Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 9 (2017), pp. 6221-6240

http://www.ripublication.com

A Symmetric 4-(σ ,)-Generalized Derivation in Prime Rings

Maryam K.Chitheer, Abdulrahman H. Majeed

*Department of Mathematics, College of Science,
University of Baghdad, Iraq.*

Abstract

Let R be a 3, 2 -torsion free prime ring with center Z, and U be a nonzero square closed Lie ideal of R and g: $R \to R$ is any mapping. Let F: $R^4 \to R$ be a symmetric 4-(**σ**, **τ**)-generalized derivation associated with D:$R^4 \to R$ be a symmetric 4-(**σ**, **τ**)-derivation, if f and d be the traces of F and D respectively satisfy one of the following .(i) $f([u,v])$-$[f(u),v]_{\sigma,\tau}$ =0; (ii) $f(uov)$ -$[f(u),v]_{\sigma,\tau}$ =0; (iii) $(f(u)ov)_{\sigma,\tau}$ -$[f(u),v]_{\sigma,\tau}$ =0; (iv) $[f(u),v]_{\sigma,\tau}$ -$[u, g(v)]$ =0; (v) $f(u)of(v)$ -$[f(u),v]_{\sigma,\tau}$ =0; (vi) $f(u)\tau(v)$ -$ug(v)$ =0; (vii) $f(uv)$- $f(u)\sigma(v)$ - $\tau(u)d(v)$ =0, for all $u,v\in U$, then either $U\subseteq Z(R)$ or d=0 .

Keywords: Prime ring, Symmetric 4-derivation, Symmetric 4-(**σ**, **τ**)-derivation, Symmetric 4-(**σ**, **τ**)-generalized derivation.

INTRODUCTION:

The concept of a permuting tri-(**σ**, **τ**)-derivation has been introduced by Ozturk in [5]. Some recent results on properties of prime rings, semiprime rings and near rings with derivations have been investigated in several ways [1-3, 5, 8]. Kyoo-Hong Park et al. in [7] have introduced the concept of permuting tri-(**σ**, **τ**)-derivation of near ring and investigated the conditions for a near ring to be commutative ring. Further Ozturk et al. in [5, 6] introduce the concepts of permuting tri- (**σ**, **τ**)-derivation and permuting tri-(**σ**, **τ**)-generalized derivation of near ring, prime and semiprime rings with gave some properties. Recently, Durna et al. in [4] studied some results on permuting tri-(σ, τ)-derivations in prime and semi-prime rings.

Throughout this paper R will be an associative ring and the center of R will be denoted by Z , σ and τ be automorphisms of R. Recall that a ring R is prime if $xRy=\{0\}$ implies $x=0$ or $y=0$. A ring R is said to be n -torsion free if $nx=0$ implies $x=0$, for all $x\in R$. For any x, y the symbol $[x,y]$ stands for the commutator $xy-yx$ and the symbol (xoy) stands for the anti-commutator $xy+yx$, an additive subgroup U of R is called Lie ideal of R if whenever $u\in U$, $r\in R$ then$[u,r]\in U$ [10], and Lie ideal U is called square close Lie ideal if $u^2\in U$ for all $u\in U$ [10]. A mapping F: $R^4 \to R$ be a symmetric 4-additive if $F(x_1,x_2,x_3,x_4)=F(x_{\pi(1)},,x_{\pi(2)},x_{\pi(3)},x_{\pi(4)})$,for all $x_1,\ldots,x_4\in R$ and every permutation$\{\pi(1),\pi(2), \pi(3), \pi(4)\}$. A mapping f: $R \to R$ is said to be trace of if $f(x)=F(x,x,x,x)$, for all $x\in R$. The trace f of satisfies the relation $f(x+y) = [f(x),+4F(x,x,x,y)+ 6F(x,x,y,y)+ 4F(x,y,y,y)+ f(y),]$, for all $x,y\in R$. An additive mapping d: $R \to R$ is called a derivation if $d(xy)=d(x)y+xd(y)$, for all $x,y\in R$. A symmetric 4-additive mapping is called a symmetric 4-derivation if is derivation in each argument[9]. An additive mapping d: $R \to R$ is called a -(σ, τ)-derivation if $d(xy)=d(x)\sigma(y)+ \tau(x)d(y)$, for all $x,y\in R$, where σ and τ to be a automorphisms of R . A symmetric 4-additive mapping is called a symmetric 4-(σ, τ)-derivation if there exists functions σ, τ: $R \to R$ such that

$$D(xu,y,z,w) = D(x,y,z,w)\alpha(u)+ \tau(x)D(u,y,z,w),$$

$$D(x,yu,z,w) = D(x,y,z,w)\alpha(u) + \tau(y)D(x,u,z,w),$$

$$D(x,y,zu,w) = D(x,y,z,w)\alpha(u) + \tau(z)D(x,y,u,w),$$

$$D(x,y,z,wu) = D(x,y,z,w) \alpha(u) + \tau(w)D(x,y,z,u),$$ for all $x,y,z,w\in R$

An additive mapping $F:R \to R$ is called a generalized derivation, if there exists $d:R \to R$ a derivation such that $F(xy)=F(x)y+xd(y)$, for all $x,y\in R$. An additive mapping $F:R \to R$ is said to be a-(σ, τ) -generalized derivation of R, if there exists a -(σ, τ) - derivation $d:R \to R$ such that $F(xy)=F(x)\sigma(y)+ \tau(x)d(y)$, for all $x,y\in R$. A symmetric 4-additive map is said to be a symmetric 4-right (resp. left) (σ, τ)-generalized derivation of R associated with D if $F(xw,y,z,u)=F(x,y,z,u)\sigma(w)+ \tau(x)D(w,y,z,u)$ (resp $F(xw,y,z,u)=D(x,y,z,u)\sigma(w)+ \tau(x)F(w,y,z,u)$.), for all $x,y,w,z\in R$. Also F is said to be a symmetric 4- (σ, τ)-generalized derivation of R associated with D if it is both a symmetric 4-right (resp. left) (σ, τ)-generalized derivation of R associated with D.

