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Interval Oscillation Criteria for Forced Fractional Differential Equations with Mixed Nonlinearities

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Abstract

In this paper, we investigate the oscillatory behavior of forced fractional differential equation with mixed nonlinearities of the form

$$T_\alpha[r(t)T_\alpha x(t)] + q(t)x(t) + \sum_{i=1}^{n} q_i(t)\,|\,x\,|^{\alpha_i}\, sgnx = e(t), \quad t \geq t_0 > 0,$$

where $0 < \alpha \leq 1$, $T_\alpha(.)$ denotes the conformable fractional derivative introduced by R. Khalil et al. [6], $r \in C^\alpha([t_0,\infty),\mathbb{R}^+)$, $e \in C([t_0,\infty),\mathbb{R})$, $q(t)$, $q_i(t) \in C([t_0,\infty),\mathbb{R})$ and $\alpha_1 > \ldots > \alpha_m > 1 > \alpha_m + 1 > \ldots > \alpha_n > 0$. By using the properties of conformable fractional derivative, a generalized Riccati transformation and the integral averaging technique, we establish some interval oscillation results. Illustrative examples are also given.

Key words and phrases. Oscillation; Conformable fractional derivative; Mixed nonlinearities; Riccati transformation.

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1. INTRODUCTION

In recent years, many researchers found that the fractional differential equations are more accurate in describing some practical models. Today it has been used widely in physics, electrochemistry, control theory and electromagnetic fields [2,5,7,9]. It should be noted that there has been a great deal of work on the oscillatory behavior of integer order differential equations; see [8,10,12]. However, there are only few papers dealing with the oscillation of fractional differential equation; see [4,3,13,11].

In this paper, we consider the forced fractional differential equation with mixed nonlinearities of the form

$$T_\alpha[r(t)T_\alpha x(t)] + q(t)x(t) + \sum_{i=1}^{n} q_i(t)\,|x|^{\alpha_i}\, sgnx = e(t), \quad t \ge t_0 > 0, \tag{1.1}$$

where $0 < \alpha \le 1$, $T_\alpha(.)$, denotes the conformable fractional derivative introduced by R. Khalil et al. [6], $r \in C^\alpha([t_0,\infty),\mathbb{R}^+)$, $e \in C([t_0,\infty),\mathbb{R})$, $q(t), q_i(t) \in C([t_0,\infty),\mathbb{R})$ and $\alpha_1 > \ldots > \alpha_m > 1 > \alpha_m + 1 > \ldots > \alpha_n > 0$.

A solution of equation (1.1) is called oscillatory if it has arbitrarily large zeros, otherwise it is called nonoscillatory. Equation (1.1) is called oscillatory if all its solutions are oscillatory.

We use the following definition introduced by R. Khalil et al. [6].

Given $f: [0,\infty) \to \mathbb{R}$. Then the "conformable fractional derivative" of f of order α is defined by

$$T_\alpha(f)(t) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon}$$

for all $t>0$, $\alpha \in (0,1]$. If f is α -differentiable in some $(0, a)$, $a>0$ and $\lim_{t\to 0^+} f^{(\alpha)}(t)$ exists, then define

$$f^{(\alpha)}(0) = \lim_{t \to 0^+} f^{(\alpha)}(t)$$

Conformable fractional derivative has the following properties :

Let $\alpha \in (0,1]$ and f, g be α - differentiable at a point $t > 0$. Then

(a) $T_\alpha(af + bg) = aT_\alpha(f) + bT_\alpha(g)$, for all $a, b \in \mathbb{R}$.

(b) $T_\alpha(t^p) = pt^{p-\alpha}$ for all $p \in \mathbb{R}$.

(c) $T_\alpha(\lambda) = 0$, for all constant functions $f(t) = \lambda$.

(d) $T_\alpha(fg) = fT_\alpha(g) + gT_\alpha(f)$.

(e) $T_\alpha\left(\frac{f}{g}\right) = \frac{gT_\alpha(f) - fT_\alpha(g)}{g^2}$.

(f) If, in addition, f is differentiable, then $T_\alpha(f)(t) = t^{1-\alpha}\frac{df}{dt}(t)$.

For proving our main results we use the followings:

(i) $D(a,\ b) = \{u \in C^1([a,b]) : u(t) \neq 0 \text{ for } t \in (a,\ b),\ u(a) = u(b) = 0\}$.

