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Edge – Even Graceful Labeling on Circulant Graphs With Different Generating Sets

Dr. K. Ameenal Bibi[1] and T.Ranjani[2]

[1,2] P.G and Research Department of Mathematics.
D.K.M College For Women (Autonomous), Vellore- 632001, India.

Abstract

Let G = (V,E) be a simple, finite undirected and connected graph with order p and size q. A graph G admits an Edge – Even graceful labeling , if there exists a bijection f from E to {2,4,6,…2q} so that the induced mapping f^+ from V to {0,1,2…n-1} given by $f^+ (X) = \sum\{f(XY/XY\epsilon\ E\}\ mod|\ 2q|$.

In this paper, we have constructed an Edge – Even graceful labeling on circulant graphs. We already published papers with generating sets (1,2),(1,2,3) and (1,2,3,4) . Now we have extended the generating sets as (1,2,3,4,5),(1,2,3,4,5,6),(1,2,3,4,5,6,7) and (1,2,3,4,5,6,7,8) for odd n , $n\epsilon$ I.

Keywords: Labeling, Graceful labeling, Even graceful labeling, Edge graceful labeling, Edge – even graceful labeling, circulant graph.

AMS Subject Classification (2010): 05c78.

INTRODUCTION

The study of graceful graphs and graceful labeling were introduced by Rosa in 1967.S.Lo introduced edge- graceful labeling and Gayathri introduced Even- Edge graceful labeling. Sin-Min Lee, Kuo-Jye Chen and Yung-Chin Wang introduced the K-edge – graceful graphs.

In this paper, we considered the problem of labeling the edges and the vertices in such a way that the labeling of the edges and the vertices should be distinct integers.

We have extended the number of generating sets and constructed an Edge- Even graceful labeling for circulant graphs.[6],[11].

MAIN RESULTS:

Edge – Even Graceful Labeling on Circulant Graphs.

Theorem 2.1:

For odd n ≥ 11, the circulant graph $G = C_n(1,2,3,4,5)$ admits an edge- even graceful labeling.

Here (1,2,3,4,5) are the generators of G.

Proof:

Let $G = C_n$ (1,2,3,4,5) be the 10- regular graph with n ≥ 11 .

Let V(G) = {V_i /i=0,1,2….n-1} here q=5n.

Define the mapping as follows $f: E(G) \rightarrow \{2,4, \dots 2q\}$ by

$f(V_iV_{i+1})$ = 2i+2 for i = 0,1....n-1

$f(V_iV_{i+2})$ = n+2i+13 for i = 0,1,….n-1

$f(V_iV_{i+3})$ = 3n+2i+13 for i = 0,1,….n-1

$f(V_iV_{i+4})$ = 5n+2i+13 for i = 0,1,….n-1

$f(V_iV_{i+5})$ = 7n+2i+13 for i = 0,1,….n-1

It can be verified that the edge labeled under the labeling f is a bijection from the set E of C_n (1,2,3,4,5) onto the set $\{2,4, \dots 2q\}$.

For every vertex $v \in V(G)$, the vertex- weight $f^+(V)$ of C_n (1,2,3,4,5) are defined as follows,

Case(i) For i = 0,1,2,3,4

a) For i=0

$\sum_{e \in N(v_0)} f(e) =$
2i+2+n+2i+13+3n+2i+13+5n+2i+13+7n+2i+13+7n+2(n+2)+3

+5n+2(n)+13+3n+2(n-3)+13+n+2(n-4)+13+2(n-5)+2.

= 2+n+13+3n+13+5n+13+7n+13+7n+2(n+2)+3+5n+2(n)+13

+3n+2(n-3)+13 +n+2(n-4)+13+2(n-5)+2.

= 42n+78.

$f(V_0) = 540.$

b) For i=1

$\sum_{e\in N(v_1)} f(e)$ = 2i+2+n+2i+13+3n+2i+13+5n+2i+13+7n+2i+13+7n+2(n+1)
+13+5n+2(n-4)+13+3n+2(n-5)+13+n+2(n-6)+13
+2(n-6)+13+2(n-7)+2.
= 2+2+n+2+13+3n+2+13+5n+2+13+7n+2+13+7n+2(n+1)
+13+5n+2(n-4)+13+3n+2(n-5)+13+n+2(n-6)+13+2(n-7)+2.
= 42n+76.