Throughout this paper, we shall make use of the some basic commutator identities:

$[x,yz] =y[x,z] + [x,y]z$; $[xy,z] =[x,z]y+ x[y,z]$; $[x,y]_{\sigma,\tau}= x\sigma(y)- \tau(y)x$;

$(xoy)= x\sigma(y)+ \tau(y)x$; $[xy,z]_{\sigma,\tau}=x[y,z]_{\sigma,\tau}+ [x, \tau(z)]y = x[y, \sigma(z)]+ [x,z]_{\sigma,\tau}y$.

In this paper we prove that if f and d be the traces of F and D respectively satisfy one of the following .(i) $f([u,v])$-[$f(u),v]_{\sigma,\tau}$ =0; (ii) $f(uov)$ -[$f(u),v]_{\sigma,\tau}$ =0; (iii) $(f(u)ov)_{\sigma,\tau}$ -[$f(u),v]_{\sigma,\tau}$ =0; (iv) [$f(u),v]_{\sigma,\tau}$ -[u, $g(v)$] =0; (v) $f(u)of(v)$ -[$f(u),v]_{\sigma,\tau}$ =0; (vi) $f(u)\tau(v)$ -$ug(v)$ =0; (vii) $f(uv)$- $f(u)\sigma(v)$ - $\tau(u)d(v)$ =0, for all $u,v\in U$, then either $U\subseteq Z(R)$ or d=0

To prove our main result we will need the following lemmas:

Remark 1: Let U be a square closed lie ideal of R . Notice that $xy+yx=(x+y)^2-x^2-y^2$ for all $x,y\in U$. Since $x^2\in U$, for all $x\in U$, $xy+yx\in U$, for all $x,y\in U$. Hence we find that $2xy\in U$. Therefore, for all $r\in R$, we get $2r[x,y]=2[x,ry]-2[x,r]y\in U$ and $2[x,y]r=2[x,yr]-2y[x,r]$, so that $2R[U,U]\subseteq U$ and $2[U,U]R\subseteq U$

Lemma 1: Let R be a 3 and 2 -torsion free prime ring and U be a nonzero square closed lie ideal of R . Let $D:R^4\rightarrow R$ be a symmetric 4-(σ, τ)-derivation and d be the trace of D such that $d(U)=0$, then either d=0 or $U\subseteq Z(R)$.

Proof:

Suppose that $U\not\subseteq Z(R)$.

We have $d(x)=0$, for all $x\in U$. (1)

We replacing x by $x+y$ in equation (1), we get

$$d(x+y)=0$$

$d(x)+4D(x,x,x,y)+6D(x,x,y,y)+4D(x,y,y,y)+d(y)=0$, for all $x,y\in U$. (2)

Using equation (1) in the equation (2), we get

$4D(x,x,x,y)+6D(x,x,y,y)+4D(x,y,y,y)=0$, for all $x,y\in U$. (3)

We replacing y by $-y$ in equation (3), we get

$-4D(x,x,x,y)+6D(x,x,y,y)-4D(x,y,y,y)=0$, for all $x,y\in U$. (4)

We subtracting equation (4) form equation (3), we get

$$4D(x,x,x,y)+4D(x,y,y,y)=0, \text{ for all } x,y\in U . \quad (5)$$

We replacing y by $y+z$ in equation (5)and Using equation (5), we get

$$12D(x,y,z,z)+ 12D(x,y,y,z)=0, \text{ for all } x,y,z \in U . \quad (6)$$

We replacing z by $-z$ in equation (6) and compare with (6), we get

$$12(x,,y,z)=0, \text{ for all } x,y,z \in U. \quad (7)$$

We replacing y by $y+u$ in equation (7)and Using equation (7), we get

$$24(x,,u,z)=0, \text{ for all } x,y,z ,u\in U. \quad (8)$$

Since is a 3 and2 -torsion free ring, we get

$$(x,,u,z)=0, \text{ for all } x,y,z,u \in U. \quad (9)$$

We replacing x by $2xv,v\in R$, and using is a2 -torsion free ring in equation (9), we get

$$(xv,,u,z)=0, \text{ for all } x,y,z,u \in U.$$

$$D(x,y,u,z)\boldsymbol{\sigma}(v) + \boldsymbol{\tau}(x) D(v,y,u,z)=0, \text{ for all } x,y,z,u \in U,v\in R. \quad (10)$$

Using equation (9)in equation (10) , we get

$$\boldsymbol{\tau}(x) D(v,y,u,z)=0, \text{ for all } x,y,z,u \in U,v\in R. \quad (11)$$

We replacing x by xr ,$r\in R$ in equation (11), we get

$$\boldsymbol{\tau}(xr) D(v, y ,u,z)=0, \text{ for all } x,y,z,u \in U,v,r\in R. \quad (12)$$

We left multiply(11) by $\boldsymbol{\tau}(r)$ in equation (8), we get

$$\boldsymbol{\tau}(r) \boldsymbol{\tau}(x) D(v,y,u,z)=0, \text{ for all } x,y,z,u \in U,v,r\in R. \quad (13)$$

We subtracting equation (12) form equation (13), we get

$$\boldsymbol{\tau}(xr-rx) D(v,y,u,z)=0, \text{ for all } x,y,z,u \in U,v,r\in R.$$

$$\tau([x,r])\, D(v,y,u,z)=0, \text{ for all } x,y,z,u \in U, v,r \in R. \quad (14)$$

We replacing r by tr ,t∈R in equation (13), we get

$$(\,[x,tr])\,(v,y,u,z)=0$$

$$(\tau(t)\,\tau([x,r])+\tau([x,t])\,\tau(r))\,(v,y,u,z)=0$$

$$\tau(t)\;\tau([x,r])\,D(v,y,u,z)+\tau([x,t])\,\tau(r)\,D(v,y,u,z)=0$$

Using equation (14) in the above equation, we get

$$([x,t])\,\tau(r)\,(v,y,u,z)=0$$

$$\tau([x,t])\,R\,D(v,y,u,z)=0, \text{ for all } x,y,z,u \in U, v,t \in R. \quad (15)$$

