

Deleting Components and Regions for Gray Images using Symmetric group S_n

Professor Hind Rustum Mohammed

*Computer Science Department,
Faculty of Computer Science and mathematics/ University of Kufa
E-mail: hindrustum.shaaban@uokufa.edu.iq*

Maani Abdul Munem Saeed

*Department of mathematics/Faculty of Computer Science and mathematics/
University of Kufa
E-mail: maani.saeed@uokufa.edu.iq*

Abstract

The paper deals with a new way to Gray Images Deleting Components and Regions Using Symmetric group S_n . The proposed system consists of two phases, Focus on finding the edges of the image .The second stage is the image encryption phase. The system was executed on a database of 50 gray images 256 x 256 with format .png and tif. The results effectively demonstrated the proposed coding system for images using Symmetric group S_n With 100% success rate.

Keywords: Gray Images, Images Components and Regions, edges detection image, Symmetric group S_n .

1. INTRODUCTION

Automatic symmetry detection has been a topic of interest for as long as computer vision has been in existence, though the primary interest had been on the detection of reflection symmetries [1]. Representation and character theories provide varied applications not only in other branches of mathematics but also in physics and chemistry.[2].

The traditional encryption algorithms used to encrypt images directly, it is not a good idea for two reasons. The first is the image size is often larger than text. Consequently, the traditional encryption algorithms need longer time to directly encrypt the image data, the second, is the decrypted text must be equal to the original text, but this requirement is not necessary for image data. Due to the characteristic of human perception, a decrypted image containing small distortion is usually acceptable [3]. The paper is organized as follows; Section 2: contain the discussion for many Theorems and Definitions of Symmetric group . Section 3: deals with algorithms and stages steps for the proposed method , section 4: gives the overview of results and last section 5: ends the paper with conclusion.

2. THEOREMS AND DEFINITIONS OF SYMMETRIC GROUP

In This section, we present some basic concepts of representation theory , character theory This section is devoted to study characters of finite group and class function.

Definition: [7]

Two elements g and h in G are said to be conjugate if

$$h = xgx^{-1} \text{ for some } x \in G.$$

This relation is an equivalence relation on G . The equivalence classes determined by this relation are referred to as the conjugate classes, denoted by $CL(g)$ to the conjugate class of the element g .

Theorem :[6]

Let G be a finite group then the number of non-equivalent irreducible matrix representations of a group G is equal to the number of conjugacy classes of G .

Theorem :[8]

Let T_1, T_2, \dots, T_h be all non-equivalent irreducible matrix representations of the group G with degrees n_1, n_2, \dots, n_k respectively, then $|G| = \sum_{i=1}^h n_i^2$, where k is the number of conjugacy classes of G .

Definition : [5]

Let T be a matrix representation of a group G over the field F , the character χ of a matrix representation T is the mapping $\chi: G \rightarrow F$ defined by $\chi(g) = \text{Tr}(T(g))$ refers to the trace of the matrix $T(g)$ (the sum of the main diagonal of $T(g)$). The degree of T is called the degree of χ .

Characters Table of Finite Groups:[7]

A finite group G has a finite number of conjugacy classes and a finite number of distinct k -irreducible representations, the group character of a group representation is constant on a conjugacy class $CL^\alpha, (1 \leq \alpha \leq k)$, the values of the characters can be written as a table known as the characters table which is denoted by $\equiv(G)$.

Thus, it is sufficient to record the value $\chi^i(g^\alpha)$, $i = 1, 2, \dots, k$ if $g^\alpha \in CL^\alpha$ denoting the number of elements in CL^α by h^α . We have the class equation $h_1 + h_2 + \dots + h_k = |G|$ and the degree of k distinct representations of G over C by $h^i ; i = 1, 2, \dots, k$.

Also, the size of the centralizer is $|C_G(CL^\alpha)| = |G| / h^\alpha = m^\alpha$. The table(1.1) presents a typical characters table, the body of the table is $K \times K$ square matrix whose rows are given by the irreducible characters and columns are given by the conjugacy classes.

The characters table $\equiv(G)$ is displayed as :

Table;(1): Characters table of finite group G .

