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μ- More Adjacency in Intuitionistic Fuzzy Graphs

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Abstract

In this paper, μ-more adjacency nodes of an intuitionistic fuzzy graph is defined. Certain properties of μ-more adjacency are studied. The relationship between μ-more adjacency and intuitionistic fuzzy bridges, intuitionistic fuzzy cut nodes are established. We introduce the concept of μ-more adjacent set, μ-more adjacent graph of an intuitionistic fuzzy graph and bounds for a minimal μ-more adjacency set are suggested.

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Keywords: μ-more adjacent node, intuitionistic fuzzy bridge, intuitionistic fuzzy cut node, μ-more adjacent set, μ-more adjacent graph

1. INTRODUCTION

Intuitionistic Fuzzy Graph theory was introduced by Atanassov in [1]. In [14] R.Parvathi and M.G.Karunambigai introduced intuitionistic fuzzy graph as a special case of Atanassov's IFG. In [8] Akram and Alshehri introduces the intuitionistic fuzzy cycle intuitionistic fuzzy trees. In [10] A.Nagoor Gani and V.T. Chandrsekaran introduced the more adjacency in fuzzy graphs. A. Nagoor Gani and H. Sheik Mujibur Rahman introduced the lower and upper truncations of intuitionistic fuzzy graphs in [13]. In this paper we introduce the μ- more adjacency in intuitionistic fuzzy graphs.

2. PRELIMINARIES

2.1 Definition: A path P in an intuitionistic fuzzy graph is a sequence of distinct vertices $v_1, v_2, \dots, v_n$ such that either one of the following conditions is satisfied.

(a) $\mu_{2ij} > 0$ and $\nu_{2ij} = 0$ for some i and j

(b) $\mu_{2ij} = 0$ and $\nu_{2ij} > 0$ for some i and j

(c) $\mu_{2ij} > 0$ and $\nu_{2ij} > 0$ for some i and j $(i, j = 1,2, \dots, n)$.

2.2 Definition: The length of the path $P = v_1, v_2, \dots, v_{n+1} (n > 0)$ is n.

2.3 Definition: A path $P = v_1, v_2, \dots, v_{n+1}$ is called a cycle if $v_1 = v_{n+1}$, and $n \geq 3$.

2.4 Definition: Two vertices that are joined by a path are said to be connected.

2.5 Definition: An intuitionistic fuzzy graph $G = (V, E)$ is said to be complete intuitionistic fuzzy graph if any two vertices are joined by a path.

2.6 Definition: An intuitionistic fuzzy graph $G = (V, E)$ is said to be strong intuitionistic fuzzy graph if $\mu_{2ij} = min(\mu_{1i}, \mu_{1j})$ and $\nu_{2ij} = max(\nu_{1i}, \nu_{1j})$ for every $v_i, v_j \in E$.

2.7 Definition: A connected intuitionistic fuzzy graph $G = (A, B)$ is an intuitionistic fuzzy tree if it has an intuitionistic fuzzy spanning sub graph $H = (A, C)$ which is a tree, where for all arcs (x, y) not in H, $\mu_B(x, y) < \mu_C{}^{\infty}(x, y)$, $\nu_B(x, y) > \nu_C{}^{\infty}(x, y)$.

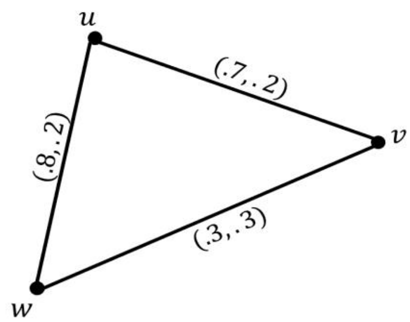
2.8 Definition: Let $G = (V, E)$ be an intuitionistic fuzzy graph. Let v_i, v_j be any two distinct vertices and $H = (V', E')$ be an intuitionistic fuzzy sub graph of G obtained by deleting the edge (v_i, v_j). That is, $H = (V', E')$, where $\mu_{2ij}{}' = 0$ and $\nu_{2ij}{}' = 0$ and $\mu_2{}' = \mu_2$, $\nu_2{}' = \nu_2$for all other edges. Now (v_i, v_j) is said to be a bridge in G, if either $\mu_{2xy}{}'^{\infty} < \mu_{2xy}$ and $\nu_{2xy}{}'^{\infty} \geq \nu_{2xy}$ (or) $\mu_{2xy}{}'^{\infty} \leq \mu_{2xy}$and $\nu_{2xy}{}'^{\infty} > \nu_{2xy}$, for some $v_x, v_y \in V$. In other words, deleting an edge (v_i, v_j) reduces the strength of connectedness between some pair of vertices.