Since R is prime, we get either $[x,t]=0$, for all $x\in U$, $t\in R$, or $D(v,y,u,z)=0$, for all $y,z,u \in U, v\in R$,

Since τ is an automorphism of R, we have either $[x,t]=0$, for all $x\in$U or $D(v,y,u,z)=0$, for all $y,z,u \in U$. Now let A= $\{x\in U/[x,t]=0\ t\in R,\}$ and B=$\{y,u,z \in U,\ D(v,y,u,z)=0$ for all $v\in R\}$. Clearly, A and B are additive proper subgroups of whose union is U . Since a group cannot be the set theoretic union of two proper subgroups. Hence either $A=U$ or $B=U$.

If $A=U$, then $[x,t]=0$, for all $x\in U$ and we get $U\subseteq Z(R)$ a contradiction. On the other hand if B=U , then $D(v,y,u,z)=0$, for all $y,z,u \in U$. If we do similar calculations, we get $D(v,s,t,n)=0$, for all $v,s,t,n \in R$, and so $D=0$, i.e.$d=0$. This completes the proof of the theorem.

Lemma 2: Let R be a 3 and 2 -torsion free prime ring and U be a nonzero square closed lie ideal of R . Let $F:R^4 \to R$ be a symmetric 4-(**σ**, **τ**)-generalized derivation associated with $D:R^4 \to R$ be a symmetric 4-(**σ**, **τ**)-derivation, f and d be the traces of F and D such that $f(U)=0$, then either $U\subseteq Z(R)$ or $d=0$.

Proof:

Suppose that $U\not\subseteq Z(R)$.

We have $f(x)=0$, for all $x\in U$. (16)

We replacing x by $x+y$ in equation (16), we get

$f(x+y)=0$

$f(x) + 4F(x,x,x,y) + 6F(x,x,y,y) + 4F(x,y,y,y) + f(y)$, for all $x,y \in U$.

Using equation (16) in above equation, we get

$4F(x,x,x,y) + 6F(x,x,y,y) + 4F(x,y,y,y)$ for all $x,y \in U$. (17)

We replacing y by $-y$ in equation (17), we get

$-4F(x,x,x,y) + 6F(x,x,y,y) - 4F(x,y,y,y)$ for all $x,y \in U$. (18)

We subtracting equation (18) form equation (17), we get

$$4F(x,x,x,y) + 4F(x,y,y,y)=0, \text{ for all } x,y \in U. \quad (19)$$

We replacing y by $y+z$ in equation (19)and Using equation (19), we get

$$12F(x,y,z,z)+ 12F(x,y,y,z)=0, \text{ for all } x,y,z \in U. \quad (20)$$

We replacing z by $-z$ in equation (20), we get

$$12F(x,y,y,z)=0, \text{ for all } x,y,z \in U. \quad (21)$$

We replacing y by $y+u$ in equation (21)and Using equation (21), we get

$$24F(x,y,u,z)=0, \text{ for all } x,y,z \in U.$$

Since is a3 and2 -torsion free ring, we get

$$F(x,y,u,z)=0, \text{ for all } x,y,z,u \in U. \quad (22)$$

We replacing x by $2xv, v \in R$, and using is a2 -torsion free ring in equation (22), we get

$$F(xv,y,u,z)=0, \text{ for all } x,y,z,u \in U.$$

$$F(x,y,u,z)\boldsymbol{\sigma}(v) + \boldsymbol{\tau}(x) D(v,y,u,z)=0, \text{ for all } x,y,z,u \in U, v \in R. \quad (23)$$

Using equation (22)in equation (23) , we get

$$\tau(x)\, D(v,y,u,z)=0, \text{ for all } x,y,z,u \in U, v\in R. \quad (24)$$

The equation (24) is same as equation (11) in Lemma 1. Thus, by same argument of Lemma 1, we can conclude the result are either $d=0$ or $U\subseteq Z(R)$.

Lemma 3: Let R be a 3 and 2 -torsion free prime ring and U be a nonzero square closed Lie ideal of R . Let $F:R^4 \to R$ be a symmetric 4-(**σ**, **τ**)-generalized derivation associated with $D:R^4 \to R$ be a symmetric 4-(**σ**, **τ**)-derivation, f and d be the traces of F and D such that $[f(x),y]_{\sigma,\tau}=0$ for all $x,y\in U$, then either $U\subseteq Z(R)$ or $d=0$.

Proof:

Suppose that U⊈Z(R).

We have

$$[f(x),y]_{\sigma,\tau}=0 \text{ for all } x,y\in U \quad (25)$$

We replacing x by $x+z$ in equation (25), we get

$[f(x+z),y]_{\sigma,\tau}=0$ for all $x,y\in U.$

$[f(x) + 4F(x,x,x,z) + 6F(x,x,z,z) + 4F(x,z,z,z) + f(z),y]_{\sigma,\tau}=0$ for all $x,y,z\in U$.