CL_α	CL_1	CL_2	...	CL_α	...	CL_K
$ CL_\alpha $	h_1	h_2	...	h_α	...	h_K
$ C_G(CL_\alpha) $	m_1	m_2	...	m_α	...	m_K
χ_1	1	1	...	1	...	1
χ_2	n_2	$\chi_2(g_2)$...	$\chi_2(g_\alpha)$...	$\chi_2(g_K)$
χ_3	n_3	$\chi_3(g_2)$...	$\chi_3(g_\alpha)$...	$\chi_3(g_K)$
\vdots	\vdots	\vdots	\ddots	\vdots	\ddots	\vdots
χ_K	n_K	$\chi_K(g_2)$...	$\chi_K(g_\alpha)$...	$\chi_K(g_K)$

Example:

The symmetric group S_3 has three conjugacy classes namely $[(1)] = \{(1)\}$, $[(12)] = \{(12), (13), (23)\}$ and $[(123)] = \{(123), (132)\}$.

The S_3 has three non-equivalent irreducible representations.

1- $T_1(g) = 1, \forall g \in S_3 \dots$

2- $T_2(g) = \begin{cases} 1 & \text{If } g \text{ is even} \\ -1 & \text{If } g \text{ is odd} \end{cases}, \forall g \in S_3.$

3- $T_3(1) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, T_3(12) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, T_3(13) = \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix}$
 $T_3(23) = \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}, T_3(123) = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, T_3(132) = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}.$

If χ_1, χ_2 and χ_3 are the characters of T_1, T_2 and T_3 respectively,

then we have $\chi_1(g) = 1, \forall g \in S_3,$

$$\chi_2((1)) = \chi_2((123)) = \chi_2((132)) = 1, \quad \chi_2((12)) = \chi_2((13)) = \chi_2((23)) = -1 \text{ and}$$

$$\chi_3((1)) = 2, \quad \chi_3((123)) = \chi_3((132)) = -1, \quad \chi_3((12)) = \chi_3((13)) = \chi_3((23)) = 0$$

Then the character table of S_3 is :

	C_1	C_2	C_3
CL_α	(1)	(12)	(123)
$ CL_\alpha $	1	3	2
$\equiv (S_3) = C_G(CL_\alpha) $	6	2	3
χ_1	1	1	1
χ_2	1	-1	1
χ_3	2	0	-1

We check that : $n_1^2 + n_2^2 + n_3^2 = 1^2 + 1^2 + 2^2 = 6.$

3. THE PROPOSED METHOD

In this section we explained the implementation steps of the proposed system agencies:

1-we took a group of S_3 number of 3 elements! $A = 6$ is $S_3 = \{(1), (12), (13), (23), ((123), (132))\}$

2-Conjugacy classes were extracted as follows

.{(and $[(123)] = \{(123), (132)\}$, $\{(23), (13), (12)\} = [(12)]$, $\{(1)\} = [(1)]$ }

3-Since the number of conjugacy classes = 3 is the number of Characters = 3

4-Each element of the group is represented by its array and I will symbolize the representation by the symbol $T(g)$ where g is any element in S_3

5-Three equal representation (formerly equal representation) will be as follows .5

1- $T_1(g) = 1, \forall g \in S_3 \dots$

2- $T_2(g) = \begin{cases} 1 & \text{If } g \text{ is even} \\ -1 & \text{If } g \text{ is odd} \end{cases}, \forall g \in S_3 .$

3- $T_3(1) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, T_3(12) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, T_3(13) = \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix}$
 $T_3(23) = \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}, T_3(123) = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, T_3(132) = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} .$

6-The characters table, whose elements are derived from $T(g)$, are created as follows

If χ_1, χ_2 and χ_3 are the characters of T_1, T_2 and T_3 respectively,

then we have $\chi_1(g) = 1, \forall g \in S_3,$

$\chi_2((1)) = \chi_2((123)) = \chi_2((132)) = 1, \chi_2((12)) = \chi_2((13)) = \chi_2((23)) = -1$ and

$\chi_3((1)) = 2, \chi_3((123)) = \chi_3((132)) = -1, \chi_3((12)) = \chi_3((13)) = \chi_3((23)) = 0$

7- Create your filter to find the edges of the gray image and then repeat it again to get the desired encryption and in less time and faster

1	1	1
1	-1	1
2	0	-1

4. EXPERIMENTAL RESULTS

In this section, the results will be discussed .The system was executed on a database of 50 gray images 256 x 256 with format .png and tif. Figure (1) shown sample of gray images.

