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Mathematical Wavelet Compression by Using Dimensions in Haar Technique

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Abstract

Wavelets are mathematical tools for hierarchically decomposing functions. Wavelet Transform has been proved to be a very useful tool for image processing in recent years. It allows a function which may be described in terms of a coarse overall shape, plus details that range from broad to narrow. This paper describes the design and implementation of Dimensional Haar wavelet transform HWT with transpose based computation and dynamic partial reconfiguration. As a result of the separability property of the multidimensional HWT, the proposed architecture has been implemented using a cascade of three point HWT and two transpose memory, suitable for medical image compression. The most distinctive feature of Haar Transform lies in the fact that it lends itself easily to simple manual calculations. 3D Haar Wavelet Transform, is one of the algorithms which can reduce the calculation work in Haar Transform (HT) and Fast 3D Haar Transform.

1. INTRODUCTION TO WAVELET

In the course of recent years, the wavelet change has increased across the board acknowledgment in flag handling all in all and in picture pressure look into specifically. In applications, for example, still picture pressure, discrete wavelets change (DWT) based plans have bleated other coding plans like the ones in light of DCT. Since there is no compelling reason to partition the info picture into noncovering 2D pieces and its premise capacities have variable length, wavelet-coding plans at higher pressure proportions abstain from blocking antiques. On account of their innate multi resolution nature, wavelet-coding plans are particularly appropriate for applications where versatility and middle of the road debasement are imperative. As of late the JPEG council has discharged its new picture coding standard, JPEG-2000, which has been founded on DWT.

Essentially we utilize Wavelet Transform (WT) to investigate non-stationary signs, i.e., signals whose recurrence reaction shifts in time, as Fourier Transform (FT) is not reasonable for such flags. To beat the impediment of FT, Short Time Fourier Transform (STFT) was proposed. There is just a minor contrast amongst STFT and FT. In STFT, the flag is separated into little sections, where these fragments (segments) of the flag can be thought to be stationary. For this reason, a window work "w" is picked. The width of this window in time should be equivalent to the section of the flag where it is still be viewed as stationary. By STFT, one can get time-recurrence reaction of a flag all the while, which can't be acquired by FT. The brief span Fourier change for a genuine persistent flag is characterized as:

$$X(f, t) = \int_{-\infty}^{\infty} [x(t) w\ (t-\tau)^{*}] e^{-2j\pi f t} dt \qquad \text{------------ (2.1)}$$

Where the length of the window is (t-□) in time with the end goal that we can move the window by changing estimation of t, and by fluctuating the esteem □ we get distinctive recurrence reaction of the flag sections.

The Heisenberg instability standard clarifies the issue with STFT. This rule expresses that one can't know the correct time-recurrence portrayal of a flag, i.e., one can't recognize what ghostly parts exist at what examples of times. What one can know are the time interims in which certain band of frequencies exists and is called determination issue. This issue needs to do with the width of the window work that is utilized, known as the support of the window. In the event that the window capacity is restricted, then it is known as minimalistically upheld. The smaller we make the window, the better the time determination, and better the suspicion of the flag to be stationary, yet poorer the recurrence determination:

Limit window ===> great time determination, poor recurrence determination

Wide window ===> great recurrence determination, poor time determination

The wavelet change (WT) has been produced as a substitute way to deal with STFT to beat the determination issue. The wavelet investigation is done to such an extent that the flag is duplicated with the wavelet work, like the window work in the STFT, and the change is processed independently for various portions of the time-area motion at various frequencies. This approach is called Multi-determination Analysis (MRA) [4], as it dissects the flag at various frequencies giving diverse resolutions.

MRA is intended to give great time determination and poor recurrence determination at high frequencies and great recurrence determination and poor time determination at low frequencies. This approach is great particularly when the flag has high recurrence segments for brief terms and low recurrence segments for long lengths, e.g., pictures and video outlines.

The wavelet change includes anticipating a flag onto a total arrangement of interpreted and expanded forms of a mother wavelet $\Psi(t)$. The strict meaning of a mother wavelet will be managed later so that the type of the wavelet change can be analyzed first. For the present, accept the free necessity that $\Psi(t)$ has smaller transient and unearthly support (constrained by the instability standard obviously), whereupon set of premise capacities can be characterized.

The premise set of wavelets is created from the mother or essential wavelet is characterized as:

$$\Psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) \quad ; a, b \in \Re \text{ and } a>0 \quad \text{------------} \ (2.2)$$

The variable ‘a’ (inverse of frequency) reflects the scale (width) of a particular basis function such that its large value gives low frequencies and small value gives high frequencies. The variable ‘b’ specifies its translation along x-axis in time. The term $1/\sqrt{a}$ is used for normalization.