2.9 Definition: A vertex v_i is said to be a cut vertex in G if deleting a vertex v_i reduces the strength of connectedness between some pair of vertices. In other words, $\mu_{2xy}{}'^{\infty} \leq \mu_{2xy}{}^{\infty}$and $\nu_{2xy}{}'^{\infty} < \nu_{2xy}{}^{\infty}$(or) $\mu_{2xy}{}'^{\infty} < \mu_{2xy}{}^{\infty}$and $\nu_{2xy}{}'^{\infty} \leq \nu_{2xy}{}^{\infty}$, for some $v_x, v_y \in V$.

3. μ -MORE ADJACENCY

3.1 Definition: Let G be an intuitionistic fuzzy graph and u, v be two distinct nodes in V. Then the node u is said to be μ-more adjacent to v if $\mu_2(u,v) \geq \mu_2(u,w)$ for any $w \in V$. It is interesting to note that adjacency is a symmetric relation while μ-more adjacency is not so.

3.2 Example: In an intuitionistic fuzzy graph given in figure 1, u is μ-more adjacent to w, v is μ-more adjacent to u and w is μ-more adjacent to u.



$Figure$ 1

3.3 Proposition: In an intuitionistic fuzzy graph, every node is either isolated or μ-more adjacent to some node of the intuitionistic fuzzy graph.

Proof: Let G be an intuitionistic fuzzy graph. If a node u is not isolated then it is adjacent to at least some other node in G. Let v be node in G such that $\mu_2(u,v) = max\ \{\mu_2(u,w): w \in V\}$. Then u is μ-more adjacent to v.

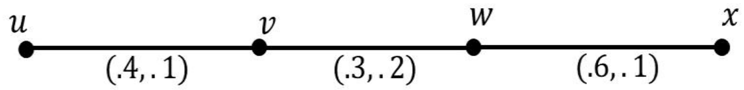
3.4 Proposition: In every intuitionistic fuzzy graph with not all nodes isolated, there are at least two nodes, which are μ-more adjacent to each other.

Proof: Let G be an intuitionistic fuzzy graph such that all of its nodes are not isolated. Hence there are some nodes u and v such that $\mu_2(u,v) \geq 0$. Let $\mu_2(x,y) = max\ \{\mu_2(u,v): uv \in V\}$. Then it is easy to observe that the nodes x and y are μ-more adjacent to each other.

3.5 Proposition: Let u and v be two nodes of an intuitionistic fuzzy graph G. If u is the only μ-more adjacent node to v, then (u, v) is an intuitionistic fuzzy bridge of G.

Proof: If u is the only node μ-more adjacent node to v then the removal of the arc (u, v) reduces the strength of connectedness between the nodes u and v. Thus (u, v) is an intuitionistic fuzzy bridge.

3.6 Remark: The converse of the above proposition need not be true. In the intuitionistic fuzzy graph given in figure 2, the arc (v, w) is an intuitionistic fuzzy bridge. But neither v is μ-more adjacent node to w nor w is μ-more adjacent node to v.

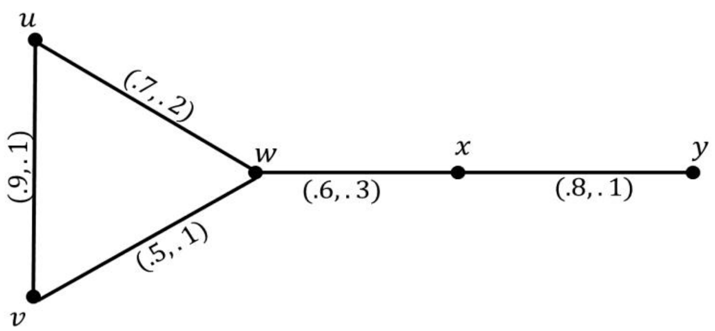


$Figure$ 2

3.7 Proposition: If there are two nodes u and v μ-more adjacent to only w in an intuitionistic fuzzy graph G then w is an intuitionistic fuzzy cut node of G.