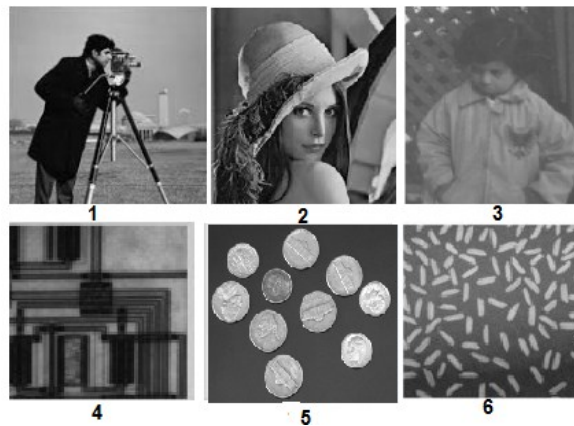


Figure (1): sample of gray images

Carried out processing discussed in section 4 where it turns out the proposed method in which we used special mathematical equations symmetric group using a character table for the filter you passed on the entire image.

Figure 1 shown Estimate the Background for image which that the background illumination is brighter in the center of the image than at the bottom use Open Morphological.

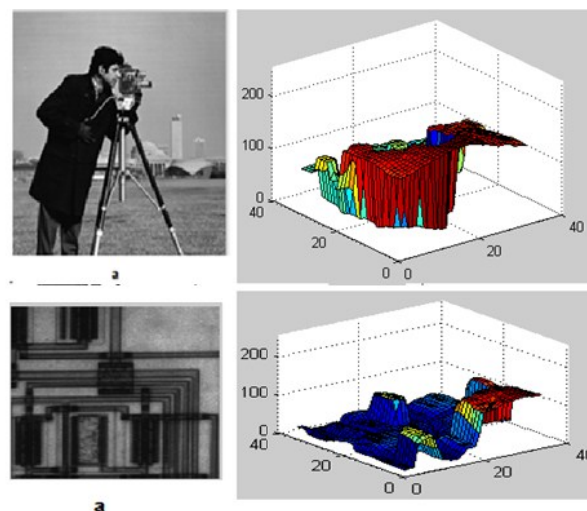


Figure 1: Estimate the Background

Figure 2 shown performed process to Deleting Large Regions for Gray Images using Symmetric group S_n

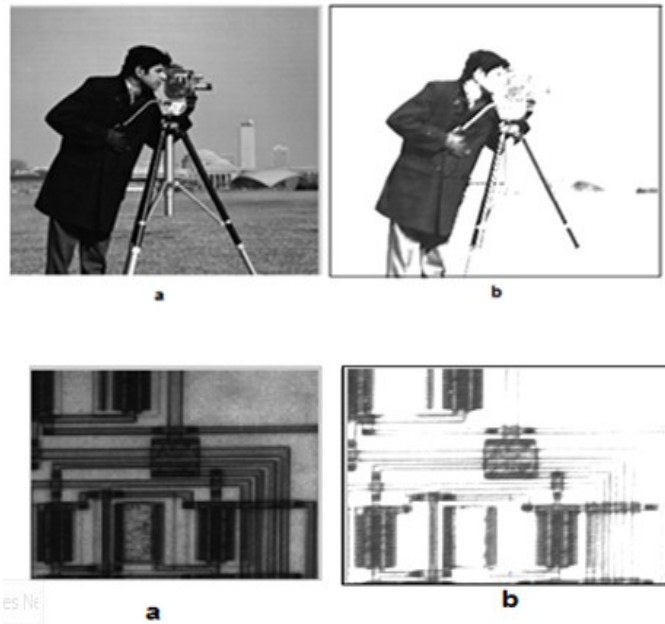


Figure 2 : Deleting Large Regions for Gray Images using Symmetric group S_n

Figure 3 shown performed process to Deleting Small Regions for Gray Images using Symmetric group S_n