$f(V_1)$ = 538.

c) For i=2

$\sum_{e\in N(v_2)} f(e)$ = 2i+2+n+2i+13+3n+2i+13+5n+2i+13+7n+2i+13+7n+2(n-1)
+13+5n+2i+13+7n+2i+13+7n+2(n-1)+13+5n+2(n-5)+13+3n
+2(n-6)+13+n+2(n-7)+13+2(n-8)+2.
= 4+2+n+4+13+3n+4+13+5n+4+13+7n+4+13+7n+2(n-1)
+13+5n+2(n-5)+13+3n+2(n-6)+13+n+2(n-7)+13+2(n-8)+2.
= 42n+74.

$f(V_2)$ = 536.

d) For i=3

$\sum_{e\in N(v_3)} f(e)$ = 2i+2+n+2i+13+3n+2i+13+5n+2i+13+7n+2i+13+7n+2(n-3)
+13+5n+2(n-6)+13+3n+2(n-7)+13+n+2(n-8)+13+2(n-9)+2.
= 42n+72.

$f(V_3)$ = 534.

e) For i=4

$\sum_{e\in N(v_4)} f(e)$ = 2i+2+n+2i+13+3n+2i+13+5n+2i+13+7n+2i+13+7n+2(n-5)
+13+5n+2(n-7)+13+3n+2(n-8)+13+n+2(n-9)+13+2(n-10)+2.
= 42n+70.

$f(V_4)$ = 532.

Case(ii) For i =5,6,…10

$\sum_{e\in N(v_i)} f(e)$ = 2i+2+n+2i+13+3n+2i+13+5n+2i+13+7n+2i+13+7n

+2(i-1)+13+5n+2(i-2)+13+3n+2(i-3)+13+n+2(i-4)

+13+2(i-5)+2.

= 32n+20i+78.

Thus, we obtained that the sum of the values assigned to all the edges incident to a given vertex $v \in V(G)$ are all distinct integers.

Thus, the vertex – weight induced mapping f^+ from V onto $\{0,1, \dots 5n-1\}$ admits an Edge- Even graceful labeling.

Theorem 2.2 :

For odd ≥ 13 , the circulant graph $G = C_n(1,2,3,4,5,6)$ admits an edge- even graceful labeling.

Here (1,2,3,4,5,6) are the generators of G.

Proof:

Let $G = C_n$ (1,2,3,4,5,6) be the 12- regular graph with n ≥ 13 .

Let V(G) = {V_i /i=0,1,2….n-1} here q=6n.

Define the mapping as follows $f: E(G) \rightarrow \{2,4, \dots 2q\}$ by

$f(V_iV_{i+1})$ = 2i+2 for i = 0,1....n-1

$f(V_iV_{i+2})$ = 2n+2i+2 for i = 0,1,….n-1

$f(V_iV_{i+3})$ = 4n+2i+2 for i = 0,1,….n-1

$f(V_iV_{i+4})$ = 6n+2i+2 for i = 0,1,….n-1

$f(V_iV_{i+5})$ = 8n+2i+2 for i = 0,1,….n-1

$f(V_iV_{i+6})$ = 10n+2i+2 for i = 0,1,….n-1

It can be verified that the edge labeled under the labeling f is a bijection from the set E of C_n (1,2,3,4,5,6) onto the set $\{2,4, \dots 2q\}$.

For every vertex $v \in V(G)$, the vertex- weight $f^+(V)$ of C_n (1,2,3,4,5,6) are defined as follows,

Case(i) For i = 0,1,2,3,4,5,6

a) For i=0

$\sum_{e \in N(v_0)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+10n+2(n-4)
+2+8n+2(n-3)+2+6n+2(n-2)+2+4n+2(n-1)+2+2n+2(n+2)+2+2(0)+2.

= 70n+8.

$f(V_0) = 918.$

b) For i=1

$\sum_{e \in N(v_1)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+10n+2(n-8)
+2+8n+2(n-7)+2+6n+2(n-6)+2+4n+2(n-5)+2+2n+2(n-4)+2+2(n+2)+2.

= 72n-20.

$f(V_1) = 916.$

C) For i=2

$\sum_{e \in N(v_2)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+10n+2(n-9)
+2+8n+2(n-8)+2+6n+2(n-7)+2+4n+2(n-6)+2+2n+2(n-5)+2+2(n)+2.

= 72n-22.

$f(V_2) = 914.$

d) For i=3

$\sum_{e \in N(v_3)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+10n+2(n-10)
+2+8n+2(n-9)+2+6n+2(n-8)+2+4n+2(n-7)+2+2n+2(n-6)+2+2(n-2)+2.

= 72n-24.

