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Some Common Fixed Point Theorems in Fuzzy Metric Spaces by using (CLRg) property

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Abstract

The aim of this paper is to prove some common fixed point theorems for a pair of weakly compatible mappings in Kromosil and Michalek and George and Veeramani fuzzy metric spaces by using (CLRg) property. Our results generalize the main results of Wutiphol Sintunavarat and Poom Kumam [10].

Keywords: Fuzzy Metric Space, Weakly Compatible Mapping, (CLRg) property.

Mathematics Subject Classification: 52H25, 47H10.

1. INTRODUCTION

The fundamental work for the fuzzy theory was first given by Zadeh [11] in 1965, who introduced the concept of fuzzy set. Kramosil and Michalek [6] developed the fuzzy metric space and later George and Veeramani [3] modified the notion of fuzzy metric spaces by introducing the concept of continuous t–norm.

Many researchers have extremely developed the theory by defining different concepts and amalgamation of many properties. Fuzzy set theory has its significance in various fields such as communication, gaming, signal processing, modeling theory, image processing, etc.

In 2002, Aamri and EI Moutawakil[1] defined the property(E.A.) requires the containment and closedness of ranges for the existence of fixed points. In 2009,Abbas et al.[2] introduced the notion of common property (E.A.).Later on, Sintunavarat and Kumam[10] give the idea of "Common limit in the range property "which does not require the closedness of the subspaces for the existence of fixed point for a pair of mappings.

The purpose of this work is to prove some common fixed point theorems for a pair of weakly compatible mappings in both Kromosil and Michalek and George and Veeramani fuzzy metric spaces by using (CLRg) property.

2. PRELIMINARIES

Definition 2.1: [10] A mapping $*$: [0, 1] × [0, 1] → [0, 1] is called a continuous t-norm if $*$ is satisfying the following conditions:

1) $*$ is commutative and associative;

2) The mapping $*$: [0, 1]× [0, 1]→ [0, 1] is continuous.

3) $a * 1 = a$ for all a∈[0, 1];

4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all a,b,c,d ∈[0, 1].

Example 2.2: [10] The following examples are the classical examples of a continuous t-norms

(TL) (the Lukasiewicz t-norm).A mapping T_L : [0,1] × [0, 1] → [0, 1] which defined through

$$T_L(a,b)=\max\{a+b-1,0\}$$

(TP) (the Product t-norm).A mapping T_P :[0,1] × [0, 1] → [0, 1] which defined through

$$T_P(a,b)=ab$$

(TM) (the minimum t-norm).A mapping T_M :[0,1] × [0, 1] → [0, 1] which defined through

$$T_M(a,b)=\min\{a,b\}$$

Definition 2.3: Kramosil and Michalek [6] A Fuzzy Metric Space is a triple (X,M, ∗) where X is a non empty set , ∗ is a continuous t-norm and M is a fuzzy set on X^2 × [0, 1] such that the followings axioms hold:

(KM-1) M(x,y,0)=0 for all x,y ∈ X;

(KM-2) M(x,y,t)=1 for all x,y ∈ X where t >0 ⇔x=y;

(KM-3) M(x,y,t)= M(y,x,t) for all x,y ∈ X

(KM-4) M(x,y,.): [0, ∞)→[0, 1]is left continuous for all x,y ∈ X;

(KM-5) M(x,z,t+s) ≥ M(x,y,t) ∗ M(y,z,s) for all x,y,z ∈ X and for all s, t > 0.

We will refer to these spaces as KM-fuzzy metric Spaces.

Lemma 2.4: Grabiec [8] For every x,y ∈ X, the mappings M(X,M, .) is non decreasing on [0,∞].

George and Veeramani [1,2] introduced and studies a notion of fuzzy metric space which constitutes a modification of one due to Kromosil and Michalek.

Definition 2.5:George and Veeramani [3,4] A Fuzzy Metric Space is a triple (X,M, ∗) where X is a nonempty set , ∗ is a continous t-norm and M is a fuzzy set on X^2 × [0, 1] such that the following axioms hold:

(GV-1) M(x,y,t) >0;

(GV-2) M(x,y,t)=1 ⇔ x=y;

(GV-3) M(x,y,t)= M(y,x,t) ;

(GV-4) M(x,y,.): [0, ∞)→[0, 1]is continuous;

(GV-5) M(x,z,t+s) ≥ M(x,y,t) ∗ M(y,z,s) for all x,y,z ∈ X and for all s, t > 0.