2. WAVELET TRANSFORM

In any case we are living in a universe of signs. Nature is conversing with us with signs: light, sounds… Men are conversing with each other with signs: music, TV, telephones… The human body is prepared to get by in this universe of signs with sensors, for example, eyes and ears, which can get and handle these signs. Consider, for example, our ears: they can segregate the volume and tone of a voice. The vast majority of the data our ears procedure from a flag is in the recurrence substance of the flag.

Researchers have created numerical techniques to mimic the preparing performed by our body and concentrate the recurrence data contained in a flag. These numerical calculations are called changes and the most prominent among them is the Fourier Transform. The second technique to break down non-stationary signs is to first channel distinctive recurrence groups, cut these groups into cuts in time, and after that examine them. The wavelet change utilizes this approach. The wavelet change or wavelet examination is presumably the latest answer for defeat the weaknesses of the Fourier change. In wavelet investigation the utilization of a completely versatile adjusted window takes care of the flag cutting issue. The window is moved along the flag and for each position the range is figured. Then this process is rehashed ordinarily with a marginally shorter (or more) window for each new cycle.

At last the outcome is a gathering of time-recurrence portrayals of the flag, all with various resolutions. In light of this gathering of portrayals, we can discuss a multi-determination investigation. On account of wavelets, we ordinarily don't talk about time-recurrence.

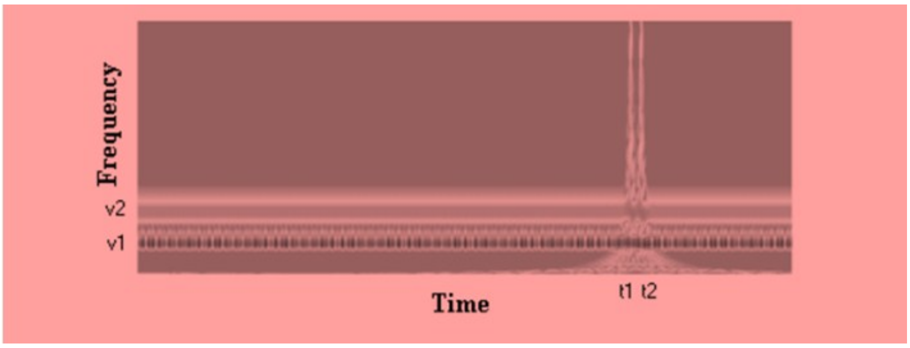


Figure-1: Continuous Wavelet Transformation Of discrete School

3. DISCRETE WAVELET TRANSFORM

The discrete wavelet change (DWT) was created to apply the wavelet change to the advanced world. Channel banks are utilized to surmised the conduct of the persistent wavelet change. The flag is disintegrated with a high-pass channel and a low-pass channel. The coefficients of these channels are registered utilizing numerical examination and made accessible to you. See Appendix B for more data about these calculations.

Where

The wavelet writing presents the channel coefficients to you in tables. An illustration is the Daubechies channels for wavelets. These channels rely on upon a parameter p called the vanishing minute

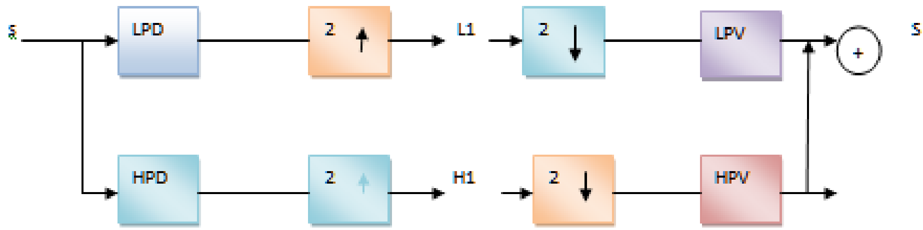


Figure 2 –Wavelet Construction

Table 1. Daubechies (p =2,3) Wavelet Coefficients

Vanishing Moment	n	_hp[n]
p = 2	0	0.48296291311445341
	1	0.8365163037378079
	2	0.2241438680420134
	3	–0.1294095225512604
p = 3	0	0.332670552950
	1	0.80691509311
	2	0.459877502118
	3	–0.135011020010
	4	–0.085441273882
	5	0.03522629291882

The hp[n] coefficients are used as the low-pass reconstruction filter (LPr).