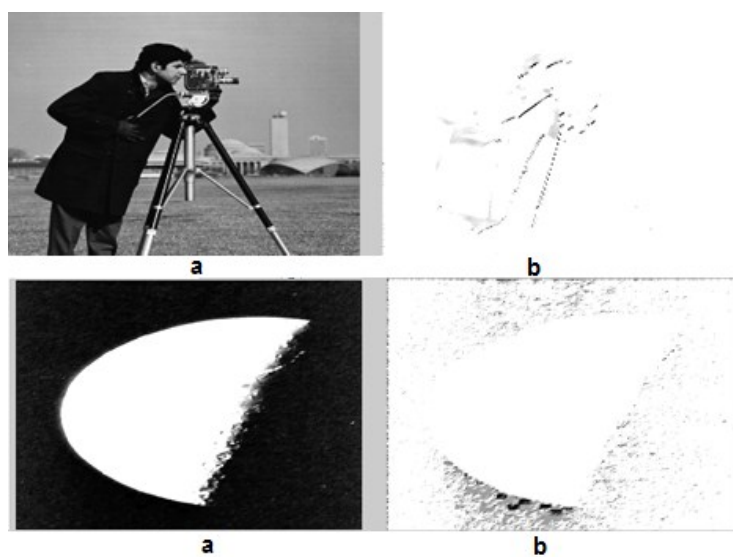


Figure 3 : Deleting Small Regions for Gray Images using Symmetric group S_n

Figures 4 and 5 shown performed process to Estimate the Background Deleting large and (Components & Small)Regions for Gray Images using Symmetric group S_n

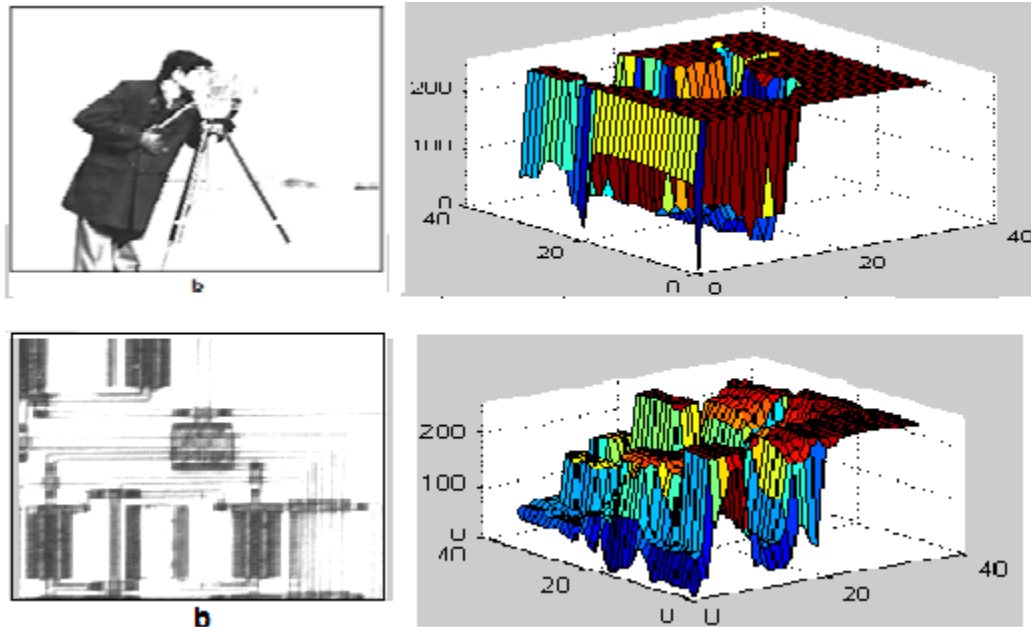


Figure 4 : Estimate the Background large Regions for Gray Images using Symmetric group S_n

