \{\phi, \{c\}\}$. The set $A = \{a, c\}$ is ${}_{g\beta^*}I$ -closed but it is not $*$ -closed.

Remark-3.5: Every semi $*$ -closed set is ${}_{g\beta^*}I$ -closed but not conversely.

Proof: Let U be a ω -open set in X and A be a semi $*$ -closed set such that $A \subseteq U$. Then $A_* \subseteq A \subseteq U$. Since A_* is semi closed, $A_* \subseteq \text{int} U$. Therefore $A_{**} \subseteq A_* \subseteq \text{int} U$.

Hence A is ${}_{g\beta^*}I$ -closed.

Example - 3.6: Let $X = \{a, b, c, d\}$ with the topology $\tau = \{\phi, X, \{a, b\}\}$ and the ideal $I = \{\phi, \{c\}\}$. The set $A = \{a, c\}$ is ${}_{g\beta^*}I$ -closed but it is not semi- $*$ -closed.

Remark -3.7:

1. Every member of I is ${}_{g\beta^*}I$ -closed in an ideal space (X, τ, I) .
2. A_{**} is ${}_{g\beta^*}I$ -closed for every subset A of (X, τ, I) .
3. If $I = \{\phi\}$, then $A_{**} = spcl(A)$ and hence ${}_{g\beta^*}I$ -closed sets coincide with β^* -closed sets.

Theorem- 3.8: Let (X, τ, I) be an ideal topological space and A a nonempty subset of X . Then the following statements (1), (2) and (3) are equivalent and (3) implies (4) and (5) which are equivalent. (3) implies (1) if (X, τ) is a T_ω -space.

- (1) A is ${}_{g\beta^*}I$ -closed

(2) $spcl(A_{**}) \subseteq \text{int} U$ for every ω -open set U containing A .

(3) For all $x \in spcl(A_{**})$, $\omega cl(\{x\}) \cap A \neq \phi$

(4) $spcl(A_{**}) - A$ contains no non empty ω -closed set.

(5) $A_{**} - A$ contains no non empty ω -closed set.

Proof:

(1) \Rightarrow (2): Let A be $g\beta^*I$ -closed. Then $A_{**} \subseteq \text{int} U$ whenever $A \subseteq U$ and U is ω -open set in X . So by Lemma 3.2, $spcl(A_{**}) \subseteq \text{int} U$ whenever $A \subseteq U$ and U is ω -open set in X . This proves (2).

(2) \Rightarrow (3): Suppose $x \in spcl(A_{**})$. If $\omega cl(\{x\}) \cap A = \phi$ then $A \subseteq X - \omega cl(\{x\})$. By (2),

$spcl(A_{**}) \subseteq \text{int}(X - \omega cl(\{x\}))$. This contradicts the fact that $x \in spcl(A_{**})$. Hence $\omega cl(\{x\}) \cap A \neq \phi$.

(3) \Rightarrow (1): Let (X, τ) be a T_ω -space. Suppose A is not $g\beta^*I$ -closed. There exists a ω -open set U in X such that $A \subseteq U$ and A_{**} is not contained in $\text{int} U$. Then there exist a point $x \in A_{**}$ such that $x \notin \text{int} U$. Then we have $\{x\} \cap \text{int} U = \phi$ and hence

$\{x\} \subseteq X - \text{int} U$ where $X - \text{int} U$ is closed and so ω -closed. Therefore

$\omega cl(\{x\}) \subseteq X - \text{int} U$ this implies $\omega cl(\{x\}) \cap A = \phi$. But by Lemma 3.2,

$spcl(A_{**}) = A_{**}$ and so it follows that (3) does not hold. Therefore A is $g\beta^*I$ -closed.

(3) \Rightarrow (4): Suppose that $F \subseteq spcl(A_{**}) - A$ where F is a nonempty ω -closed set.

Then there exists $x \in F$. Since $F \subseteq X - A$ and $\{x\} \subseteq F$, $\omega cl(\{x\}) \subseteq F$ then

$\omega cl(\{x\}) \cap A = \phi$. Since $x \in spcl(A_{**})$, by (3) $\omega cl(\{x\}) \cap A \neq \phi$. This is a

contradiction. This proves (4). It follows from Lemma 3.2 that (4) and (5) are equivalent.