The coefficients for the filters HPd, LPd and HPr are computed from the h[n] coefficients as follows:

- High-pass decomposition filter (HPd) coefficients

g[n] = (–1)n h[L–n] (L: length of the filter)

- Low-pass reconstruction filter (LPr) coefficients h[n] = h[L–n] (L: length of the filter)

- High-pass reconstruction filter (HPr) coefficients

g[n] = g[L–n] (L: length of the filter)

The Daubechies filters for Wavelets are provided in the C55x IMGLIB for $2 \leq p \leq 10$. Since there are several sets of filters, we may ask ourselves what are the advantages and disadvantages to using one set or another.In the first place we have to comprehend that we will have culminate reproduction regardless of what the channel length is. In any case, longer channels give smoother, littler moderate outcomes. In this manner, if transitional preparing is required, we are less inclined to lose data because of fundamental edge or immersion. Be that as it may, longer channels clearly include all the more handling.

Wavelets and Perfect Reconstruction Filter Banks

Channel banks break down the flag into high-and low-recurrence parts. The low-recurrence segment more often than not contains the greater part of the recurrence of the flag. This is known as the estimation. The high-recurrence part contains the points of interest of the flag.

Wavelet deterioration can be executed utilizing a two channel bank. Two channel banks are talked about in this area quickly. The principle thought is that immaculate remaking channel banks execute arrangement developments of discrete-time signals.

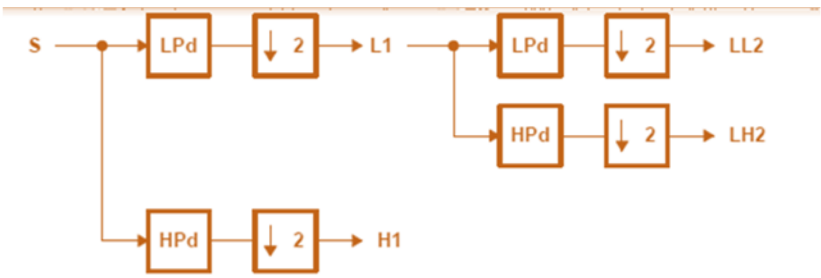


Figure-3: Two Wavelet Decomposition

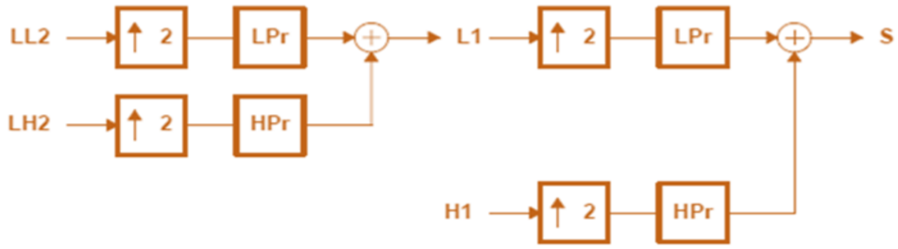


Figure-4: Two Wavelet Reconstruction

The information and the reproduction are indistinguishable; this is called idealize recreation. Two prevalent disintegration structures are pyramid and wavelet parcel. The first deteriorates just the estimate (low-recurrence segment) part while the second one breaks down both the estimation and the detail (high-recurrence part).