(ii) For any $t \geq t_0$, there exist $a_1,\ b_1,\ a_2,\ b_2$ such that $T \leq a_1 < b_1 \leq a_2 < b_2$ and

$$\begin{cases} q_i(t) \geq 0 & for\ t \in [a_1, b_1] \cup [a_2, b_2], \quad i = 1, 2, ..., n, \\ e(t) \leq 0 & for\ t \in [a_1, b_1], \\ e(t) \geq 0 & for\ t \in [a_2, b_2]. \end{cases}$$

In this paper, we established some interval oscillation criteria for equation (1.1) in Section 2. Further in Section 3, we have given some examples to illustrate our main results.

2. MAIN RESULTS

We need the following lemmas to prove our main results.

Lemma 2.1. [12]*Let* $\alpha_i,\ i = 1, 2, ..., n,$ *be the* n*-tuple satisfying* $\alpha_1 > ... > \alpha_m > 1 > \alpha_m + 1 > ... > \alpha_n > 0$. *Then there exists an* n*-tuple* $(\eta_1, \eta_2, ..., \eta_n)$ *satisfying*

$$\sum_{i=1}^{n} \alpha_i \eta_i = 1, \tag{2.1}$$

which also satisfies either

$$\sum_{i=1}^{n} \eta_i < 1, \quad 0 < \eta_i < 1 \quad or \tag{2.2}$$

$$\sum_{i=1}^{n} \eta_i = 1, \quad 0 < \eta_i < 1 \tag{2.3}$$

Lemma 2.2. [8] *Let* u, B *and* C *be positive real numbers and* l, m *be ratio of odd positive integers. Then*

(i) $l > m + 1, \quad 0 < m \leq 1, \quad u^{l-1} + M_1 B^{\frac{l-1}{l-m-1}} \geq B u^m,$

(ii) $0 < l + m < 1,\ u^{l+m-1} + M_2 C^{\frac{1-l}{1-l-m}} u^m \geq C,$

where

$$M_1 = \left(\frac{m}{l-1}\right)^{\frac{m}{l-m-1}} \left(\frac{l-m-1}{l-1}\right), \quad M_2 = \left(\frac{1-l-m}{1-l}\right)\left(\frac{m}{l-1}\right)^{\frac{m}{1-l-m}}.$$

Theorem 2.1. *Suppose that condition (ii) holds. If there exists a function* $G \in D(a_{i,} b_i)$ *such that the inequality*

$$\int_{a_i}^{b_i} G^2(t)Q(t)dt > \frac{1}{4}\int_{a_i}^{b_i}\left((1-\alpha)+2t\frac{|G'(t)|}{|G(t)|}\right)^2 t^{-2\alpha} r(t)G^2(t)dt, \tag{2.4}$$

holds for $i = 1, 2,$ *where*

$$Q(t) = q(t) + \kappa_0 |e(t)|^{\eta_0} \prod_{i=1}^{n} q_i^{\eta_i}(t), \tag{2.5}$$

$\kappa_0 = \prod_{i=0}^{n} \eta_i^{-\eta i}$, $\eta_0 = 1 - \sum_{i=1}^{n} \eta_i$ *and* $\eta_1, \eta_2, ..., \eta_n$ *are positive constants satisfying (2.1) and (2.2) in Lemma* 2.2, *then equation (1.1) is oscillatory.*

Proof. Suppose that $x(t)$ is a nonoscillatory solution of equation (1.1). Then $x(t)$ eventually must have one sign, i.e., $x(t) \neq 0$ on $[T_0, \infty)$ for some large $T_0 \geq t_0$. Define

$$w(t) := \frac{r(t)T_\alpha x(t)}{x(t)} \tag{2.6}$$

for $t \geq T_0$. Then we get

$$T_\alpha w(t) = -\left(q(t) - e(t)x^{-1}(t) + \sum_{i=1}^{n} q_i(t)\frac{|x|^{\alpha_i}}{x(t)}\operatorname{sgn} x\right) - \frac{w^2(t)}{r(t)} \tag{2.7}$$