$f(V_3) = 912.$

e) For i = 4

$\sum_{e\in N(v_4)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+10n+2(n-11)

+2+8n+2(n-10)+2+6n+2(n-9)+2+4n+2(n-8)+2+2n+2(n-7)+2+2(n-4)+2.

= 72n-26.

$f(V_4) = 910.$

f) For i = 5

$\sum_{e\in N(v_5)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+10n+2(n-12)

+2+8n+2(n-11)+2+6n+2(n-10)+2+4n+2(n-9)+2+2n+2(n-8)+2+2(n-6)+2.

= 72n-28.

$f(V_5) = 908.$

g) For i = 6

$\sum_{e\in N(v_6)} f(e) =$

2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+10n+2(n-13)

+2+8n+2(n-12)+2+6n+2(n-11)+2+4n+2(n-10)+2+2n+2(n-9)+2+2(n-8)+2.

= 72n-30.

$f(V_6) = 906.$

Case(ii) For i = 7,8…

$\sum_{e\in N(v_i)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+10n+2(i-6)

+2+8n+2(i-5)+2+6n+2(i-4)+2+4n+2(i-3)+2+2n+2(i-2)+2+

2(i-1)+2.

= 60n+24i-18.

Thus, we obtained that the sum of the values assigned to all the edges incident to a given vertex $v \in V(G)$ are all distinct integers.

Thus, the vertex – weight induced mapping f^+ from V onto $\{0,1, \ldots 6n - 1\}$ admits an Edge- Even graceful labeling.

Theorem 2.3 :

For odd ≥ 17 , the circulant graph $G = C_n\,(1,2,3,4,5,6,7)$ admits an edge-even graceful labeling.

Here (1,2,3,4,5,6,7) are the generators of G.

Proof:

Let $G = C_n\,(1,2,3,4,5,6,7)$ be the 14- regular graph with n ≥ 17 .

Let V(G) = {V_i /i=0,1,2….n-1} here q=7n.

Define the mapping as follows $f: E(G) \rightarrow \{2,4, \ldots 2q\}$ by

$f(V_iV_{i+1})$ = 2i+2 for i = 0,1....n-1

$f(V_iV_{i+2})$ = 2n+2i+2 for i = 0,1,….n-1

$f(V_iV_{i+3})$ = 4n+2i+2 for i = 0,1,….n-1

$f(V_iV_{i+4})$ = 6n+2i+2 for i = 0,1,….n-1

$f(V_iV_{i+5})$ = 8n+2i+2 for i = 0,1,….n-1

$f(V_iV_{i+6})$ = 10n+2i+2 for i = 0,1,….n-1

$f(V_iV_{i+7})$ = 12n+2i+2 for i = 0,1,….n-1

It can be verified that the edge labeled under the labeling f is a bijection from the set E of C_n (1,2,3,4,5,6,7) onto the set $\{2,4, \ldots 2q\}$.

For every vertex $v \in V(G)$, the vertex- weight $f^+(V)$ of C_n (1,2,3,4,5,6,7) are defined as follows,

Case(i) For i = 0,1,2,3,4,5,6

a) For i=0

$\sum_{e \in N(v_0)} f(e) =$

2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+12n+2i+2+12n

+2(n-7) +2+10n+2(n-6)+2+8n+2(n-5)+2+6n+2(n-4)+2+4n+2(n-3)

+2+2n+2(n-2)+2+2(n-1)+2.

= 98n-28.

$f(V_0) = 1638.$

b) For i=1

$\sum_{e\in N(v_1)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+12n+2i+2+12n
+2(n-9) +2+10n+2(n-8)+2+8n+2(n-7)+2+6n+2(n-6)+2+4n+2(n-5)
+2+2n+2(n-4)+2+2(n+1)+2.
= 98n-34.

$f(V_1) = 1632.$

c) For i=2

$\sum_{e\in N(v_2)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+12n+2i+2+12n
+2(n-11) +2+10n+2(n-10)+2+8n+2(n-9)+2+6n+2(n-8)+2+4n+2(n-7)
+2+2n+2(n-3)+2+2(n)+2.
= 98n-40.

$f(V_2) = 1626.$

d) For i=3

$\sum_{e\in N(v_3)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+12n+2i+2+12n
+2(n-12) +2+10n+2(n-11)+2+8n+2(n-10)+2+6n+2(n-9)+2+4n+2(n-8)
+2+2n+2(n-7)+2+2(n-1)+2.
= 98n-46.

$f(V_3) = 1620.$

e) For i=4

$\sum_{e\in N(v_4)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+12n+2i+2+12n
+2(n-13) +2+10n+2(n-12)+2+8n+2(n-11)+2+6n+2(n-10)+2+4n+2(n-9)+2+2n+2(n-8)+2+2(n-5)+2.
= 98n-52.