Lemma -3.9: Let $\{A_\alpha : \alpha \in \Omega\}$ be locally finite family of sets in (X, τ, I) . Then

$$\bigcup_{\alpha \in \Omega} (A_\alpha)_{**} = \left(\bigcup_{\alpha \in \Omega} A_\alpha \right)_{**}$$

Proof: $A_\alpha \subset \bigcup_{\alpha \in \Omega} A_\alpha$ implies $(A_\alpha)_{**} \subset \left(\bigcup_{\alpha \in \Omega} A_\alpha \right)_{**} \forall \alpha \in \Omega$. So $\bigcup_{\alpha \in \Omega} (A_\alpha)_{**} \subset \left(\bigcup_{\alpha \in \Omega} A_\alpha \right)_{**}$.

Conversely, let $x \in \left(\bigcup_{\alpha \in \Omega} A_\alpha \right)_{**}$ and V be any semi-preopen set of X containing x . Since

$\{A_\alpha : \alpha \in \Omega\}$ is locally finite, there exists an ω -open set U in X containing x that intersects only a finite number of members says $A_{\alpha_1}, A_{\alpha_2}, \dots, A_{\alpha_n}$ of $\{A_\alpha : \alpha \in \Omega\}$. But

$x \in \left(\bigcup_{\alpha \in \Omega} A_\alpha \right)_{**}$ implies $(V \cap U) \cap \left(\bigcup_{\alpha \in \Omega} A_\alpha \right) = \bigcup_{\alpha \in \Omega} ((V \cap U) \cap A_\alpha) \notin I$ for every

$V \in SPO(X, x)$. This gives

$\bigcup_{i=1}^n ((V \cap U) \cap A_{\alpha_i}) \notin I$ for every $V \in SPO(X, x)$. There exists atleast one

$A_{\alpha_j} \in \{A_{\alpha_1}, A_{\alpha_2}, \dots, A_{\alpha_n}\}$ such that $(V \cap U) \cap A_{\alpha_j} \notin I$, hence $V \cap A_{\alpha_j} \notin I$. This gives

$x \in (A_{\alpha_j})_{**} \Rightarrow x \in \bigcup_{i=1}^n (A_{\alpha_i})_{**}$ and hence $x \in \bigcup_{\alpha \in \Omega} (A_\alpha)_{**}$. This completes the proof.

Theorem- 3.10: Let (X, τ, I) be an ideal topological space. If $\{A_\alpha : \alpha \in \Omega\}$ is a locally finite family of sets and each A_α is $g\beta^*I$ -closed then $\bigcup_{\alpha \in \Omega} A_\alpha$ is $g\beta^*I$ -closed in (X, τ, I) .

Proof: Let $\bigcup_{\alpha \in \Omega} A_\alpha \subseteq U$ where U is ω -open in X . Since A_α is $g\beta^*I$ -closed for each $\alpha \in \Omega$, then $(A_\alpha)_{**} \subseteq \text{int} U$. Hence $\bigcup_{\alpha \in \Omega} (A_\alpha)_{**} \subseteq \text{int} U$. By Lemma 3.9,

$\left(\bigcup_{\alpha \in \Omega} A_\alpha \right)_{**} \subseteq \text{int} U$. Therefore $\bigcup_{\alpha \in \Omega} A_\alpha$ is $g\beta^*I$ -closed in X .

Remark-3.11: The intersection of two $g\beta^*I$ -closed sets need not be $g\beta^*I$ -closed set as seen from the following example. Let $X = \{a, b, c, d\}$ with the topology $\tau = \{\emptyset, X, \{a, b\}\}$ and the ideal $I = \{\emptyset, \{c\}\}$. We see that, the set $A = \{a, b, c\}$, $B = \{a, b, d\}$ are $g\beta^*I$ -closed sets but $A \cap B = \{a, b\}$ is not a $g\beta^*I$ -closed set.

Lemma -3.12: If A and B are subsets of (X, τ, I) then $(A \cap B)_{**} \subseteq A_{**} \cap B_{**}$

Theorem- 3.13: Let (X, τ, I) be an ideal topological space where (X, τ) is a T_ω - space. If A is $g\beta^*I$ -closed set and B is ω -closed in X , then $A \cap B$ is $g\beta^*I$ -closed.