for $t \geq T_0$. By assuming (ii), if $x(t) > 0$, then we can choose $a_1, b_1 \geq T_0$ such that $e(t) \leq 0$ for $t \in [a_1, b_1]$. Similarly if $x(t) < 0$, then we can choose $a_2, b_2 \geq T_0$ such that $e(t) \geq 0$ for $t \in [a_2, b_2]$. So $\frac{e(t)}{x(t)} \leq 0$ $\left(i.e., -\frac{e(t)}{x(t)} = \left|\frac{e(t)}{x(t)}\right|\right)$ for $t \in [a_i, b_i]$, $i = 1, 2.$
Also $q_i(t) \geq 0$ for $t \in [a_1, b_1] \cup [a_2, b_2]$, $i = 1, 2, ..., n.$
Therefore equation (2.7) becomes

$$T_\alpha w(t) = -\left(q(t) - \left|\frac{e(t)}{x(t)}\right| + \sum_{i=1}^{n} q_i(t)|x|^{\alpha_i - 1}\right) - \frac{w^2(t)}{r(t)}. \tag{2.8}$$

Now, recall the arithmetic-geometric mean inequality [1]

$$\sum_{i=1}^{n} \eta_i u_i \geq \prod_{i=1}^{n} u_i^{\eta_i}, \quad u_i \geq 0, \quad \eta_i \geq 0. \tag{2.9}$$

Choose $\eta_1,\eta_2,...,\eta_n$ according to given $\alpha_1,\alpha_2,...,\alpha_n$ satisfying (2.1) and (2.2). Then identify $u_0=\eta_0^{-1}\left|\frac{e(t)}{x(t)}\right|$ and $u_i=\eta_i^{-1}q_i(t)\,|\,x\,|^{\alpha_i-1}$ from equation (2.8) and using the inequality (2.9), we obtain

$$T_\alpha w(t)\le -Q(t)-\frac{w^2(t)}{r(t)}, \tag{2.10}$$

where $Q(t)$ is defined by equation (2.5). Hence equation (2.10) holds for $t\in[a_{1,}b_1]$ or $t\in[a_{2,}b_2]$.

By using the properties of conformable fractional derivative [6], we get

$$Q(t)\le -t^{1-\alpha}w'(t)-\frac{w^2(t)}{r(t)} \tag{2.11}$$

for $t\in[a_i,b_i],\quad i=1,2$.

Now multiplying the inequality (2.11) by $G^2(t)$ and integrating from a_i to b_i, we obtain

$$\int_{a_i}^{b_i}G^2(t)Q(t)dt\le -\int_{a_i}^{b_i}G^2(t)t^{1-\alpha}w'(t)dt-\int_{a_i}^{b_i}\frac{G^2(t)}{r(t)}w^2(t)dt$$
$$=\int_{a_i}^{b_i}\big((1-\alpha)G(t)+2tG'(t)\big)t^{-\alpha}G(t)w(t)dt$$
$$-\int_{a_i}^{b_i}\frac{w^2(t)}{r(t)}G^2(t)dt \tag{2.12}$$

for $t\in[a_i,b_i],\quad i=1,2$.

Setting

$$m(v)=\left((1-\alpha)+2t\frac{|G'(t)|}{|G(t)|}\right)t^{-\alpha}v-\frac{v^2}{r(t)},\quad v>0,$$

we have $m'(v)=0$ and $m''(v^*)<0$, where $v^*=\frac{r(t)t^{-\alpha}}{4}\left((1-\alpha)+2t\frac{|G'|}{|G|}\right)$,

which implies that $m(v)$ obtains its maximum at v^*. So we have

$$m(v)\le m(v^*)=\frac{r(t)t^{-2\alpha}}{4}\left((1-\alpha)+2t\frac{|G'(t)|}{|G(t)|}\right)^2. \tag{2.13}$$

Then, by using (2.13) in (2.12), we obtain

$$\int_{a_i}^{b_i}G^2(t)Q(t)dt\le\frac{1}{4}\int_{a_i}^{b_i}\left((1-\alpha)+2t\frac{|G'(t)|}{|G(t)|}\right)^2t^{-2\alpha}r(t)G^2(t)dt, \tag{2.14}$$

which contradicts the inequality (2.4).

Hence the proof is complete.