$f(V_4) = 1614.$

f) For i=5

$\sum_{e\in N(v_5)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+12n+2i+2+12n

+2(n-15) +2+10n+2(n-14)+2+8n+2(n-13)+2+6n+2(n-12)+2+4n

+2(n-11) +2+2n+2(n-10)+2+2(n-3)+2.

= 98n-58.

$f(V_5) = 1608.$

g) For i=6

$\sum_{e\in N(v_6)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+12n+2i+2+12n

+2(n-16) +2+10n+2(n-15)+2+8n+2(n-14)+2+6n+2(n-13)+2+4n

+2(n-12) +2+2n+2(n-11)+2+2(n-7)+2.

= 98n-64.

$f(V_6) = 1602.$

Case(ii) For i = 7,8…

$\sum_{e\in N(v_i)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+12n+2i+2+12n

+2(i-7) +2+10n+2(i-6)+2+8n+2(i-5)+2+6n+2(i-4)+2+4n

+2(i-3) +2+2n+2(i-2)+2+2(i-1)+2.

= 84n+28i-28.

Thus, we obtained that the sum of the values assigned to all the edges incident to a given vertex $v \in V(G)$ are all distinct integers.

Thus, the vertex – weight induced mapping f^+ from V onto $\{0,1, \dots 7n-1\}$ admits an Edge- Even graceful labeling.

Theorem 2.4:

For odd ≥ 19 , the circulant graph $G = C_n\,(1,2,3,4,5,6,7,8)$ admits an edge-even graceful labeling.

Here (1,2,3,4,5,6,7,8) are the generators of G.

Proof:

Let $G = C_n$ (1,2,3,4,5,6,7,8) be the 16- regular graph with n ≥ 19 .

Let V(G) = {V_i /i=0,1,2….n-1} here q=8n.

Define the mapping as follows $f: E(G) \rightarrow \{2,4, \dots 2q\}$ by

$f(V_iV_{i+1})$ = 2i+2 for i = 0,1....n-1

$f(V_iV_{i+2})$ = 2n+2i+2 for i = 0,1,….n-1

$f(V_iV_{i+3})$ = 4n+2i+2 for i = 0,1,….n-1

$f(V_iV_{i+4})$ = 6n+2i+2 for i = 0,1,….n-1

$f(V_iV_{i+5})$ = 8n+2i+2 for i = 0,1,….n-1

$f(V_iV_{i+6})$ = 10n+2i+2 for i = 0,1,….n-1

$f(V_iV_{i+7})$ = 12n+2i+2 for i = 0,1,….n-1

$f(V_iV_{i+8})$ = 14n+2i+2 for i = 0,1,….n-1

It can be verified that the edge labeled under the labeling f is a bijection from the set E of C_n (1,2,3,4,5,6,7,8) onto the set $\{2,4, \dots 2q\}$.

For every vertex $v \in V(G)$, the vertex- weight $f^+(V)$ of C_n (1,2,3,4,5,6,7,8) are defined as follows,

Case(i) For i = 0,1,2,3,4,5,6,7

a) For i=0

$\sum_{e \in N(v_0)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+12n+2i+2+14n
+2i+2+14n +2(n-8) +2+12n+2(n-7)+2+10n+2(n-6)+2+8n
+2(n-5)+2+6n+2(n-4)+2+4n+2(n-3)+2+2n+2(n-2)+2+2(n-1)+2.
= 128n-40.

$f(V_0) = 2392.$

b) For i=1

$\sum_{e\in N(v_1)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+12n+2i+2+14n
+2i+2+14n +2(n-10) +2+12n+2(n-9)+2+10n+2(n-8)+2+8n
+2(n-7)+2+6n+2(n-6)+2+4n+2(n-5)+2+2n+2(n-4)+2+2(n+2)+2.
= 128n-46.

$f(V_1) = 2386.$

c) For i=2

$\sum_{e\in N(v_2)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+12n+2i+2+14n
+2i+2+14n +2(n-11) +2+12n+2(n-10)+2+10n+2(n-9)+2+8n
+2(n-8)+2+6n+2(n-7)+2+4n+2(n-6)+2+2n+2(n-5)+2+2(n-1)+2.
= 128n-50.