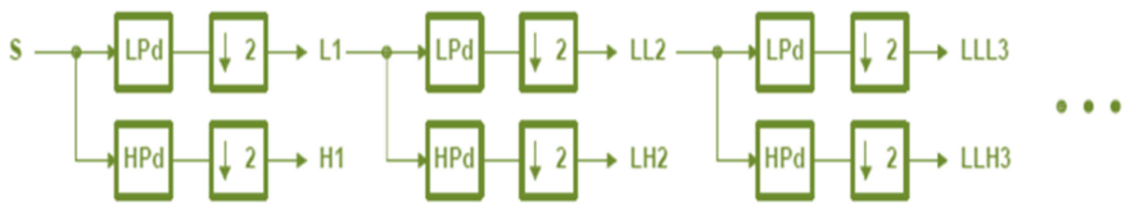


Figure-5 : Packet Transformation

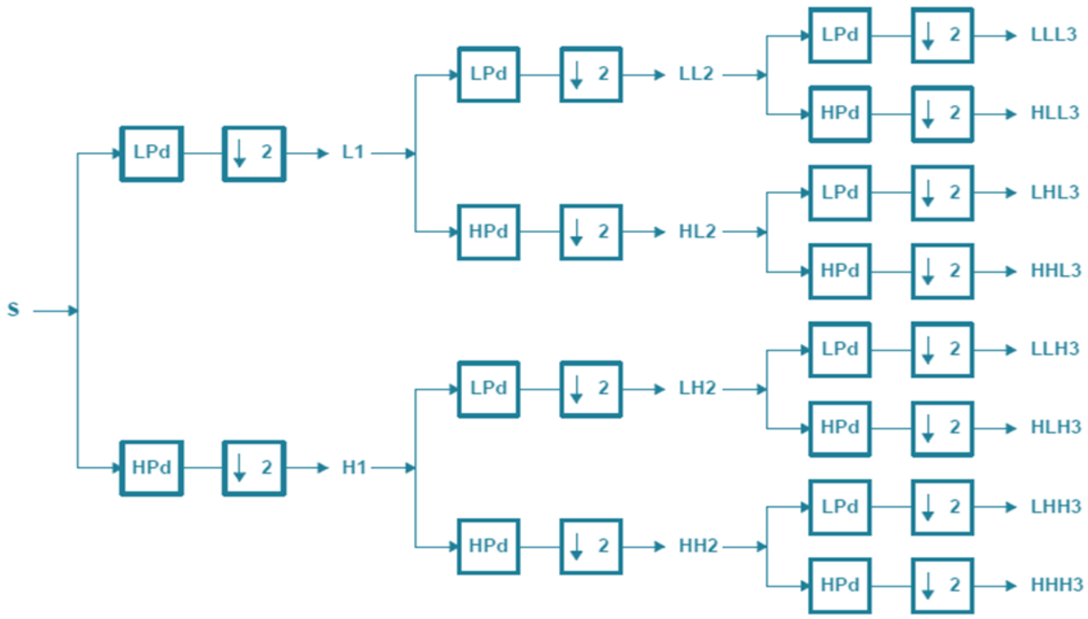


Figure 6- Wavelet Packet Decomposition

The C55x IMGLIB provides the following functions for one dimension pyramid and packet decomposition and reconstruction. Complete information about these functions can be found in the C55x IMGLIB.

1-D discrete wavelet transform

void IMG_wave_decom_one_dim(short *in_data, short *wksp, int *wavename, int length,int level);

• **1-D inverse discrete wavelet transform**

void IMG_wave_recon_one_dim(short *in_data, short *wksp, int *wavename, int length,int level);

• **1-D discrete wavelet package transform**

void IMG_wavep_decom_one_dim(short *in_data, short *wksp, int *wavename, int length, int level);

• **1-D inverse discrete wavelet package transform**

void IMG_wavep_recon_one_dim(short *in_data, short *wksp, int *wavename, int length,int level);

4. WAVELETS IMAGE PROCESSING

Wavelets have found a large variety of applications in the image processing field. The JPEG 2000 standard uses wavelets for image compression. Other image processing applications such as noise reduction, edge detection, and finger print analysis have also been investigated in the literature.

Wavelet Decomposition of Images

In wavelet breaking down of a picture, the disintegration is done line by line and afterward segment by segment. For example, here is the methodology for a N x M picture. You channel each line and after that down-specimen to acquire two N x (M/2) pictures. At that point channel every segment and subsample the channel yield to get four (N/2) x (M/2) pictures. Of the four sub-pictures acquired as found in Figure 12, the one gotten by low-pass separating the lines and sections is alluded to as the LL picture.

The one acquired by low-pass sifting the lines and high-pass separating the segments is alluded to as the LH pictures. The one gotten by high-pass separating the lines and low-pass sifting the sections is known as the HL picture. The sub-picture acquired by high-pass sifting the lines and segments is alluded to as the HH picture. Each of the sub-pictures gotten in this mold can then be sifted and sub-tested to acquire four more sub-pictures. This procedure can be proceeded until the coveted sub-band structure is gotten.

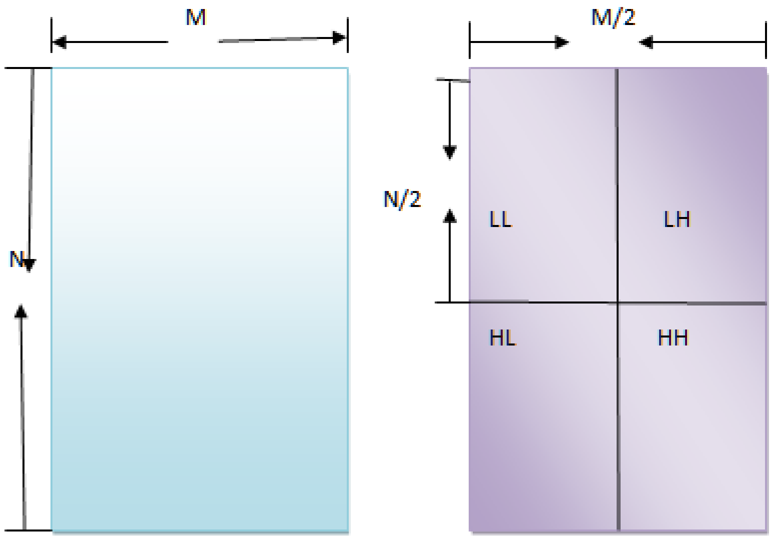


Figure 8- Original image one level 2D composition

Three of the most popular ways to decompose an image are: pyramid, spacl, and wavelet packet, as shown in Figure 9.