The following theorem gives an interval oscillation criteria for equation (1.1) with $e(t) \equiv 0.$

Theorem 2.2. *Assume that for any* $t \geq t_0$, *there exists* a, b *such that* $T \leq a < b$ *and* $q(t), q_i(t) \geq 0$ *for* $t \in [a,b]$ *and if there exists a function* $G \in D(a,b)$ *such that the inequality*

$$\int_a^b G^2(t)\bar{Q}(t)dt > \frac{1}{4}\int_a^b \left((1-\alpha) + 2t\frac{|G'(t)|}{|G(t)|} \right)^2 t^{-2\alpha} r(t) G^2(t)dt, \tag{2.15}$$

holds, where

$$\bar{Q}(t) = q(t) + \kappa_1 \prod_{i=1}^{n} q_i^{\eta_i}(t), \tag{2.16}$$

$\kappa_1 = \prod_{i=1}^{n} \eta_i^{-\eta_i}$, *and* $\eta_1, \eta_2, ..., \eta_n$ *are positive constants (2.1) and (2.3) in Lemma 2.2, then equation (1.1) is oscillatory.*

Proof. The proof is immediate from Theorem 2.1, if we put $e(t) = 0$, $\eta_0 = 0$ and applying conditions (2.1) and (2.3) of Lemma 2.2.

Next we discuss the oscillatory behavior of the equation

$$T_\alpha(r(t)T_\alpha x(t)) + q(t)x(t) + q_1(t)x^{\alpha_1}(t) = 0, \quad t \geq t_0, \tag{2.17}$$

where α_1 is a ratio of odd positive integers.

Theorem 2.3. *Assume that for any* $t \geq t_0$, *there exists* a, b *such that* $T \leq a < b$ *and* $q(t), q_1(t) \geq 0$ *for* $t \in [a,b]$ *and if there exists a function* $G \in D(a,b)$ *such that the inequality*

$$\int_a^b G^2(t)Q_1(t)dt > \frac{1}{4}\int_a^b \left((1-\alpha) + 2t\frac{|G'(t)|}{|G(t)|} \right)^2 t^{-2\alpha} r(t) G^2(t)dt, \tag{2.18}$$

where $\alpha_1 > \beta + 1, \quad 0 < \beta \leq 1, \quad Q_1(t) = q(t) - M_1 q_1(t)(\rho(t))^{\frac{\alpha_1 - 1}{\alpha_1 - \beta - 1}}$

where $M_1 = \left(\frac{\beta}{\alpha_1 - 1} \right)^{\frac{\beta}{\alpha_1 - \beta - 1}} \left(\frac{\alpha_1 - \beta - 1}{\alpha_1 - 1} \right)$ *and* $\rho(t)$ *is a positive continuous function, then equation (2.17) is oscillatory.*

Proof. Suppose that $x(t)$ is a nonoscillatory solution of equation (2.17). Without loss of generality, we may assume that $x(t) > 0$, for $t \in [a,b]$. Define

$$w(t) := -\frac{r(t)T_\alpha x(t)}{x(t)}, \quad t \in [a,b].$$

Then for $t \in [a,\ b]$, we get

$$T_\alpha w(t) = q(t) + q_1(t)x^{\alpha_1 - 1}(t) + \frac{w^2(t)}{r(t)} \tag{2.19}$$

or

$$\begin{aligned} T_\alpha w(t) &\geq q(t) + q_1(t)x^{\alpha_1 - 1}(t) - q_1(t)\rho(t)x^{\beta}(t) + \frac{w^2(t)}{r(t)} \\ &= q(t) + q_1(t)\left(x^{\alpha_1 - 1}(t) - \rho(t)x^{\beta}(t)\right) + \frac{w^2(t)}{r(t)}. \end{aligned}$$

Thus by Lemma 2.3(i), we have

$$T_\alpha w(t) \geq Q(t) + \frac{w^2(t)}{r(t)}. \tag{2.20}$$

Then proceeding as in Theorem 2.1, we obtain a contradiction to (2.18). Hence the proof is complete.