From (GV-1) to (GV-2), it follows that $x \neq y$, then $0 < M(x,y,t) < 1$ for all $t > 0$.

Example 2.6:[10] Let (X, d) be a metric space $a*b = T_M(a,b)$ and for all $x,y \in X$ and $t > 0$

$$M(x,y,t) = \frac{t}{t+d(x,y)}$$

Then (X, M, ∗) is a GV-fuzzy metric space called standard fuzzy metric space induced by (X,d).

Definition 2.7:[10] Let (X,M, ∗) be a (KM or GV) fuzzy metric space .A sequence { x_n } in X is said to be convergent to $x \in X$ if

$$\lim_{n\to\infty} M(x_n, x, t) = 1$$

for all t>0

Definition 2.8:[10] Let (X, M, ∗) be a KM or GV fuzzy metric space .A sequence { x_n } in X is said to be G- Cauchy sequence if

$$\lim_{n\to\infty} M(x_n, x_{n+m}, t) = 1$$

for all t>0 and $m \in N$.

Definition 2.9 [10]A fuzzy metric space (X,M, ∗) is called complete if every Cauchy sequence converges to a point in X.

Lemma 2.10: Schweizer and Sklar [9] If (X,M, ∗) is a KM fuzzy Metric Space and { x_n }and

{ y_n } are sequences in X such that

$\lim_{n\to\infty} x_n = x$, $\lim_{n\to\infty} y_n = y$

then

$$\lim_{n\to\infty} M(x_n, y_n, t) = M(x, y, t)$$

for every continuity point t of $M(x, y, .)$

Definition 2.11 :Jungck and Rhoades [5] Two self mappings f and g of a fuzzy metric space $(X,M,*)$ are said to weakly compatible if they commute at their coincidence points i.e fx=gx implies fgx=gfx .

Definition 2.12:[10] Let Φ be class of all mappings $\varphi: [0,1] \to [0,1]$ satisfying the following properties

$(\varphi 1)$ φ is continuous and non decreasing on [0,1];

$(\varphi 2)$ $\varphi(x) > x$ for all $x \in (0, 1)$.

Definition 2.13 Aamri ,M. and Moutawakil, D. E :[10] Let f and g be self mappings of a fuzzy metric space $(X,M,*)$.We say that f and g satisfy E.A. property if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$$

for some $t \in X$.

Definition 2.14:[10]Suppose that $(X,M,*)$ is a fuzzy metric space f, g : X→X .Two mappings f and g are said to satisfy the common limit in the range of g property if

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gx$$

for some $x \in X$.

The common limit in the range property will be denoted by the (CLRg) property .

Example 2.15:[10] Let X = [0,∞) be the usual metric space .Define f,g: X->X by fx=x/4 and gx=3x/4 for all x∈ X.We consider the sequence $\{x_n\}=\{1/n\}$.Since

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = 0 = g0,$$

Therefore f and g satisfy the (CLRg) property.

Example 2.16:[10]Let X=[0,∞) be the usual metric space .Define f,g: X->X by fx=x+1 and gx=2x for all x∈ X. Consider the sequence {xn}={1+1/n}.Since

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = 2 = g1,$$

Therefore f and g satisfy the (CLRg) property.

***Lemma* .2.17:[7]** Let (X, M,∗) be a fuzzy metric space. If there exists a number k ∈ (0, 1)

$$M(x, y, kt) \geq M(x, y, t) \text{ for all } x, y \in X \ \& \ t > 0 \text{ then } x=y.$$

3. MAIN RESULTS

Theorem 3.1

Let (X, M,∗) be a KM- fuzzy metric space satisfying the following property:

for all x,y ∈ X, $x \neq y$,∃ t > 0 :0 < M(x,y,t) < 1 **(3.1.1)**

and let f, g be weakly compatible self mappings of X such that , for some $\varphi \in \Phi$,