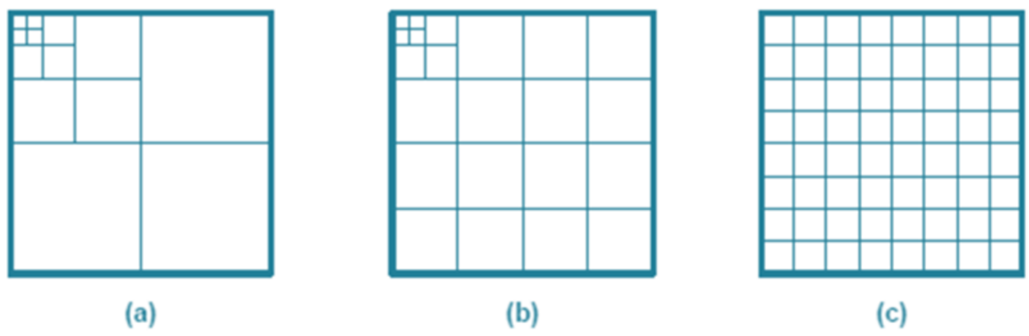


Figure-9: Three Popular Wavelet Decomposition Structures On Image: (a)Pyramid , (b) Spacl , (c) Wavelet packet

In the structure of pyramid decomposition, only the LL subimage is decomposed after each decomposition into four more subimages.

- In the structure of wavelet packet decomposition, each subimage(LL, LH,HL, HH) is decomposed after each decomposition.

- In the structure of spacl, after the first level of decomposition, each subimage is decomposed into smaller subimages, and then only the LL subimage is decomposed.

In the part I development stage, the JPEG 2000 standard supports the pyramid decomposition structure. In the future all three structures will be supported.

For two dimensions, the C55x IMGLIB provides functions for pyramid and packet decomposition and reconstruction. Complete information about these functions can be found in the C55x IMGLIB.

• 2-D discrete wavelet transform

void IMG_wave_decom_two_dim(short **image, short * wksp, int width, int height, int *wave name, int level);

• 2-D inverse discrete wavelet transform

void IMG_wave_recon_two_dim(short **image, short * wksp, int width, int height, int *wave name, int level);

• 2-D discrete wavelet package transform

void IMG_wavep_decom_two_dim(short **image, short * wksp, int width, int height, int *waven ame, int level);

• 2-D inverse discrete wavelet package transform

void IMG_wavep_recon_two_dim(short **image, short * wksp, int width, int height, int *wave name, int level);

Wavelets Applications

TI provides several one dimension and two dimension wavelets applications, which illustrate how to use the wavelets functions provided in the C55x IMGLIB.

2.2.1. 1-D Continuous wavelet transform

The 1-D continuous wavelet transform is given by:

$$W_f(a, b) = \int_{-\infty}^{\infty} x(t)\psi_{a,b}(t)dt \quad \text{------------ (2.3)}$$

The inverse 1-D wavelet transform is given by:

$$x(t) = \frac{1}{C}\int_{0}^{\infty}\int_{-\infty}^{\infty} W_f(a,b)\psi_{a,b}(t)db\frac{da}{a^2} \quad \text{------------ (2.4)}$$

$$\text{Where } C = \int_{-\infty}^{\infty}\frac{|\psi\omega)|^2}{\omega}d\omega < \infty$$

$\Psi(\omega)$ is the Fourier transform of the mother wavelet $\Psi(t)$. C is required to be finite, which leads to one of the required properties of a mother wavelet. Since C must be finite, then $\Psi(0) = 0$ to avoid a singularity in the integral, and thus the $\Psi(t)$ must have zero mean. This condition can be stated as

$\int_{-\infty}^{\infty}\psi(t)dt = 0$ and known as the admissibility condition.

1-D Discrete wavelet transform

The discrete wavelets transform (DWT), which transforms a discrete time signal to a discrete wavelet representation. The first step is to discretize the wavelet parameters, which reduce the previously continuous basis set of wavelets to a discrete and orthogonal / orthonormal set of basis wavelets.

$\psi_{m,n}(t) = 2^{m/2}\,\psi(2^m t - n)$; m, n $\in$ Z such that $-\infty < m, n < \infty$ -------- (2.5)

The 1-D DWT is given as the inner product of the signal x(t) being transformed with each of the discrete basis functions.