$[f(x),y]_{\sigma,\tau} + 4[F(x,x,x,z),y]_{\sigma,\tau} + 6[F(x,x,z,z),y] + 4[F(x,z,z,z),y]_{\sigma,\tau} +[f(z),y]_{\sigma,\tau}=0$ for all $x,y,z\in U$

Using equation (25) and using R is a2 -3torsion free ring in the above equation, we get

$$[F(x,x,x,z),y]_{\sigma,\tau} + [F(x,x,z,z),y]_{\sigma,\tau} + [F(x,z,z,z),y]_{\sigma,\tau} =0 \text{ for all } x,y,z\in U. \quad (26)$$

We replacing z by $-z$ in equation (26) and compare with(26) , we get

$$[F(x,x,x,z),y]_{\sigma,\tau} + [F(x,z,z,z),y]_{\sigma,\tau} =0 \text{ for all } x,y,z\in U. \quad (27)$$

We replacing z by $z+w$ in equation (27), and using (27) we get

$$3[F(x,z,w,w),y]_{\sigma,\tau} +3[F(x,z,z,w),y]_{\sigma,\tau} =0 \text{ for all } x,y,z,w\in U. \quad (28)$$

We replacing z by $-z$ in equation (28), and compare with(28) we get

$$3[F(x,z,w,w),y]_{\sigma,\tau}=0 \text{ for all } x,y,z,w\in U. \quad (29)$$

We replacing w by $w+u$ in equation (29), we get

$$6[F(x,z,w,u),y]_{\sigma,\tau}=0 \text{ for all } x,y,z,w,u\in U. \quad (30)$$

We replacing x by $2xv, v\in$R , and using is a 3,2 -torsion free ring in equation (9), we get

$$[F(xv,z,w,u),y]_{\sigma,}=0, \text{ for all } x,y,z,u,w,v\in U.$$

$$[F(x,z,w,u)\sigma(v)+\tau(x)D(v,z,w,u),y]_{\sigma,\tau}=0, \text{ for all } x,y,z,u,w,v\in U.$$

$$[F(x,z,w,u)\sigma(v),y]_{\sigma,\tau}+[\tau(x)D(v,z,w,u),y]_{\sigma,\tau}=0, \text{ for all } x,y,z,u,v\in U.$$

$F(x,z,w,u)[\sigma(v),\sigma(y)]+[F(x,z,w,u),y]_{\sigma,\tau}\sigma(v)+\tau(x)[D(v,z,w,u),y]_{\sigma,\tau}+[\tau(x),\tau(y)]D(v,z,w,u)=0$, for all $x,y,z,u,v,w\in U$.

We replacing v by y and using equation (30) in the above equation, we get

$$\tau(x)[D(y,z,w,u),y]_{\sigma,\tau}+[\tau(x),\tau(y)]D(y,z,w,u)=0, \text{ for all } x,y,z,u,w\in U. \quad (31)$$

We replacing x by rx, $r\in R$, in equation (31), we get

$\tau(rx)[D(y,z,w,u),y]_{\sigma,\tau}+[\tau(rx),\tau(y)]D(y,z,w,u)=0$, for all $x,y,z,u,w\in U, r\in R$.

$\tau(r)\tau(x)[D(y,z,w,u),y]_{\sigma,\tau}+\tau(r)[\tau(x),\tau(y)]D(y,z,w,u)+[\tau(r),\tau(y)]\tau(x)D(y,z,w,u)=0$, for all $x\, y,z,u,w\in U, r\in R$. (32)

Left multiplying by $\tau(r)$ in equation (31), we get

$\tau(r)\tau(x)[D(y\, z,w,u),y]_{\sigma,\tau}+\tau(r)[\tau(x),\tau(y)]D(y,z,w,u)=0$, for all $x,y,z,u,w\in U$, $r\in R$. (33)

We subtracting equation (32) from equation (33), we get

$$[\tau(r),\tau(y)]\tau(x)D(y,z,w,u)=0, \text{ for all } x,y,z,u,w\in U, r\in R.$$

We replacing x by sx, $s\in R$, in the above equation, we get

$$[\tau(r), \tau(y)]\, \tau(s)\ (x)\, D(y, z,w,u) = 0$$

$$[\tau(r), \tau(y)]\, R\ (x)\, D(y, z,w,u) = 0, \text{ for all } x ,y,z,u, w \in U, r\in R.$$

Since R is prime, we get either $\tau([r,y]) = 0$, for all $y\in U$, $r\in R$, or $\tau(x)\, D(y, z,w,u) = 0$, for all $x ,y,z,u, w \in U$.

Since is an automorphism of R, we have either $[r,y] = 0$, for all $y \in U$ or $\tau(x)\, D(y, z,w,u) = 0$, for all $x ,y,z,u, w \in U$. If $[r,y] = 0$, for all $y \in U$ and we get $U\subseteq Z(R)$ a contradiction. On the other hand, if

$$\tau(x)\, D(y, z,w,u) = 0, \text{ for all } x ,y,z,u, w \in U, \tag{34}$$

We replacing x by xr, in equation (34), we get

$$\tau(xr)\, D(y, z,w,u) = 0, \text{ for all } x ,y,z,u, w \in U, , r\in R. \tag{35}$$

Left multiplying by $\tau(r)$ in equation (34), we get

$$\tau(r)\, \tau(x)\, D(y, z,w,u) = 0, \text{ for all } x ,y,z,u, w \in U, , r\in R. \tag{36}$$

We subtracting equation (36) from equation (35), we get

$$\tau([x, r])\, D(y, z,w,u) = 0, \text{ for all } x ,y,z,u, w \in U, r\in R. \tag{37}$$

We replacing r by sr, $s\in R$, in equation (28), we get

$$\tau([x,s r])\, D(y, z,w,u) = 0, \text{ for all } x ,y,z,u, w \in U, r\in R.$$

$$\tau(s)\tau([x, r])D(y, z,w,u)+ \tau([x, s])\, \tau(r)\, D(y, z,w,u) = 0, \text{ for all } x ,y,z,u, w \in U, r,s\in R.$$

Using equation (37) in the above equation, we get

$$\tau([x, s])\, \tau(r)\, D(y, z,w,u) = 0$$

$$\tau([x, s])\, R\, D(y, z,w,u) = 0, \text{ for all } x ,y,z,u, w \in U, s\in R. \tag{38}$$

Since R is prime, we get either $\tau([x, s])$,=0 , $x \in U$, $s \in R$, or , $D(y, z,w,u) = 0$, for all $y,z,u, w \in U$. Since τ is an automorphism of R , we have either $[x, s] = 0$, $x \in U$, or , $D(y, z,w,u) = 0$, for all ,$y,z,u, w \in U$. If $[x, s] = 0$,for all $x \in U$ and we get U⊆Z(R) a contradiction.