M(fx,fy,t) ≥

$$\varphi\left\{min\left\{\begin{array}{l}\left(\frac{M(gx,gy,t)+M(fy,gy,t)}{M(fx,gy,t)+M(fy,gy,t)}\right)M(gx,gy,t),\left(\frac{M(fy,gy,t)+M(gy,gx,t)}{M(fx,gx,t)+M(fy,gx,t)}\right)M(fy,gy,t),\\ \left(\frac{M(gx,gy,t)+M(fx,gy,t).M\big((fy,gy,t)\big)}{M(gx,gy,t)+M(fy,gx,t)}\right),\left(\frac{M(gx,gy,t)+M(fx,gx,t)+M\big((fy,gy,t)\big)}{M(fx,gx,t)+M(fy,gy,t)+M(fx,gy,t)}\right)\end{array}\right\}\right\}$$

(3.1.2)

for all x,y ∈ X , where t>0.If f and g satisfy (CLRg) property then f and g have a unique common fixed point.

Proof: It follows f and g satisfy (CLRg) property then we can find a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gx \quad \textbf{(3.1.3)}$$

for some x ∈ X.

Let t be a continuity point of (X, M,∗) then

$$M(fx_n,fx,t) \geq \varphi\left\{min\left\{\begin{array}{c}\left(\frac{M(gx_n,gx,t)+M(fx,gx,t)}{M(fx_n,gx,t)+M(fx,gx,t)}\right)M(gx_n,gx,t),\left(\frac{M(fx,gx,t)+M(gx,gx_n,t)}{M(fx_n,gx_n,t)+M(fx,gx_n,t)}\right)M(fx,gx,t),\\ \left(\frac{M(gx_n,gx,t)+M(fx_n,gx,t).M((fx,gx,t))}{M(gx_n,gx,t)+M(fx,gx_n,t)}\right),\left(\frac{M(gx_n,gx,t)+M(fx_n,gx_n,t)+M((fx,gx,t))}{M(fx_n,gx_n,t)+M(fx,gx,t)+M(fx_n,gx,t)}\right)\end{array}\right\}\right\}$$

(3.1.4)

for all n ∈ N. By taking the limit n tends to infinity in (3.1.4), we have

$$M(gx,fx,t) \geq$$

$$\varphi\left\{min\left\{\begin{array}{c}\left(\frac{M(gx,gx,t)+M(fx,gx,t)}{M(gx,gx,t)+M(fx,gx,t)}\right)M(gx,gx,t),\left(\frac{M(fx,gx,t)+M(gx,gx,t)}{M(gx,gx,t)+M(fx,gx,t)}\right)M(fx,gx,t),\\ \left(\frac{M(gx,gx,t)+M(gx,gx,t).M((fx,gx,t))}{M(gx,gx,t)+M(fx,gx,t)}\right),\left(\frac{M(gx,gx,t)+M(gx,gx,t)+M((fx,gx,t))}{M(gx,gx,t)+M(fx,gx,t)+M(gx,gx,t)}\right)\end{array}\right\}\right\}$$

$$= \varphi\{min\{1, M(fx,gx,t), 1,1\}\}$$

$$= \varphi(M(gx,fx,t)).$$

for every t>0 .Now, we claim that gx=fx if not, then

$$\exists\ t_0 > 0 : 0 < M(gx,fx,t_0) < 1 \quad \textbf{(3.1.5)}$$

It follows from condition of (φ2) that, $\varphi(M(gx,fx,t_0)) > M(gx,fx,t_0)$ which is a contradiction therefore gx=fx.

Next, we let fx=gx=z (say).Since f and g are weakly compatible mappings fgx=gfx which implies that fz=fgx=gfx=gz **(3.1.6)**

We claim that fz=z assume not, then by (3.1.1) it implies that 0< $M(fz,z,t_1)$ <1 for some $t_1 > 0$.

By condition (φ2) we have $\varphi(M(fz,z,t_1)) > M(fz,z,t_1)$

It follows from condition (3.1.2) that

M(fz,z,t)=M(fz,fx,t) ≥

$$\varphi\left\{min\left\{\begin{array}{c}\left(\frac{\mathrm{M(gz,gx,t)+M(fx,gx,t)}}{\mathrm{M(fz,gx,t)+M(fx,gx,t)}}\right)\mathrm{M(gz,gx,t)},\left(\frac{\mathrm{M(fx,gx,t)+M(gx,gz,t)}}{\mathrm{M(fz,gz,t)+M(fx,gz,t)}}\right)\mathrm{M(fx,gx,t)},\\ \left(\frac{\mathrm{M(gz,gx,t)+M(fz,gx,t).M((fx,gx,t))}}{\mathrm{M(gz,gx,t)+M(fx,gz,t)}}\right),\left(\frac{\mathrm{M(gz,gx,t)+M(fz,gz,t)+M((fx,gx,t))}}{\mathrm{M(fz,gz,t)+M(fx,gx,t)+M(fz,gx,t)}}\right)\end{array}\right\}\right\}$$