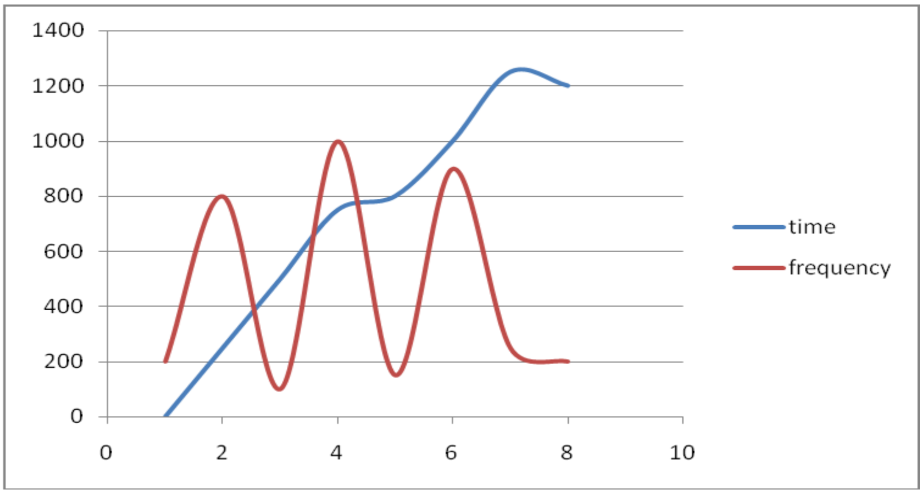
$$W_{m,n} = \langle x(t), \psi_{m,n}(t) \rangle \quad ; \quad m, n \in \mathbb{Z} \qquad \text{------------} \quad (2.6)$$

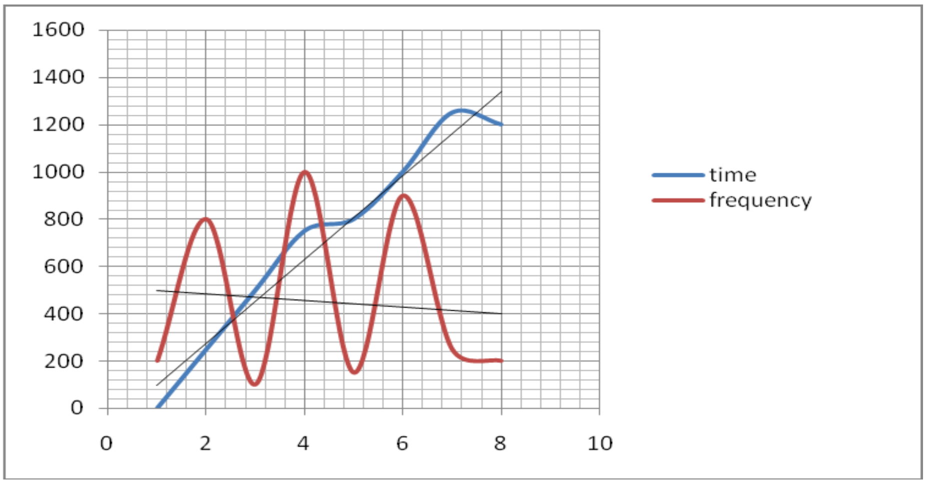
The 1-D inverse DWT is given as:

$$x(t) = \sum_{m} \sum_{n} W_{m,n} \psi_{m,n}(t) \quad ; m, n \in \mathbb{Z} \qquad \text{-------------} \quad (2.7)$$

One Dimension Wavelet Applications

The 1D_Demo.c file presents applications of the one-dimension wavelet. A 128-point sine wave is used as input for all these applications as shown in Figure 15:





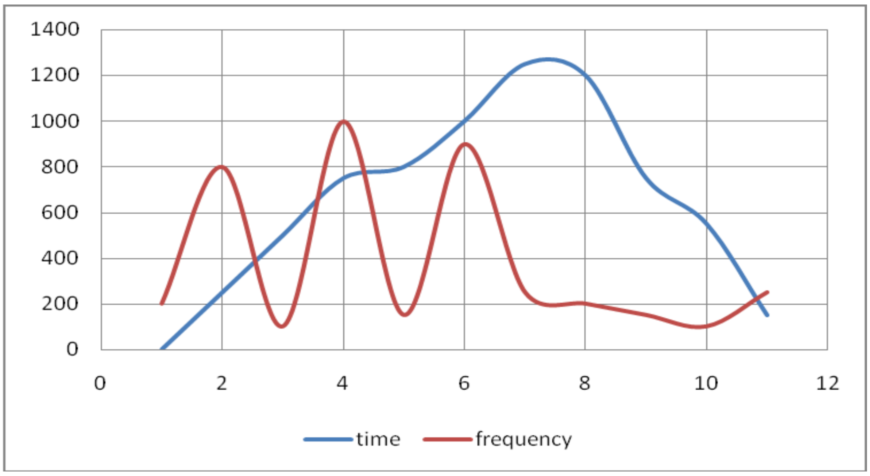


Figure 12- Level 3 Pyramid Decomposition Of The Original Sine Wave Signal

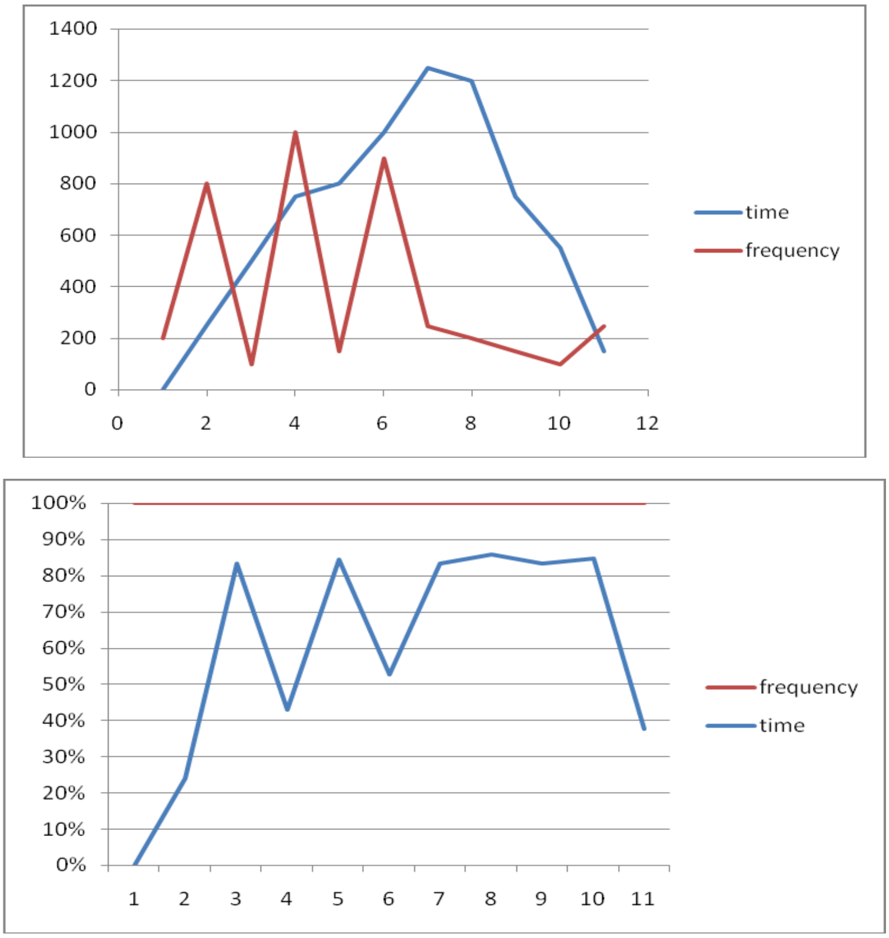


Figure 13,14 –LLL3(Left) and HLL3(Right) Components Of Three-Level Decomposition

The error signal shown in Figure 24 represents the difference between the original signal and the reconstructed signal. This error signal is not zero because of the dynamic range of the 16-bit fixed-point data.