On the other hand, if , $D(y, z,w,u) = 0$, for all $y,z,u, w \in U$. (39)

The equation (39) is same as equation (9) in lemma 1. Thus, by same argument of lemma 1, we can conclude the result are either $U \subseteq Z(R)$ or $d=0$.

Theorem 1: Let R be a 3 and 2 -torsion free prime ring and U be a nonzero square closed lie ideal of R . Let $F{:}R^4 \to R$ be a symmetric 4-(σ, τ)-generalized derivation associated with $D{:}R^4 \to R$ be a symmetric 4-(σ, τ)-derivation, f and d be the traces of F and D such that $f([u,v])-[f(u),v]_{\sigma,\tau} = 0$, for all $u,v \in U$, then either $U \subseteq Z(R)$ or $d=0$.

Proof:

Suppose that $U \not\subseteq Z(R)$.

We have

$$f([u,v])-[f(u),v]_{\sigma,\tau} = 0, \text{ for all } u,v \in U. \quad (40)$$

We substitute v by $v+w$ in equation (40), we get

$$f([u,v+w])-[f(u),v+w]_{\sigma,\tau} = 0, \text{ for all } u,v \in U.$$

$$f([u,v]+ [u,w])-[f(u),v+w]_{\sigma,\tau} = 0, \text{ for all } u,v \in U.$$

$f([u,v])+ 4F([u,v], [u,v], [u,v], [u,w])+ 6F([u,v], [u,v], [u,w], [u,w])+ 4F([u,v], ([u,w], ([u,w], [u,w])+f([u,w])-[f(u), v]_{\sigma,\tau} -[f(u), +w]_{\sigma,\tau} = 0$, for all $u,v,w \in U$.

Using equation (40) in the above equation, we get

$4F([u,v], [u,v], [u,v], [u,w])+ 6F([u,v], [u,v], [u,w], [u,w])+ 4F([u,v], ([u,w], ([u,w], [u,w]) = 0$ for all $u,v,w \in U$.

Since is a 2-torsion free ring, we get

$F([u,v], [u,v], [u,v], [u,w])+F([u,v], [u,v], [u,w], [u,w])+F([u,v], [u,w], [u,w], [u,w]) = 0$ for all $u,v,w \in U$.

We substitute w by v in the above equation, we get

$$3F([u,v],[u,v],[u,v],[u,v])=0 \text{ ,for all } u,v\in U.$$

Since be a 3 -torsion free ring, we get

$$F([u,v],[u,v],[u,v],[u,v])=0, \text{ for all } u,v\in U.$$

$$f([u,v])=0 \text{ ,for all } u,v\in U. \quad (41)$$

We subtracting equation (41) form equation (40), we get

$$[f(u),v]_{\sigma,\tau}=0 \text{ for all } u,v\in U.$$

By using lemma 3, we get the required result are either $U\subseteq Z(R)$ or d=0 .

Theorem 2: Let R be a 3 and 2 -torsion free prime ring and U be a nonzero square closed lie ideal of R . Let $F:R^4\rightarrow R$ be a symmetric 4-(**σ**, **τ**)-generalized derivation associated with $D:R^4\rightarrow R$ be a symmetric 4-(**σ**, **τ**)-derivation, f and d be the traces of F and D such that $f(uov)-[f(u),v]_{\sigma,\tau}=0$, for all $u,v\in U$, then either $U\subseteq Z(R)$ or d=0 .

Proof:

Suppose that $U\nsubseteq Z(R)$.

We have

$$f(uov)-[f(u),v]_{\sigma,\tau}=0, \text{ for all } u,v\in U. \quad (42)$$

We substitute v by $v+w$ in equation (42), we get

$$f(u\,o(v+w))-[f(u),v+w]_{\sigma,\tau}=0, \text{ for all } u,v,w\in U.$$

$$f((uov)+(uow))-[f(u),v+w]_{\sigma,\tau}=0, \text{ for all } u,v,w\in U.$$

$f(uov)+4F((uov),(uov),(uov),(uow))+6F((uov),(uov),(uow),(uow))+4F((uov),(uow),(uow),(uow))+f(uow)-[f(u),v]_{\sigma,\tau}-[f(u),w]_{\sigma,\tau}=0$, for all $u,v,w\in U$.

Using equation (42) in the above equation, we get
$4F((uov),(uov),(uov),(uow))+6F((uov),(uov),(uow),(uow))+4F((uov),(uow),(uow),(uow))=0$, for all $u,v,w\in U$.

Since is a 3-2torsion free ring, we get

$F((uov), (uov), (uov), (uow))+F((uov), (uov), (uow), (uow))+F((uov), (uow), (uow), (uow))=0$, for all $u,v,w\in U$.

We substitute w by v in the above equation, we get

$$3F((uov), (uov), (uov), (uov))=0 \text{ ,for all } u,v,w \in U.$$

Since be a3 -2torsion free ring, we get

$$F((uov), (uov), (uov), (uov)) =0, \text{ for all } u,v,w \in U.$$

$$f(uov)=0 \text{ ,for all } u,v \in U. \quad (43)$$

We subtracting equation (42) form equation (43), we get

$$[f(u),v]_{\sigma,\tau}=0 \text{ for all } u,v\in.$$

By using lemma 3, we get the required result are either $U\subseteq Z(R)$ or d=0 .

Theorem 3: Let R be a 3 and 2 -torsion free prime ring and U be a nonzero square closed lie ideal of R . Let $F:R^4\rightarrow R$ be a symmetric 4-(**σ**, **τ**)-generalized derivation associated with $D:R^4\rightarrow R$ be a symmetric 4-(**σ**, **τ**)-derivation, f and d be the traces of F and D such that $f(u)ov)_{\sigma,\tau}$ -$[f(u),v]_{\sigma,\tau}$ =0, for all $u,v\in U$, then either $U\subseteq Z(R)$ or d=0.

Proof:

Suppose that $U\not\subseteq Z(R)$.