$$= \varphi\{min\{\mathrm{M(fz,fx,t)},1,1,1\}\}$$

$$= \varphi\{\mathrm{M(fz,z,t)}\}$$

for all t>0 which is a contradiction .Hence fz=z ,that is z = fz = gz therefore z is a common fixed point of f and g .For uniqueness of a common fixed point ,we suppose that w is another common fixed point in which w≠z.

It follows from (3.1.1) that there exists $t_2>0$, such that 0< M(w,z, t_2) <1.Since M(w,z, t_2) ∈ (0,1) ,we have φ (M(w,z, t_2)) > M(w,z, t_2) by virtue of (φ2) .Now, from condition (3.1.2) we have,

M(z,w,t)=M(fz,fw,t) ≥

$$\varphi\left\{min\left\{\begin{array}{c}\left(\frac{\mathrm{M(gz,gw,t)+M(fw,gw,t)}}{\mathrm{M(fz,gw,t)+M(fw,gw,t)}}\right)\mathrm{M(gz,gw,t)},\left(\frac{\mathrm{M(fw,gw,t)+M(gw,gz,t)}}{\mathrm{M(fz,gz,t)+M(fw,gz,t)}}\right)\mathrm{M(fw,gw,t)},\\ \left(\frac{\mathrm{M(gz,gw,t)+M(fz,gw,t).M((fw,gw,t))}}{\mathrm{M(gz,gw,t)+M(fw,gz,t)}}\right),\left(\frac{\mathrm{M(gz,gw,t)+M(fz,gz,t)+M((fw,gw,t))}}{\mathrm{M(fz,gz,t)+M(fw,gw,t)+M(fz,gw,t)}}\right)\end{array}\right\}\right\}$$

$$= \varphi\{min\{\mathrm{M(z,w,t)},1,1,1\}\}$$

$$= \varphi\{\mathrm{M(z,w,t)}\}$$

for all t>0,which is a contradiction therefore w=z which implies that f and g have a unique a common fixed point .

Theorem 3.2 Let (X, M,∗) be a KM- fuzzy metric space and let f,g be weakly compatible self mappings of X such that ,

M(fx,fy,kt) ≥

$$min\left\{\begin{array}{c}\left(\frac{M(gx,gy,t)+M(fy,gy,t)}{M(fx,gy,t)+M(fy,gy,t)}\right)M(gx,gy,t),\left(\frac{M(fy,gy,t)+M(gy,gx,t)}{M(fx,gx,t)+M(fy,gx,t)}\right)M(fy,gy,t),\\ \left(\frac{M(gx,gy,t)+M(fx,gy,t).M\left((fy,gy,t)\right)}{M(gx,gy,t)+M(fy,gx,t)}\right),\left(\frac{M(gx,gy,t)+M(fx,gx,t)+M\left((fy,gy,t)\right)}{M(fx,gx,t)+M(fy,gy,t)+M(fx,gy,t)}\right)\end{array}\right\}$$

(3.2.1)

for all x,y ∈ X , where t>0.If If f and g satisfy (CLRg) property then f and g have a unique common fixed point.

Proof: It follows f and g satisfy (CLRg) property then we can find a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty}fx_n=\lim_{n\to\infty}gx_n=gx \tag{3.2.2}$$

for some x ∈ X. Let t be a continuity point of (X, M,∗) then

$M(fx_n,fx,kt) \geq$

$$min\left\{\begin{array}{c}\left(\frac{M(gx_n,gx,t)+M(fx,gx,t)}{M(fx_n,gx,t)+M(fx,gx,t)}\right)M(gx_n,gx,t),\left(\frac{M(fx,gx,t)+M(gx,gx_n,t)}{M(fx_n,gx_n,t)+M(fx,gx_n,t)}\right)M(fx,gx,t),\\ \left(\frac{M(gx_n,gx,t)+M(fx_n,gx,t).M\left((fx,gx,t)\right)}{M(gx_n,gx,t)+M(fx,gx_n,t)}\right),\left(\frac{M(gx_n,gx,t)+M(fx_n,gx_n,t)+M\left((fx,gx,t)\right)}{M(fx_n,gx_n,t)+M(fx,gx,t)+M(fx_n,gx,t)}\right)\end{array}\right\}$$