Proof: Let U be a ω -open set in X containing $A \cap B$. Then $A \subseteq U \cup (X - B)$ where $U \cup (X - B)$ is ω -open. Since A is $g\beta^*I$ -closed, we have

$A_{**} \subseteq \text{int}(U \cup (X - B)) \subseteq U \cup (X - B)$ and so $B \cap A_{**} \subseteq U = \text{int} U$. Using Lemma-3.12 $(A \cap B)_{**} \subseteq A_{**} \cap B_{**} \subseteq A_{**} \cap B \subseteq \text{int} U$ because B is ω -closed.

Hence $A \cap B$ is $g\beta^*I$ -closed.

Definition-3.14: A subset A of an ideal space (X, τ, I) is said to be semi-pre *-closed if $A_{**} \subseteq A$.

Remark-3.15 : Every *-closed set is semi-pre *-closed but not conversely, since $A_{**} \subseteq A_* \subseteq A^* \subseteq A$.

Example - 3.16: Let $X = \{a, b, c, d\}$ with the topology $\tau = \{\emptyset, X, \{a, b\}\}$ and the ideal $I = \{\emptyset, \{c\}\}$. The set $A = \{a\}$ is semi-pre *-closed but it is not *-closed.

Definition-3.17: A subset A of an ideal space (X, τ, I) is said to be *-semi-pre dense in itself if $A \subseteq A_{**}$.

Remark-3.18 : Every *-semi-pre dense set in itself is *-dense in itself.

Theorem- 3.19: In an ideal topological space (X, τ, I) , a $g\beta^*I$ -closed and *-semi-pre dense set in itself set is β^* -closed .

Proof: Suppose A is *-semi-pre dense set in itself and $g\beta^*I$ -closed in X . Let U be any ω -open set containing A . Since A is $g\beta^*I$ -closed $A_{**} \subseteq \text{int} U$ and by Lemma-3.9, $spcl(A_{**}) \subseteq \text{int} U$. Since A is *-semi-pre dense $A \subseteq A_{**}$ and hence $spcl A \subseteq spcl(A_{**}) \subseteq \text{int} U$ whenever $A \subseteq U$. Therefore A is β^* -closed.

Theorem -3.20: Let (X, τ, I) be an ideal topological space and A a $g\beta^*I$ -closed in X . If B is a subset of X such that $A \subseteq B \subseteq A_{**}$ then B is $g\beta^*I$ -closed.

Proof: Let U be any ω -open set in X containing B . Then $A \subseteq U$. Since A is $g\beta^*I$ -closed $A_{**} \subseteq \text{int} U$. By Lemma 3.2, $B_{**} \subseteq (A_{**})_{**} \subseteq A_{**} \subseteq \text{int} U$ hence B is $g\beta^*I$ -closed.

Theorem- 3.21: Let (X, τ, I) be an ideal space and $A \subset Y \subset X$, where Y is open in X . Then $A_{**}(I_Y, \tau/Y) = A_{**}(I, \tau) \cap Y$.

Proof: Assume that $x \in X - (A_{**}(I, \tau) \cap Y)$. Then either $x \in Y$ or $x \notin Y$.

Case-1: $x \notin Y$. Since $A_{**}(I_Y, \tau/Y) \subset Y$, $x \notin A_{**}(I_Y, \tau/Y)$.

Case-2: $x \in Y$. Since $x \notin A_{**}(I, \tau)$, there exists a semi-pre-open set V in X containing x such that $V \cap A \in I$. Since $x \in Y$ and Y is open in X , we have by Lemma-2.7 $Y \cap V \in SPO(Y, \tau/Y)$ such that $x \in Y \cap V$ and $(Y \cap V) \cap A \in I$ hence $(Y \cap V) \cap A \in I_Y$. Consequently $x \notin A_{**}(I_Y, \tau/y)$. Hence we get

$A_{**}(I_Y, \tau/Y) \subset A_{**}(I, \tau) \cap Y$. Conversely, consider $x \notin A_{**}(I_Y, \tau/y)$.