We have

$$f(u)ov)_{\sigma,\tau}-[f(u),v]_{\sigma,\tau}=0, \text{ for all } u,v\in U . \quad (44)$$

$$f(u)\,\sigma(v)+\tau(v)f(u)-(f(u)\,\sigma(v)-\tau(v)f(u))=0, \text{ for all } u,v\in U .$$

$$2(v)f(u)=0, \text{ for all } u,v\in U .$$

Since be a 2-torsion free ring, we get

$$(v)f(u)=0, \text{ for all } u,v\in U . \quad (45)$$

We substitute v by vt in the above equation, we get

$$(vt)f(u)=0, \text{ for all } u,v\in U,t\in R. \quad (46)$$

Left multiplying by (t) in equation (45), we get

$$(t)(v)f(u)=0, \text{ for all } u,v\in U,t\in R. \quad (47)$$

We subtracting equation (47) form equation (46), we get

$$([v,t])f(u)=0, \text{ for all } u,v\in U,t\in R. \quad (48)$$

We substitute t by st in the above equation, we get

$$([v,st])f(u)=0, \text{ for all } u,v\in U,t,s\in R.$$

$$\boldsymbol{\tau}(s)\,\boldsymbol{\tau}([v,t])f(u)+\boldsymbol{\tau}([v,s])\,\boldsymbol{\tau}(t)f(u)=0, \text{ for all } u,v\in U,t,s\in R.$$

Using equation (33) in the above equation, we get

$$([v,s])(t)f(u)=0.$$

$$([v,s])Rf(u)=0, \text{ for all } u,v\in U,s\in R. \quad (49)$$

Since R is prime, we get either $\boldsymbol{\tau}([v,s])=0$, for all $v\in U,s\in R$, or $f(u)=0$, for all $u\in U$.

Since $\boldsymbol{\tau}$ is an automorphism of R , we have either $[v,s]=0$, for all $u\in$U or $f(u)=0$, for all $u\in U$. Now let $A=\{v\in U/[v,s]=0, s\in R\}$ and $B=\{u\in U/ f(u)=0\}$. Clearly, A and B are additive proper subgroups of whose union is U . Since a group cannot be the set theoretic union of two proper subgroups. Hence either $A=U$ or $B=U$.
If $A=U$, then $[v, s]=0$, for all $v\in U$ and we get $U\subseteq Z(R)$ a contradiction. On the other hand if $B=U$, then $f(u)=0$, for all $u\in U$, then by lemma(2) we get the required result are either $U\subseteq Z(R)$ or $d=0$.

Theorem 4: Let R be a 3,2 -torsion free prime ring, U be a nonzero square closed lie ideal of R and $g:R\rightarrow R$ is any mapping. Let $F:R^4\rightarrow R$ be a symmetric 4-($\boldsymbol{\sigma}$, $\boldsymbol{\tau}$)-generalized derivation associated with $D:R^4\rightarrow R$ be a symmetric 4-($\boldsymbol{\sigma}$, $\boldsymbol{\tau}$)-derivation, and f and d be the traces of F and D such that $[f(u),v]_{\boldsymbol{\sigma},\boldsymbol{\tau}}-[u, g(v)]=0$, for all $u,v\in U$, then either U⊆Z(R) or d=0 .

Proof:

Suppose that U⊈Z(R).

We have

$$[f(u),v]_{\sigma,\tau} - [u, g(v)] = 0, \text{ for all } u,v \in U. \quad (50)$$

We substitute u by $u+w$ in equation (50), we get

We have

$$[f(u+w),v]_{\sigma,\tau} - [u+w, g(v)] = 0, \text{ for all } u,v,w \in U.$$

$[f(u) + 4F(u, u, u,w) + 6F(u, u, w, w) + 4F(u, w,w,w) + f(w),v]_{\sigma,\tau} - [u, g(v)] - [w, g(v)] = 0$, for all $u,v \in U$.

$[f(u),v]_{\sigma,\tau} + 4[F(u, u,u,w),v]_{\sigma,\tau} + 6[F(u, u, w, w),v]_{\sigma,\tau} + 4[F(u, w,w,w),v]_{\sigma,\tau} + [f(w),v]_{\sigma,\tau} - [u, g(v)] - [u, g(v)] = 0$, for all $u,v,w \in U$.

Using equation (50) in the above equation, we get

$4[F(u, u,u,w),v]_{\sigma,\tau} + 6[F(u, u, w, w),v]_{\sigma,\tau} + 4[F(u, w,w,w),v]_{\sigma,\tau} = 0$, for all $u,v,w \in U$.

Since R is a 2,3 -torsion free ring, we get

$$[F(u, u,u,w),v]_{\sigma,\tau} + [F(u, u, w, w),v]_{\sigma,\tau} + [F(u, w,w,w),v]_{\sigma,\tau} = 0, \text{ for all } u,v,w \in U. \quad (51)$$

The equation (51) is same as equation (26) in lemma 3. by same argument of lemma 3, we can conclude the result are either $U \subseteq Z(R)$ or $d=0$.

Theorem 5: Let R be a 3 and 2 -torsion free prime ring and U be a nonzero square closed lie ideal of R . Let $F:R^4 \to R$ be a symmetric 4-(σ, τ)-generalized derivation associated with $D:R^4 \to R$ be a symmetric 4-(σ, τ)-derivation, f and d be the traces of F and D such that $f(u) \circ f(v) - [f(u),v]_{\sigma,\tau} = 0$, for all $u,v \in U$, then either $U \subseteq Z(R)$ or $d=0$.

Proof: Suppose that $U \nsubseteq Z(R)$.