(3.2.3)

for all n ∈ N.By taking the limit n tends to infinity in (3.2.3), we have

$M(gx,fx,kt) \geq$

$$min\left\{\begin{array}{c}\left(\frac{M(gx,gx,t)+M(fx,gx,t)}{M(gx,gx,t)+M(fx,gx,t)}\right)M(gx,gx,t),\left(\frac{M(fx,gx,t)+M(gx,gx,t)}{M(gx,gx,t)+M(fx,gx,t)}\right)M(fx,gx,t),\\ \left(\frac{M(gx,gx,t)+M(gx,gx,t).M\left((fx,gx,t)\right)}{M(gx,gx,t)+M(fx,gx,t)}\right),\left(\frac{M(gx,gx,t)+M(gx,gx,t)+M\left((fx,gx,t)\right)}{M(gx,gx,t)+M(fx,gx,t)+M(gx,gx,t)}\right)\end{array}\right\}$$

$$= min\{1,M(fx,gx,t),1,1\}$$

$$= M(fx,gx,t).$$

Hence from the lemma (2.17) we have fx=gx

Next ,we let fx=gx=z(say).Since f and g are weakly compatible mappings fgx=gfx

which implies that fz=fgx=gfx=gz **.** **(3.2.4)**

We claim that fz=z assume not, then it follows from condition (3.2.1) that

M(fz,z,kt)=M(fz,fx,kt) ≥

$$min\left\{\begin{matrix}\left(\frac{M(gz,gx,t)+M(fx,gx,t)}{M(fz,gx,t)+M(fx,gx,t)}\right)M(gz,gx,t),\left(\frac{M(fx,gx,t)+M(gx,gz,t)}{M(fz,gz,t)+M(fx,gz,t)}\right)M(fx,gx,t),\\ \left(\frac{M(gz,gx,t)+M(fz,gx,t).M\big((fx,gx,t)\big)}{M(gz,gx,t)+M(fx,gz,t)}\right),\left(\frac{M(gz,gx,t)+M(fz,gz,t)+M\big((fx,gx,t)\big)}{M(fz,gz,t)+M(fx,gx,t)+M(fz,gx,t)}\right)\end{matrix}\right\}$$

$$= min\{M(fz,fx,t),1,1,1\}$$

$$= M(fz,z,t)$$

Hence from the lemma (2.17) we have fz=z.

That is z = fz = gz .

Therefore z is a common fixed point of f and g .

For uniqueness of a common fixed point, we suppose that w is another common fixed point in which w≠z.From condition (3.2.1) we have,

M(z,w,kt)=M(fz,fw,kt) ≥

$$min\left\{\begin{matrix}\left(\frac{M(gz,gw,t)+M(fw,gw,t)}{M(fz,gw,t)+M(fw,gw,t)}\right)M(gz,gw,t),\left(\frac{M(fw,gw,t)+M(gw,gz,t)}{M(fz,gz,t)+M(fw,gz,t)}\right)M(fw,gw,t),\\ \left(\frac{M(gz,gw,t)+M(fz,gw,t).M\big((fw,gw,t)\big)}{M(gz,gw,t)+M(fw,gz,t)}\right),\left(\frac{M(gz,gw,t)+M(fz,gz,t)+M\big((fw,gw,t)\big)}{M(fz,gz,t)+M(fw,gw,t)+M(fz,gw,t)}\right)\end{matrix}\right\}$$

$$= min\{M(z,w,t),1,1,1\}$$

$$= M(z,w,t)$$

Hence from the lemma (2.17) we have z=w, which implies that f and g have a unique a common fixed point.