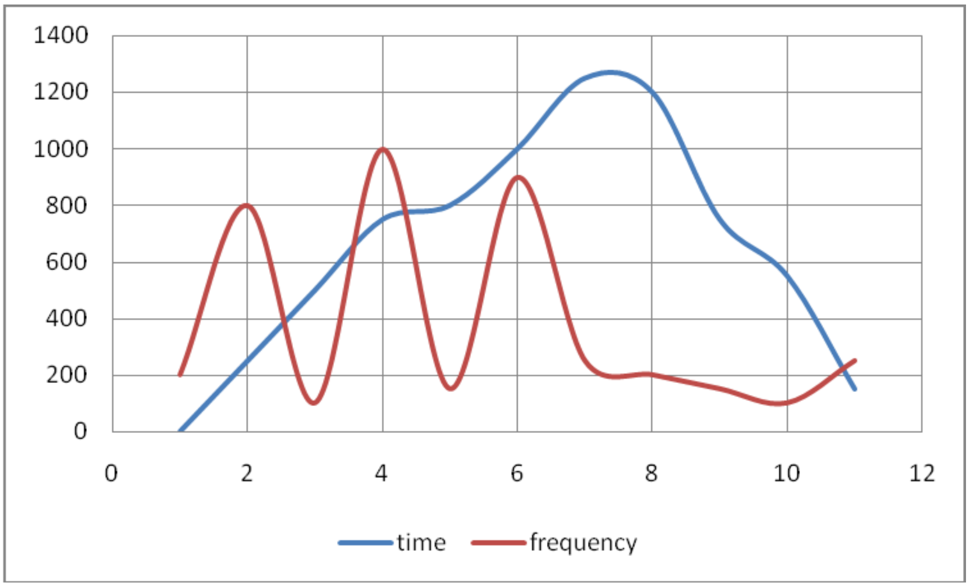


Figure 15-Reconstructed Error

2-D wavelet transform

The 1-D DWT can be stretched out to 2-D change utilizing distinct wavelet channels. With distinct channels, applying a 1-D change to every one of the lines of the info and after that rehashing on the greater part of the segments can figure the 2-D change. When one-level 2-D DWT is connected to a picture, four change coefficient sets are made. As portrayed in Figure 2.1(c), the four sets are LL, HL, LH, and HH, where the main letter relates to applying either a low pass or high pass channel to the lines, and the second letter alludes to the channel connected to the sections.

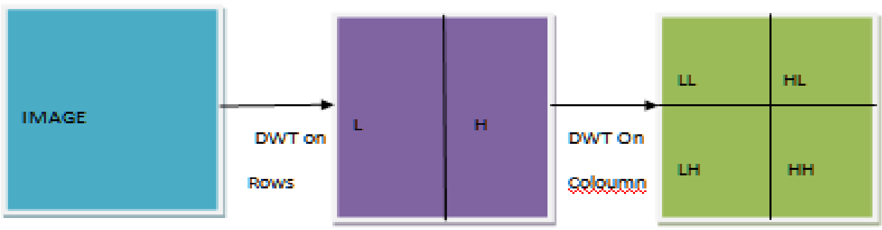


Figure 16- Block Diagram of DWT (a)Original Image (b) Output image after the 1-D applied on Row input (c) Output image after the second 1-D applied on row input

DWT for Lena image (a) Original Image (b) Output image after the 1-D applied on column input (c) Output image after the second 1-D applied on row input

The Two-Dimensional DWT (2D-DWT) changes over pictures from spatial area to recurrence space. At each level of the wavelet deterioration, every section of a picture is first changed utilizing a 1D vertical examination channel bank. A similar channel bank is then connected evenly to each line of the separated and sub sampled information. One-level of wavelet disintegration produces four separated and sub sampled pictures, alluded to as sub bands. The upper and lower territories of Fig. 2.2(b), individually, speak to the low pass and high pass coefficients after vertical 1D-DWT and sub examining. The after effect of the even 1D-DWT and sub examining to shape a 2D-DWT yield picture is shown. We can utilize various levels of wavelet changes to pack information vitality in the most reduced tested groups. In particular, the LL sub band in fig 2.1(c) can be changed again to frame LL2, HL2, LH2, and HH2 sub bands, delivering a two-level wavelet change. A (R-1) level wavelet decay is related with R determination levels numbered from 0 to (R-1), with 0 and (R-1) comparing to the coarsest and finest resolutions.

The straight forward convolution execution of 1D-DWT requires a lot of memory and extensive calculation intricacy. An option execution of the 1D-DWT, known as the lifting plan, gives critical decrease in the memory and the calculation unpredictability. Lifting likewise permits set up calculation of the wavelet coefficients. By the by, the lifting approach registers an in distinguishable coefficients from the immediate channel bank convolution.

Two Dimension Wavelet Applications

The 2D_Demo.c file, provided in the C55x IMGLIB, presents applications of the two-dimension wavelet. A 128x128 image is used as input for all these applications.

2-D Perfect Decomposition and Reconstruction Example

In this application, the image is one-level decomposed and reconstructed. You notice no difference between the original picture and the reconstructed picture .

Edge Detection Example

In the second application, a 2-D edge detection is performed for the picture in Figure 17:

Figure 17-picture plate Used In Image performation

Figure 18- 2 D one Level Decomposition detects the edges

The HH part of the picture has a vertical line. This happens because a row by row processing was performed first and then a column by column.

CONCLUSION

Basis selection and optimization of the mother wavelet through parameterization lead to improvement of performance in signal compression with respect to random selection of the mother wavelet and logarithmic DWT. The method provides an adaptive approach for optimal signal representation for the purpose of compression and can thus be applied to any type of 1-D biomedical signal. correlation between different lossy and lossless segments in 1−D image channels was explored in this paper and a technique to make use of this redundancy was suggested. Image compression can be grouped based on certain features for manual classification and abnormality detection. This grouping of similar segments is applied indirectly in the proposed scheme in the usage of reference lists. In addition, the dynamic update of the reference lists enables the algorithm to adapt to the changes in image recordings. The reference lists and the dynamic update of these lists make this algorithm an innovative technique in compressing image compression process.

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