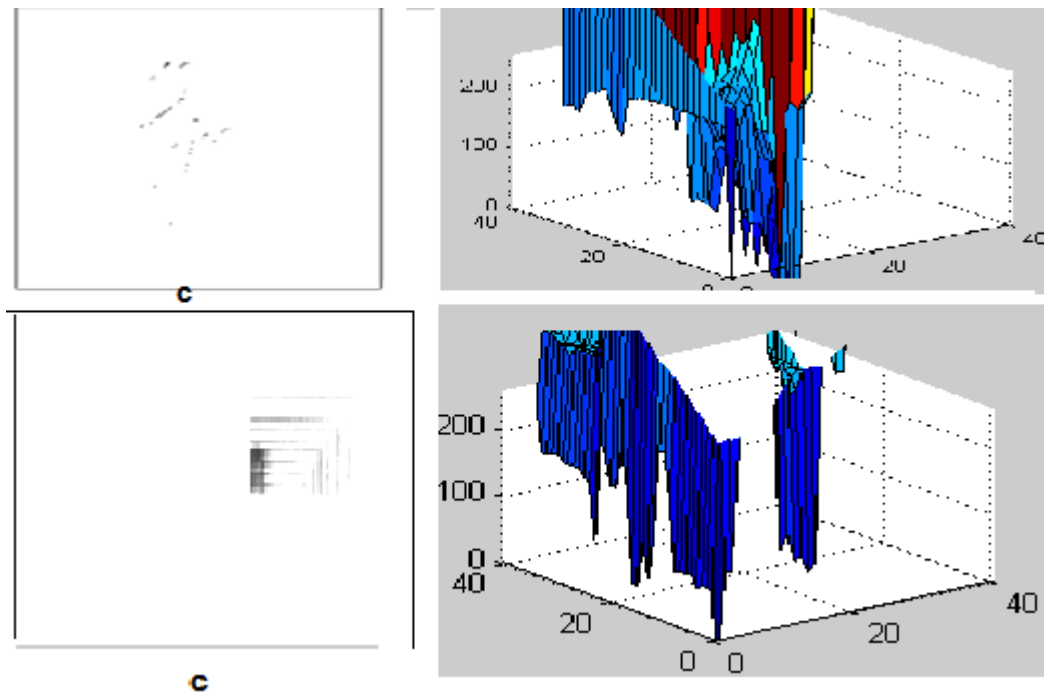


Figure 5: Estimate the Background Components and Small Regions for Gray Images using Symmetric group S_n

Figure 6 shown performed process to Deleting Components and Small Regions for Gray Images using Symmetric group S_n

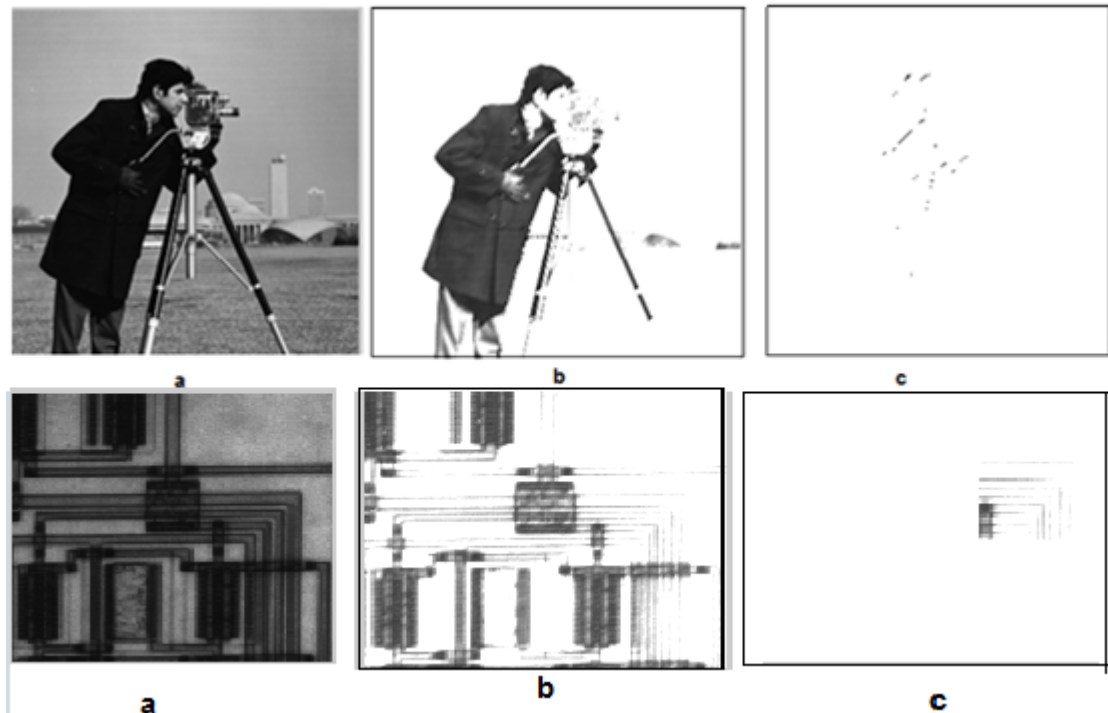


Figure 6: Deleting Components and Small Regions for Gray Images using Symmetric group S_n

5. CONCLUSION

In this work we used a symmetric group and we represented the elements of the group and extracted the table of the characteristics of the team and from the character table we used the sample matrix as a mask to encrypt the images in this research..

Note Symmetric group using a character table Its efficiency in deleting and hiding components and regions in gray images is very high.

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