Theorem 3.3

Let (X, M,∗) be a GV- fuzzy metric space and f, g are weakly compatible self mappings of X such that, for some $\varphi \in \Phi$, M(fx,fy,t) ≥

$$\varphi\left\{min\left\{\begin{matrix}\left(\frac{M(gx,gy,t)+M(fy,gy,t)}{M(fx,gy,t)+M(fy,gy,t)}\right)M(gx,gy,t),\left(\frac{M(fy,gy,t)+M(gy,gx,t)}{M(fx,gx,t)+M(fy,gx,t)}\right)M(fy,gy,t),\\ \left(\frac{M(gx,gy,t)+M(fx,gy,t).M\big((fy,gy,t)\big)}{M(gx,gy,t)+M(fy,gx,t)}\right),\left(\frac{M(gx,gy,t)+M(fx,gx,t)+M\big((fy,gy,t)\big)}{M(fx,gx,t)+M(fy,gy,t)+M(fx,gy,t)}\right)\end{matrix}\right\}\right\}$$

(3.3.1)

for all x,y ∈ X,where t >0 .If f and g satisfy (CLRg) property then f and g have a unique common fixed point.

Proof: It follows f and g satisfy (CLRg) property then we can find a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n=\lim_{n\to\infty} gx_n=gx \qquad \textbf{(3.3.2)}$$

for some x ∈ X.

Let t be a continuity point of (X, M,∗) then

$M(fx_n,fx,t) \geq$

$$\varphi\left\{min\left\{\begin{matrix}\left(\frac{M(gx_n,gx,t)+M(fx,gx,t)}{M(fx_n,gx,t)+M(fx,gx,t)}\right)M(gx_n,gx,t),\left(\frac{M(fx,gx,t)+M(gx,gx_n,t)}{M(fx_n,gx_n,t)+M(fx,gx_n,t)}\right)M(fx,gx,t),\\ \left(\frac{M(gx_n,gx,t)+M(fx_n,gx,t).M\big((fx,gx,t)\big)}{M(gx_n,gx,t)+M(fx,gx_n,t)}\right),\left(\frac{M(gx_n,gx,t)+M(fx_n,gx_n,t)+M\big((fx,gx,t)\big)}{M(fx_n,gx_n,t)+M(fx,gx,t)+M(fx_n,gx,t)}\right)\end{matrix}\right\}\right\}$$

(3.3.3)

for all n ∈ N. By taking the limit n tends to infinity in **(3.3.3)**, we have

M(gx,fx,t) ≥

$$\varphi\left\{min\left\{\begin{matrix}\left(\frac{M(gx,gx,t)+M(fx,gx,t)}{M(gx,gx,t)+M(fx,gx,t)}\right)M(gx,gx,t),\left(\frac{M(fx,gx,t)+M(gx,gx,t)}{M(gx,gx,t)+M(fx,gx,t)}\right)M(fx,gx,t),\\ \left(\frac{M(gx,gx,t)+M(gx,gx,t).M\big((fx,gx,t)\big)}{M(gx,gx,t)+M(fx,gx,t)}\right),\left(\frac{M(gx,gx,t)+M(gx,gx,t)+M\big((fx,gx,t)\big)}{M(gx,gx,t)+M(fx,gx,t)+M(gx,gx,t)}\right)\end{matrix}\right\}\right\}$$

$$= \varphi\{min\{1, M(fx,gx,t), 1,1\}\}$$

$$= \varphi(M(fx,gx,t)).$$

for every t>0 .Now, we show that gx=fx. If gx≠fx, then from (GV-1) and (GV-2)

$$0< M(gx,fx,t) <1 \qquad \textbf{(3.3.4)}$$

for all t>0. From condition of ($\varphi 2$), $\varphi(\mathrm{M}(\mathrm{gx},\mathrm{fx},\mathrm{t})) > \mathrm{M}(\mathrm{gx},\mathrm{fx},\mathrm{t})$ which is a contradiction

Hence fx=gx .

Let fx=gx=z.(say).

Since f and g are weakly compatible mappings fgx=gfx which implies that fz=fgx=gfx=gz .

Next , we will show that fz=z. We will suppose that fz $\neq z$. By (GV-1) and (GV-2) it implies that 0< M(fz,z,t) <1 for all t > 0 .By condition ($\varphi 2$) we know that $\varphi(\mathrm{M}(\mathrm{fz},\mathrm{z},\mathrm{t})) > \mathrm{M}(\mathrm{fz},\mathrm{z},\mathrm{t})$