We have $f(u) o f(v) - [f(u),v]_{\sigma,\tau} = 0$, for all $u,v \in U$. (52)

We substitute v by $v+w$ in equation (51), we get

$$f(u) o f(v+w) - [f(u),v+w]_{\sigma,\tau} = 0,$$

$f(u) o (f(v) + 4F(v,v,v,w) + 6F(v,v,w,w) + 4F(v,w,w,w) + f(w)) - [f(u),v]_{\sigma,\tau} - [f(u),w]_{\sigma,\tau} = 0.$

$f(u) o f(v) + 4 f(u) o F(v, v, v,w) + 6 f(u) o F(v, v, w, w) + 4 f(u) o F(u,v,v,v) + f(u) o f(w)) - [f(u),v]_{\sigma,\tau} - [f(u),w]_{\sigma,\tau} = 0$, for all $u,v \in U$.

Using equation (52) in the above equation, we get

$4 f(u) o F(v, v, v,w) + 6 f(u) o F(v, v, w, w) + 4 f(u) o F(u,v,v,v) = 0$, for all $u,v \in U$.

Since R is a3,2 -torsion free ring, we get

$f(u) o F(v, v, v,w) + f(u) o F(v,v,w,w) + f(u) o F(u,v,v,v) = 0$, for all $u,v \in U$.

We substitute w by v in the above equation, we get

$$3f(u) o F(v, v, v,v) = 0, \text{ for all } u,v \in U.$$

Since is a 2,3 -torsion free ring, we get

$$f(u) o f(v) = 0, \text{ for all } u,v \in U. \quad (53)$$

We subtracting equation (52) form equation (53), we get

$$[f(u),v]_{\sigma,\tau} = 0, \text{ for all } u,v \in U.$$

By using lemma 3, we get the required result are either $U \subseteq Z(R)$ or d=0.

Theorem 6: Let R be a 3,2 -torsion free prime ring, U be a nonzero square closed lie ideal of R and $g:R \to R$ is any mapping. Let $F:R^4 \to R$ be a symmetric 4-($\boldsymbol{\sigma}$, $\boldsymbol{\tau}$)-generalized derivation associated with $D:R^4 \to R$ be a symmetric 4-($\boldsymbol{\sigma}$, $\boldsymbol{\tau}$)-derivation, and f and d be the traces of F and D such that $f(u)\boldsymbol{\tau}(v) - ug(v) = 0$, for all $u,v \in U$, then either $U \subseteq Z(R)$ or d=0.

Proof:

Suppose that $U \nsubseteq Z(R)$.

We have

$$f(u)\boldsymbol{\tau}(v) - u\, g(v) = 0, \text{ for all } u,v \in U . \quad (54)$$

We substitute u by $u+w$ in equation (44), we get

$f(u+w)\boldsymbol{\tau}(v) - (u+w)\, g(v) = 0$, for all $u,v,w \in U$.

$(f(u) + 4F(u, u, u,w) + 6F(u, u, w, w) + 4F(u, w,w,w) + f(w))\,(v) - ug(v) - wg(v) = 0$, for all $u,v,w \in U$.
$f(u)\boldsymbol{\tau}(v) + 4F(u, u, u,w)\boldsymbol{\tau}(v) + 6F(u, u,w, w)\boldsymbol{\tau}(v) + 4F(u, w,w,w)\boldsymbol{\tau}(v) + f(w)\boldsymbol{\tau}(v) - ug(v) - wg(v) = 0$, for all $u,v,w \in U$.

Using equation (54) in the above equation, we get

$4F(u, u, u,w)\boldsymbol{\tau}(v) + 6F(u, u,w, w)\boldsymbol{\tau}(v) + 4F(u, w,w,w)\boldsymbol{\tau}(v) = 0$, for all $u,v,w \in U$.

Since R be a 3,2 -torsion free ring, we get

$F(u,u,u,w)\boldsymbol{\tau}(v) + F(u, u,w,w)\boldsymbol{\tau}(v) + F(u,w,w,w)\boldsymbol{\tau}(v) = 0$, for all $u,v,w \in U$.

We substitute w by u in the above equation, we get

$$3F(u,u,u,u)\boldsymbol{\tau}(v) = 0, \text{ for all } u,v \in U.$$

Since is a 3 -torsion free ring, we get

$$f(u)\boldsymbol{\tau}(v) = 0, \text{ for all } u,v \in U. \quad (55)$$

We substitute v by tv in the above equation, we get

$$f(u)\boldsymbol{\tau}(vt) = 0, \text{ for all } u,v \in U, t \in R. \quad (56)$$

Right multiplying by (t) in equation (55), we get

$$f(u)\boldsymbol{\tau}(v)\,\boldsymbol{\tau}(t) = 0, \text{ for all } u,v \in U, t \in R. \quad (57)$$

We subtracting equation (56) form equation (57), we get

$$f(u)\boldsymbol{\tau}([v,t]) = 0, \text{ for all } u,v \in U, t \in R. \quad (58)$$

We substitute t by ts in the above equation, we get

$$f(u)\boldsymbol{\tau}([v,ts]) = 0$$

$$f(u)\boldsymbol{\tau}([v,t])\,\boldsymbol{\tau}(s) + f(u)\,\boldsymbol{\tau}(t)\,\boldsymbol{\tau}([v,s]) = 0 \text{ for all } u,v \in U, t,s \in R.$$

Using equation (58) in the above equation, we get

$$f(u)\,\boldsymbol{\tau}(t)\,\boldsymbol{\tau}([v,s]) = 0 \text{ for all } u,v \in U, t \in R.$$

$$f(u)\,R\,\boldsymbol{\tau}([v,t]) = 0 \text{ for all } u,v \in U, t,s \in R. \quad (59)$$

The equation (59) is same as equation (49) in theorem 3. Thus, by same argument of theorem 3, we get the required result.

Theorem 7: Let R be a 3 and2 -torsion free prime ring and U be a nonzero square closed lie ideal of R . Let $F{:}R^4 \rightarrow R$ be a symmetric 4-(**σ**, **τ**)-generalized derivation associated with $D{:}R^4 \rightarrow R$ be a symmetric 4-(**σ**, **τ**)- derivation, f and d be the traces of

F and D such that $f(uv)-f(u)\,\boldsymbol{\sigma}(v) - \boldsymbol{\tau}(u)d(v)=0$, for all $u,v\in U$, then either $U\subseteq Z(R)$ or d=0.