Proof: If w is the only node μ-more adjacent to u and v, then the removal of w reduces the strength of connectedness between the nodes u and v. Hence w is an intuitionistic fuzzy cut node.

3.8 Remark: The converse of the above proposition need not be true. No node is μ-more adjacent to w in the intuitionistic fuzzy graph given in figure 3, still w is an intuitionistic fuzzy cut node of G.



$Figure$ 3

4. μ -MORE ADJACENCT SET

4.1 Definition: Let G be an intuitionistic fuzzy graph. A set $D \subseteq V$ is called a μ-more adjacent set of G if every node in $V - D$ has a μ-more adjacent node in D. A μ-more adjacent set D is called a minimal μ-more adjacent set if no proper subset of D is a μ-more adjacent set. The smallest number of nodes in any μ-more adjacent set of G is called its μ-more adjacency number and is denoted by $\omega_\mu(G)$or simply ω_μ. A μ-more adjacent set D of an intuitionistic fuzzy graph G such that $|D| = \omega_\mu$ is called a minimum μ-more adjacent set.

4.2 Example: The set $\{v, w\}$ is a minimal μ-more adjacent set of the intuitionistic fuzzy graph given in figure 1 while $\{u\}$ is a minimum μ-more adjacent set.

4.3 Definition: The μ-more adjacent neighbourhood of u is $N_m(u) = \{v \in V: v$ is μ-more adjacent to $u\}$. $N_m[u] = N_m(u) \cup \{u\}$ is the closed μ-more adjacent neighbourhood of u. The minimum cardinality of μ-more adjacent neighbourhood $\delta_{\mu m}(G) = min\{|N_m(u)|: u \in V(G)\}$ and the maximum cardinality of μ-more adjacent neighbourhood $\Delta_{\mu m}(G) = max\{|N_m(u)|: u \in V(G)\}$.

4.4 Example: For the intuitionistic fuzzy graph given in figure 3, $\delta_{\mu m}(G) = 0$ and $\Delta_{\mu m}(G) = 2$.

4.5 Proposition: If G is a complete intuitionistic fuzzy graph then $\omega_\mu = 1$.

Proof: Let u be a node of G such that $\mu_1(u) = max\{\mu_1(v): v \in V\}$. Then u is μ-more adjacent all other nodes of the intuitionistic fuzzy graph G and hence $\omega_\mu = 1$.

4.6 Remark: It is easy to observe that for an intuitionistic fuzzy graph with all nodes isolated $\omega_\mu = p$, where $p = |\mu_1^*|$.

4.7 Proposition: A μ-more adjacent set D of an intuitionistic fuzzy graph G is a minimal μ-more adjacent set if for each $u \in D$ one of the following is holds.

(i) u is not μ-more adjacent set to any node in D

(ii) There is a node $v \notin D$ such that v is μ-more adjacent to u only

Proof: Suppose D is minimal μ-more adjacent set of G. Then for each node $u \in D$, the set $D' = D - \{u\}$ is not a μ-more adjacent set. Thus, there is a node $v \in V - D'$ which is not μ-more adjacent to any node in D'. Now either $u = v\ (or)\ u \in V - D$. If $u = v$, then u is not μ-more adjacent to any node in D. If $v \in V - D$ and v is not μ-more adjacent to any node in $D - \{u\}$, but is μ-more adjacent to a node in D, then v is μ-more adjacent only to the node u in D.

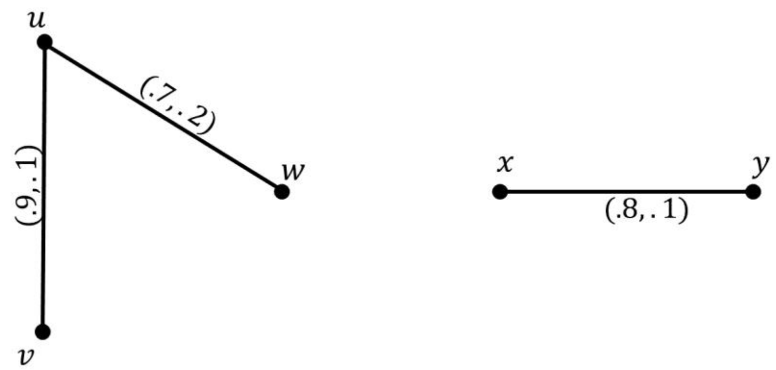
Conversely, suppose D is a μ-more adjacent set and each node $u \in D$, one of the two stated conditions holds. Now we prove that D is a minimal μ-more adjacent set. Suppose D is not a minimal μ-more adjacent set. Then there exists a node $u \in D$ such that $D - \{u\}$ is a μ-more adjacent set. Thus u is μ-more adjacent to at least one node in $D - \{u\}$. Therefore condition (i) does not hold. Also if $D - \{u\}$ is a μ-more adjacent set, then every node in $V - D$ is μ-more adjacent to at least one node in $D - \{u\}$. Therefore for condition (ii) does not hold. Hence neither condition (i) nor (ii) holds, which is a contradiction.