It follows from condition (3.3.1) that

M(fz,z,t)=M(fz,fx,t) ≥

$$\varphi\left\{min\left\{\begin{array}{c}\left(\frac{\mathrm{M(gz,gx,t)+M(fx,gx,t)}}{\mathrm{M(fz,gx,t)+M(fx,gx,t)}}\right)\mathrm{M(gz,gx,t)},\left(\frac{\mathrm{M(fx,gx,t)+M(gx,gz,t)}}{\mathrm{M(fz,gz,t)+M(fx,gz,t)}}\right)\mathrm{M(fx,gx,t)},\\ \left(\frac{\mathrm{M(gz,gx,t)+M(fz,gx,t).M\big((fx,gx,t)\big)}}{\mathrm{M(gz,gx,t)+M(fx,gz,t)}}\right),\left(\frac{\mathrm{M(gz,gx,t)+M(fz,gz,t)+M\big((fx,gx,t)\big)}}{\mathrm{M(fz,gz,t)+M(fx,gx,t)+M(fz,gx,t)}}\right)\end{array}\right\}\right\}$$

$$= \varphi\big\{min\{\mathrm{M(fz,fx,t)},1,1,1\}\big\}$$

$$= \varphi\{\mathrm{M(fz,z,t)}\}$$

for all t>0 which is a contradicting the above inequality therefore fz=z and we have z = fz = gz consequently f and g have a common fixed point that is z.

Finally we will prove that a common fixed point of f and g is unique, for that let us suppose that w is a common fixed point of f and g in which w≠z. It follows from condition of (GV-1) and (GV-2) that for every t > 0,we have M(w,z,t) ∈(0,1) which implies that φ(M(w,z,t))> M(w,z,t).On the other hand ,we know that

M(z,w,t)= M(fz,fw,t) ≥

$$\varphi\left\{min\left\{\begin{array}{c}\left(\frac{\mathrm{M(gz,gw,t)+M(fw,gw,t)}}{\mathrm{M(fz,gw,t)+M(fw,gw,t)}}\right)\mathrm{M(gz,gw,t)},\left(\frac{\mathrm{M(fw,gw,t)+M(gw,gz,t)}}{\mathrm{M(fz,gz,t)+M(fw,gz,t)}}\right)\mathrm{M(fw,gw,t)},\\ \left(\frac{\mathrm{M(gz,gw,t)+M(fz,gw,t).M\big((fw,gw,t)\big)}}{\mathrm{M(gz,gw,t)+M(fw,gz,t)}}\right),\left(\frac{\mathrm{M(gz,gw,t)+M(fz,gz,t)+M\big((fw,gw,t)\big)}}{\mathrm{M(fz,gz,t)+M(fw,gw,t)+M(fz,gw,t)}}\right)\end{array}\right\}\right\}$$

$$= \varphi\big\{min\{\mathrm{M(z,w,t)},1,1,1\}\big\}$$

$$= \varphi\{\mathrm{M(z,w,t)}\}$$

for all t>0,which is a contradiction .

Hence we conclude that w=z which implies that f and g have a unique a common fixed point.

Theorem 3.4 Let (X, M,∗) be a GV- fuzzy metric space and f, g are weakly compatible self mappings of X such that,

M(fx,fy,kt) ≥

$$min\left\{\begin{array}{c}\left(\frac{M(gx,gy,t)+M(fy,gy,t)}{M(fx,gy,t)+M(fy,gy,t)}\right)M(gx,gy,t),\left(\frac{M(fy,gy,t)+M(gy,gx,t)}{M(fx,gx,t)+M(fy,gx,t)}\right)M(fy,gy,t),\\ \left(\frac{M(gx,gy,t)+M(fx,gy,t).M\big((fy,gy,t)\big)}{M(gx,gy,t)+M(fy,gx,t)}\right),\left(\frac{M(gx,gy,t)+M(fx,gx,t)+M\big((fy,gy,t)\big)}{M(fx,gx,t)+M(fy,gy,t)+M(fx,gy,t)}\right)\end{array}\right\}$$

(3.4.1)

for all x,y ∈ X,where t > 0 .If f and g satisfy (CLRg) property then f and g have a unique common fixed point.

Proof: Given that f and g satisfy (CLRg) property then we can find a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gx$ for some x ∈ X.