Proof:

Suppose that $U\nsubseteq Z(R)$.

We have

$$f(uv)-f(u)\,\boldsymbol{\sigma}(v) - \boldsymbol{\tau}(u)d(v)=0, \text{ for all } u,v\in U. \quad (60)$$

We substitute u by $u+w$ in equation (60), we get

$f((u+w)v)-f(u+w)\,\boldsymbol{\sigma}(v) - \boldsymbol{\tau}(u+w)d(v)=0$, for all $u,v,w\in U$.

$f((uv+wv))-f(u+w)\,\boldsymbol{\sigma}(v) - \boldsymbol{\tau}(u+w)d(v)=0$

$f(uv)+4F(uv, uv, uv,wv)+6F(uv, uv, wv, wv)+4F(uv, wv,wv,wv)+f(wv)-(f(u)+4F(u, u, u,w)+6F(u, u, w, w)+4F(u, w,w,w)+f(w))\,\boldsymbol{\sigma}(v)-\boldsymbol{\tau}(u)d(v)-\boldsymbol{\tau}(w)d(v)=0$, for all $u,v,w\in U$.

$f(uv)-f(u)\boldsymbol{\sigma}(v)-\boldsymbol{\tau}(u)d(v)+4F(uv, uv, uv,wv)+6F(uv, uv, wv, wv)+4F(uv, wv,wv,wv)+f(wv)-f(w)\,\boldsymbol{\sigma}(v)-\boldsymbol{\tau}(w)d(v)-4F(u, u,u,w)\,\boldsymbol{\sigma}(v)-6F(u, u,w,w)\boldsymbol{\sigma}(v)-4F(u,w,w,w)\boldsymbol{\sigma}(v)=0$, for all $u,v,w\in U$.

Using equation (60) in the above equation, we get

$4F(uv, uv, uv,wv)+6F(uv, uv, wv, wv)+4F(uv, wv,wv,wv)-4F(u, u, u,w)\,\boldsymbol{\sigma}(v)-6F(u, u, w, w)\,\boldsymbol{\sigma}(v)-4F(u, w,w,w)\,\boldsymbol{\sigma}(v)=0$, for all $u,v\in U$.

Since is a3,2 -torsion free ring, we get

$F(uv, uv, uv,wv)+F(uv, uv, wv, wv)+F(uv, wv,wv,wv)-F(u, u, u,w)\,\boldsymbol{\sigma}(v)-F(u, u, w, w)\,\boldsymbol{\sigma}(v)-F(u, w,w,w)\,\boldsymbol{\sigma}(v)=0$, for all $u,v\in U$.

We substitute w by u in the above equation, we get

$$3(f(uv)-f(u)\boldsymbol{\sigma}(v))=0, \text{ for all } u,v\in U.$$

Since R is a 3 -torsion free ring, we get

$$f(uv)- f(u)\boldsymbol{\sigma}(v) =0, \text{ for all } u,v\in U. \quad (61)$$

We substitute v by $v+w$ in equation (61), we get

$$f(u(v+w)- f(u)\boldsymbol{\sigma}(v+w) =0.$$

$f(uv) +4F(uv, uv, uv,wv) + 6F(uv, uv, wv, wv)+ 4F(uv, wv,wv,wv) + f(wv)- f(u)\boldsymbol{\sigma}(v) - f(u)\boldsymbol{\sigma}(w) =0$

Using equation (61) in the above equation, we get

$4F(uv, uv, uv,wv) + 6F(uv, uv, wv, wv)+ 4F(uv, wv,wv,wv) =0$, for all $u,v,w\in U$.

Since is a2,3 -torsion free ring, we get

$F(uv, uv, uv,wv) + F(uv, uv, wv, wv)+ F(uv, wv,wv,wv) =0$, for all $u,v,w\in U$.

We substitute w by v in the above equation, we get

$$f(uv)=0 , \text{ for all } u,v\in U. \quad (62)$$

We subtracting equation (61) from equation (62), we get

$$f(u)\boldsymbol{\sigma}(v) =0, \text{ for all } u,v\in U. \quad (63)$$

The equation (63) is same as equation (53) in theorem 6. Thus, by same argument of theorem 6, we get the required result.

REFERENCES

[1] Bell,H.E., Mason, G.: On derivations in near-rings and near-fields, North-Holland, Math. Studies, 137(1987), 31-35.

[2] Bresar,M.: Community Maps, a survey, Taiwanese, J. Math., 8(3)(2004), 361-397.

[3] Ceven,Y.,Ozturk,M.A.: Some properties of symmetric bi- -derivations in near-rings, Commun. Korean Math. Soc., 22(4)(2007), 487-491.

[4] Durna,H.,Oguz,S.: Permuting tri-derivations in prime and semi-prime rings, International Journal of Algebra and Statistics, Vol.5(1)(2016), 52-58.

[5] Ozturk,M.A.: Permuting tri-derivations in prime and semi-prime rings, East Asian Math. J., 15(2)(1999), 177-190.

[6] Ozturk,M.A.,Yazarli,H.: A note on permuting tri-derivation in near ring, Gazi University Journal of Science., 24(4)(2011), 723-729.

[7] Park,K.H.,Jung,Y.S.: On permuting tri-derivations and commutativity in prime near rings, Commun. Korean Math. Soc., 25(1)(2010), 1-9.

[8] Uckun,M.,Ozturk,M.A.: On the trace of symmetric bi-gamma-derivations in gamma-near-rings, Houston. Math., 33(2)(2007), 323-339.

[9] FaizaShujat, Abuzaid Ansari:Symmetric 4-derivationson Prime rings, J. Math. Comput. Sci. 4(2014), No. 4, 649-656.

[10] C.Haetinger, Higher Derivations on Lie Ideals,Tendˆencias em Matem´atica Aplicada e Computacional, 3, No. 1 (2002), 141-145.