Then for some semi-pre-open set V in $(Y, \tau/Y)$ containing x , there exist

$U \in SPO(X, x)$ such that

$V = U \cap Y$ and we have $(U \cap Y) \cap A \in I_Y$. Since $A \subset Y$, $U \cap A \in I_Y \subset I$ gives

$U \cap A \in I$ for some semi-preopen set U in (X, τ) containing x . This proves

$x \notin A_{**}(I, \tau)$.

Theorem -3.22: Let (X, τ, I) be an ideal space where (X, τ) is a T_ω -space and $A \subset Y \subset X$. If A is a $g\beta^*I$ -closed in $(Y, \tau/Y, I_Y)$ and Y is open and semi-pre*-closed in X , then A is $g\beta^*I$ -closed in X .

Proof: Let U be any ω -open set in X and $A \subseteq U$. Then $A_{**}(I_Y, \tau/Y) = A_{**}(I, \tau) \cap Y \subset U \cap Y$. Then we have $Y \subset U \cup (X - A_{**}(I, \tau))$.

Since Y is semi-pre*-closed we have $A_{**} \subset Y_{**} \subset Y \subset U \cup (X - A_{**}(I, \tau))$. This proves $A_{**}(I, \tau) \subset U = \text{int}U$. Hence A is $g\beta^*I$ -closed.

Theorem- 3.23: Let (X, τ, I) be an ideal space where (X, τ) is a T_ω -space and $A \subset Y \subset X$. If A is a $g\beta^*I$ -closed in (X, τ, I) and $Y \in \tau$, then A is a $g\beta^*I$ -closed in $(Y, \tau/Y, I_Y)$.

Proof: Let U be any ω -open subset of $(Y, \tau/Y)$ and $A \subseteq U$. Since $Y \in \tau$, U is ω -open in X . Thus $A_{**}(I, \tau) \subset \text{int}U$. By Theorem 3.21,

$A_{**}(I_Y, \tau/Y) = A_{**}(I, \tau) \cap Y \subset U \cap Y = U$ and $A_{**}(I_Y, \tau/Y) \subset \text{int}U$. Hence A is a $g\beta^*I$ -closed in $(Y, \tau/Y, I_Y)$.

Corollary- 3.24: Let (X, τ, I) be an ideal space where (X, τ) is a T_ω -space and $A \subset Y \subset X$ where Y is a regular open subset of X . Then A is $g\beta^*I$ -closed in $(Y, \tau/Y, I_Y)$ if and only if A is a $g\beta^*I$ -closed in X .

Theorem- 3.25: Let (X, τ, I) be an ideal space and $A \subseteq X$. If A is $g\beta^*I$ -closed then $A \cup (X - A_{**})$ is $g\beta^*I$ -closed.

Proof: Suppose A is $g\beta^*I$ -closed in X . Let U be a ω -open set such that $A \cup (X - A_{**}) \subseteq U$. Then $X - U \subseteq X - (A \cup (X - A_{**})) = A_{**} - A$. Since A is $g\beta^*I$ -closed, by Theorem 3.8, it follows that $A_{**} - A$ contains no nonempty ω -closed set. This implies $X - U = \emptyset$ or $X = U$. Hence X is the only open set containing $A \cup (X - A_{**})$. This gives $(A \cup (X - A_{**}))_{**} \subseteq X = \text{int}X$. Therefore $A \cup (X - A_{**})$ is $g\beta^*I$ -closed.

Theorem- 3.26: Let (X, τ, I) be an ideal space. Then $A \cup (X - A_{**})$ is $g\beta^*I$ -closed if and only if $A_{**} - A$ is $g\beta^*I$ -open.

Proof: Since $X - (A_{**} - A) = A \cup (X - A_{**})$, the proof follows immediately.

Theorem 3.27: Let (X, τ, I) be an ideal space. Then every subset of X is $g\beta^*I$ -closed if every ω -open set is open and semi-pre*-closed.

Proof: Suppose that every ω -open set is open and semi-pre*-closed. If $A \subset X$ and U is an ω -open set such that $A \subset U$, then $A_{**} \subset U_{**} \subset U = \text{int} U$. Hence A is $g\beta^*I$ -closed.

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