Let t be a continuity point of (X, M,∗) then

$M(fx_n,fx,kt) \geq$

$$min\left\{\begin{array}{c}\left(\frac{M(gx_n,gx,t)+M(fx,gx,t)}{M(fx_n,gx,t)+M(fx,gx,t)}\right)M(gx_n,gx,t),\left(\frac{M(fx,gx,t)+M(gx,gx_n,t)}{M(fx_n,gx_n,t)+M(fx,gx_n,t)}\right)M(fx,gx,t),\\ \left(\frac{M(gx_n,gx,t)+M(fx_n,gx,t).M\big((fx,gx,t)\big)}{M(gx_n,gx,t)+M(fx,gx_n,t)}\right),\left(\frac{M(gx_n,gx,t)+M(fx_n,gx_n,t)+M\big((fx,gx,t)\big)}{M(fx_n,gx_n,t)+M(fx,gx,t)+M(fx_n,gx,t)}\right)\end{array}\right\}$$

(3.4.2)

for all n ∈ N. By taking the limit n tends to infinity in equation (3.4.2), we have

M(gx,fx,kt) ≥

$$min\left\{\begin{matrix}\left(\frac{M(gx,gx,t)+M(fx,gx,t)}{M(gx,gx,t)+M(fx,gx,t)}\right)M(gx,gx,t),\left(\frac{M(fx,gx,t)+M(gx,gx,t)}{M(gx,gx,t)+M(fx,gx,t)}\right)M(fx,gx,t),\\ \left(\frac{M(gx,gx,t)+M(gx,gx,t).M\big((fx,gx,t)\big)}{M(gx,gx,t)+M(fx,gx,t)}\right),\left(\frac{M(gx,gx,t)+M(gx,gx,t)+M\big((fx,gx,t)\big)}{M(gx,gx,t)+M(fx,gx,t)+M(gx,gx,t)}\right)\end{matrix}\right\}$$

$$= min\{1, M(fx,gx,t), 1,1\}$$

$$= M(gx,fx,t).$$

Hence from the lemma (2.17) we have gx=fx.

Let fx=gx=z(say). Since f and g are weakly compatible mappings fgx=gfx which implies that fz=fgx=gfx=gz .Next we will show that fz=z.

We will suppose that fz $\neq z$.

Now it follows from condition (3.4.1) that

M(fz,z,kt)=M(fz,fx,kt) ≥

$$min\left\{\begin{matrix}\left(\frac{M(gz,gx,t)+M(fx,gx,t)}{M(fz,gx,t)+M(fx,gx,t)}\right)M(gz,gx,t),\left(\frac{M(fx,gx,t)+M(gx,gz,t)}{M(fz,gz,t)+M(fx,gz,t)}\right)M(fx,gx,t),\\ \left(\frac{M(gz,gx,t)+M(fz,gx,t).M\big((fx,gx,t)\big)}{M(gz,gx,t)+M(fx,gz,t)}\right),\left(\frac{M(gz,gx,t)+M(fz,gz,t)+M\big((fx,gx,t)\big)}{M(fz,gz,t)+M(fx,gx,t)+M(fz,gx,t)}\right)\end{matrix}\right\}$$

$$= min\{M(fz,fx,t), 1,1,1\}$$

$$= M(fz,z,t)$$

Hence from the lemma (2.17) we have fz=z and z = fz = gz consequently f and g have a common fixed point that is z.

Finally we will prove that a common fixed point of f and g is unique, for that let us suppose that w is a common fixed point of f and g in which w≠z. It follows from condition (3.4.1) that

M(z,w,t)=M(fz,fw,t) ≥

$$min\left\{\begin{matrix}\left(\frac{M(gz,gw,t)+M(fw,gw,t)}{M(fw,gw,t)+M(fw,gz,t)}\right)M(gz,gw,t),\left(\frac{M(fw,gw,t)+M(gw,gz,t)}{M(fz,gz,t)+M(fw,gz,t)}\right)M(fw,gw,t),\\ \left(\left(\frac{M(gz,gw,t)+M(fz,gw,t).M\big((fw,gw,t)\big)}{M(gz,gw,t)+M(fw,gz,t)}\right),\left(\frac{M(gz,gw,t)+M(fz,gz,t)+M\big((fw,gw,t)\big)}{M(fz,gz,t)+M(fw,gw,t)+M(fz,gw,t)}\right)\right)\end{matrix}\right\}$$

$$= min\{M(z,w,t),1,1,1\}$$

$$= M(z,w,t)$$

Hence from the lemma (2.17) we have w=z which implies that f and g have a unique a common fixed point.

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