5. μ -MORE ADJACENCT GRAPH

5.1 Definition: Let G be an intuitionistic fuzzy graph. A path $u_0, u_1, u_2, \ldots, u_n$ is said to be a μ-more adjacent path in G if u_i is μ-more adjacent to $u_{i+1}$for all $0 \leq i \leq n - 1$. The node u_0 is called origin of the path and the node u_n is called terminus of the path. We also say that there is a μ-more adjacent path from u_0 to u_n.

5.2 Example: The path w, u, v is a μ-more adjacent path of the intuitionistic fuzzy graph given in figure 3.

5.3 Definition: The intuitionistic fuzzy graph obtained by retaining the μ-more adjacent paths and deleting all other paths of an intuitionistic fuzzy graph is known as μ-more adjacent intuitionistic fuzzy graph.



$Figure$ 4

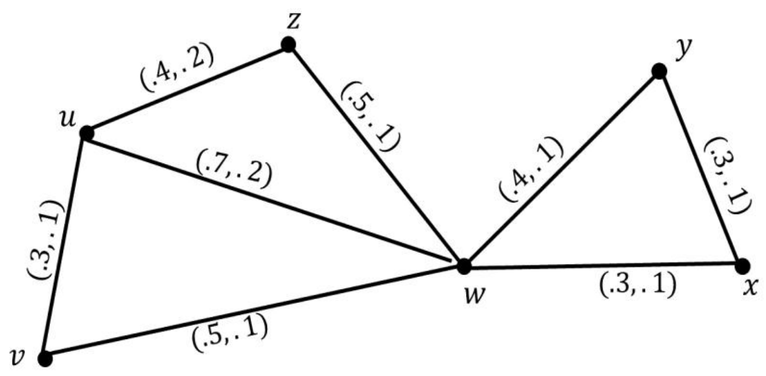
5.4 Example: The μ-more adjacent graph of the intuitionistic fuzzy graph given in figure 3, is given in figure 4.

5.5 Proposition: μ-more adjacent graph of an intuitionistic fuzzy graph G is connected if and only if there is a μ-more adjacent path between any two nodes of G.

Proof: The proof easily follows from the definition of μ-more adjacent path and μ-more adjacent graph. The statement of proposition 5.5, leads us to define root node of an intuitionistic fuzzy graph.

5.6 Definition: A node u of an intuitionistic fuzzy graph is said to be a root node if there is a μ-more adjacent path from u to v where v is any other node of G. That is keeping u as origin, it is possible to find a μ-more adjacent path to any other node v of G. Clearly a root node, if exists, is unique.

5.7 Example: The intuitionistic fuzzy graph given in figure 5 has x as the root node.



$Figure\ 5$

5.8 Proposition: μ-more adjacent graph of an intuitionistic fuzzy graph G is an intuitionistic fuzzy tree if and only if there is a unique μ-more adjacent path between any two nodes of G.

Proof: If there is a unique μ-more adjacent path between any two nodes of G then the μ-more adjacent graph cannot have an intuitionistic fuzzy cycle. Hence the μ-more adjacent graph is an intuitionistic fuzzy tree. Conversely if there is more than one μ-more adjacent paths between at least a pair of nodes of G, then all of them will present in the μ-more adjacent graph, forming an intuitionistic fuzzy cycle.

6. BOUNDS

In this section, we obtain some upper and lower bounds for ω_μ one can observe that $\omega_\mu = 1$ if $\Delta_{\mu m}(G) = p - 1$ and conversely. Also for a totally disconnected intuitionistic fuzzy graph $\omega_\mu = p$ and conversely.