$f(V_2) = 2382.$

d) For i=3

$\sum_{e\in N(v_3)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+12n+2i+2+14n
+2i+2+14n +2(n-13) +2+12n+2(n-12)+2+10n+2(n-11)+2+8n
+2(n-10)+2+6n+2(n-9)+2+4n+2(n-8)+2+2n+2(n-7)+2+2(n+1)+2.
= 128n-58.

$f(V_3) = 2374.$

e) For i=4

$\sum_{e\in N(v_4)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+12n+2i+2+14n
+2i+2+14n +2(n-15) +2+12n+2(n-14)+2+10n+2(n-13)+2+8n
+2(n-12)+2+6n+2(n-11)+2+4n+2(n-10)+2+2n+2(n-9)+2+2(n+4)+2.
= 128n-64.

$f(V_4) = 2368.$

f) For i=5

$\sum_{e\in N(v_5)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+12n+2i+2+14n
+2i+2+14n +2(n-16) +2+12n+2(n-15)+2+10n+2(n-14)+2+8n
+2(n-13)+2+6n+2(n-12)+2+4n+2(n-11)+2+2n+2(n-10)+2+2(n)+2.
= 128n-70.
$f(V_5) = 2362.$

g) For i=6

$\sum_{e\in N(v_6)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+12n+2i+2+14n
+2i+2+14n +2(n-17) +2+12n+2(n-16)+2+10n+2(n-15)+2+8n
+2(n-14)+2+6n+2(n-13)+2+4n+2(n-12)+2+2n+2(n-11)+2+2(n-4)+2.
= 128n-76.
$f(V_6) = 2356.$

h) For i=7

$\sum_{e\in N(v_7)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+12n+2i+2+14n
+2i+2+14n +2(n-18) +2+12n+2(n-17)+2+10n+2(n-16)+2+8n
+2(n-15)+2+6n+2(n-14)+2+4n+2(n-13)+2+2n+2(n-12)+2+2(n-8)+2.
= 128n-82.
$f(V_7) = 2350.$

Case(ii) For i = 8,9….

$\sum_{e\in N(v_i)} f(e) =$
2i+2+2n+2i+2+4n+2i+2+6n+2i+2+8n+2i+2+10n+2i+2+12n+2i+2+14n
+2i+2+14n +2(i-8) +2+12n+2(i-7)+2+10n+2(i-6)+2+8n
+2(i-5)+2+6n+2(i-4)+2+4n+2(i-3)+2+2n+2(i-2)+2+2(i-1)+2.
= 112n+32i-40.

Thus, we obtained that the sum of the values assigned to all the edges incident to a given vertex $v \in V(G)$ are all distinct integers.

Thus, the vertex – weight induced mapping f^+ from V onto $\{0,1, \dots 8n - 1\}$ admits an Edge- Even graceful labeling.

CONCLUSION:

In this paper, we obtained an Edge – Even graceful labeling on circulant graphs with generating sets (1,2,3,4,5)(1,2,3,4,5,6)(1,2,3,4,5,6,7) and (1,2,3,4,5,6,7,8).

REFERENCES:

[1] Baskar Babujee.J, On edge bi-magic labeling,J.Combin.Inf.Syst.Sci.,28(2004)239-244.

[2] Baskaro.M,M.Miller, Slamin and W.D.Wallis, Edge – magic total labeling, Austral,J.Combin.,22(2000)177-190.

[3] Gallian J.A. A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics,16,# Ds6(2013).

[4] Gayathri .B, Duraisamy.M and Tamilselvi.M, Even graceful labeling of some cycle related graphs,Int.J.Math.Comput.Sci.,2(2007)179 – 187.

[5] Gnanajothi.R.B, Topics in Graph Theory Ph.D Thesis Madurai Kamaraj University 1991.

[6] Kalaimurugan.G "On Edge – odd graceful labeling on circulant graphs".Journal of Computer and Mathematical Sciences.

[7] Lo.S., On Edge graceful labeling of graphs, Congress number , 50,231-241(1985).

[8] Rosa.A, On certain valuation of the vertices of a graph, Theory of graphs (International Symposium,Rome) July(1966).

[9] Singhun.S, Graphs with Edge – odd graceful labeling International Mathematical Forum, Vol.8,2013 no.12,577-582.

[10] Sin – Min Lee, Kue – Jye Chen and Yung – Chin Wang. On the edge – graceful soectra of cycles with one chord and dumbbell graphs, Congressus. Numerantium, 170(2004)171-183.

[11] Solairaju.A and Chithra.K. Edge – odd graceful labeling of some graphs, Electronics Notes in Discrete Mathematics, 33,15-20(2009).

[12] West.D.B. Introduction to graph theory, Prentice- Hall Inc, (2000).