6.1 Proposition: For any connected intuitionistic fuzzy graph $\omega_\mu \leq p - \Delta_{\mu m}(G)$.

Proof: Let u be node of G such that $N_m(u) = |\Delta_{\mu m}(G)|$. Then $V - N_m(u)$ is a μ-more adjacent set of G. Therefore $\omega_\mu \leq |\ V - N_m(u)| = p - \Delta_{\mu m}(G)$.

6.2 Proposition: For any intuitionistic fuzzy graph $[p/(1 + \Delta_{\mu m}(G))] \leq\ \omega_\mu$.

Proof: Let D be a minimal μ-more adjacent set. Each node in D is μ-more adjacent to at most $\Delta_{\mu m}(G)$ other nodes in G. Thus $p \leq\ \omega_\mu[(1 + \Delta_{\mu m}(G))]$ and hence the result follows.

REFERENCES

[1] Atanassov, K., 1999, "Intuitionistic Fuzzy Sets: Theory and Applications", Physica Verlag, New York.

[2] Chris Cornelis, Glad Deschrijver, and Etienne E.Kerre, 2004, "Implication in intuitionistic fuzzy and interval valued fuzzy set theory: construction classification, application", International Journal of Approximate Reasoning, Science Direct, 35(1), 55-95.

[3] Guo-jun wang, and Ying- Yu He, 2000, "Intuitionistic Fuzzy Sets and L-Fuzzy Sets", Fuzzy Sets and systems, Science Direct, 110(2), 271-274.

[4] Karunambigai, M.G., Palanivel, K., and Sivasankar, S., 2015, "Edge Regular intuitionistic fuzzy graph", J. Chinese Fuzzy System Assoc., 5(2), 1-7.

[5] Mordeson, J.N., and Peng, C.S., 1994, "Operations on fuzzy graphs", Inform. Sci., 79, 159-170.

[6] Muhammad Akram, and Wieslaw A. Dudek, 2012, "Regular bipolar fuzzy graphs", Neural Computing and Applications, Springer, 21, 197-205.

[7] Muhammad Akram, and Parvathi, R., 2012, "Properties of intuitionistic fuzzy line graphs", Notes on intuitionistic fuzzy sets, 18(3), 52-60.

[8] Muhammed Akram , and Alshehri, 2014, "The Intuitionistic Fuzzy Cycle Intuitionistic Fuzzy Trees", The Scientific World Journal, Hindawi, 305836.

[9] Nagoor Gani, A., and Malarvizhi, J., 2010, "Truncations on Special Fuzzy Graphs", Advances in Fuzzy Mathematics, 5(2), 135-145.

[10] Nagoor Gani, A., and Chandrasekaran, V.T., 2009, "More Adjacent Graph of a Fuzzy Graph", Proceedings of International Conference on Mathematical Modeling and Computation, 158-161.

[11] Nagoor Gani, A., and Shajitha Begum, S., 2010, Degree, order, size in intuitionistic fuzzy graphs, International Journal of Algorithms, Computing and Mathematics, 3, 11-16.

[12] Nagoor Gani, A., and Sheik Mujibur Rahman, H., 2015, Total Degree of a Vertex in Union and Join of Some Intuitionistic Fuzzy Graphs, International Journal of Fuzzy Mathematical Archive , 7(2), 233-241.

[13] Nagoor Gani, A., and Sheik Mujibur Rahman, H., "Truncations on Special Intuitionistic Fuzzy Graphs", communicated to International Journal of Pure and Applied Mathematics, Bulgaria.

[14] Parvathi, R., Karunambigai, M.G., and Atanassov, K., 2009, “Operations on intuitionistic fuzzy graphs”, Proceedings of IEEE International Conference on Fuzzy Systems, 1396-1401.

[15] Radha, K., and Kumaravel, N., 2016, “The Edge Degree and the Edge Regular Properties of Truncations of Fuzzy Graphs”, Bulletin of Mathematics and Statistics Research,4(3), 7-16

[16] Rosenfeld, A., Fuzzy Graphs, in: LA. Zadeh, Fu. K. S.Shimura (eds), 1975, “Fuzzy sets and their application to cognitive and decision processes”, Acadamic Press, New York, 77-95.