Theorem 2.4. *Assume that for any* $t \geq t_0$, *there exists* $a,\ b$ *such that* $T \leq a < b$ *and* $q(t), q_i(t) \geq 0$ *for* $t \in [a,b]$ *and if there exists a function* $G \in D(a,b)$ *such that the inequality*

$$\int_a^b G^2(t)Q_2(t)dt > \frac{1}{4}\int_a^b \left((1-\alpha) + 2t\frac{|G'(t)|}{|G(t)|}\right)^2 t^{-2\alpha} r(t)G^2(t)dt, \tag{2.21}$$

where $0 < \alpha_1 + \alpha_2 < 1, \quad Q_2(t) = q(t) - M_2 q_1(t)(\rho(t))^{\frac{1-\alpha_1}{1-\alpha_1-\alpha_2}}$

where $M_2 = \left(\dfrac{1-\alpha_1-\alpha_2}{1-\alpha_1}\right)\left(\dfrac{\alpha_2}{1-\alpha_1}\right)^{\frac{\alpha_2}{1-\alpha_1-\alpha_2}}$ *and* $\rho(t)$ *is a positive continuous function, then equation (2.17) is oscillatory.*

Proof. Proceeding as in the proof of Theorem 2.3, we obtain (2.19) or

$$T_\alpha w(t) \geq q(t) + q_1(t)\left(x^{\alpha_1 - 1}(t) - \rho(t)x^{-\alpha_2}(t)\right) + \frac{w^2(t)}{r(t)}.$$

Now apply Lemma 2.3(ii) and then proceed as in the proof of Theorem 2.1. Thus we obtain a contradiction to condition (2.21). This completes the proof.

3. EXAMPLES

Example 3.1. Consider the fractional differential equation

$$T_{1/2}\left(sin^2 t T_{1/2}x(t)\right) + l\, cos\, t\, x(t) + m\, sin\, t\, |x|^2\, sgn\, x$$

$$+n\cos t\,|x|^{1/2}\,sgnx = -\sin 4t, \quad t \geq 2, \tag{3.1}$$

where l, m and n are positive constants.

Here $r(t) = \sin^2 t,\ q(t) = l\cos t,\ q_1(t) = m\sin t,\ q_2(t) = n\cos t$ and $e(t) = -\sin\ 4t.$

Now, if we choose $\eta_0 = 1/3,\ \eta_1 = 4/9$ and $\eta_2 = 2/9,$ then

$$Q(t) = l\cos t + \kappa_0\,|\sin\ 4t|^{1/3}\,(m\sin t)^{4/9}(n\cos t)^{2/9},$$

where $\kappa_0 = 3^{1/3}\left(\frac{9}{4}\right)^{4/9}\left(\frac{9}{2}\right)^{2/9}.$

Next by choosing $G(t) = \sin^2(4t),\ a_1 = 0,\ b_1 = a_2 = \pi/4$ and $b_2 = \pi/2,$ then it is easy to verify that

$$\int_{a_1}^{b_1} G^2(t)Q(t)dt = 0.27043l + 0.519051m^{4/9}n^{2/9}$$

and

$$\frac{1}{4}\int_{a_1}^{b_1}\left((1-\alpha) + 2t\frac{|G'(t)|}{|G(t)|}\right)^2 t^{-2\alpha}r(t)G^2(t)dt = 0.568488.$$

Also

$$\int_{a_2}^{b_2} G^2(t)Q(t)dt = 0.112016l + 0.632953m^{4/9}n^{2/9}$$

and

$$\frac{1}{4}\int_{a_2}^{b_2}\left((1-\alpha) + 2t\frac{|G'(t)|}{|G(t)|}\right)^2 t^{-2\alpha}r(t)G^2(t)dt = 2.61308.$$

If we choose the constants l, m and n such that

$$0.27043l + 0.519051m^{4/9}n^{2/9} > 0.568488 \tag{3.2}$$

and

$$0.112016l + 0.632953m^{4/9}n^{2/9} > 2.61308, \tag{3.3}$$

then inequality (2.4) will be satisfied for $i = 1, 2.$

In fact, for $l = m = 5$ and $n = 3,$ inequalities (3.2) and (3.3) hold.

Thus by Theorem 2.1, equation (3.1) is oscillatory.

Example 3.2. Consider the fractional differential equation

$$T_{1/2}\left(\sin^2 t T_{1/2}x(t)\right) + l\cos t\,x(t) + m\sin t\,|x|^2\,sgn\ x$$

$$+n\cos t\,|x|^{2/3}\,sgnx = 0, \quad t \geq 2, \tag{3.4}$$

where l, m and n are positive constants.

Here $r(t) = \sin^2 t,\ q(t) = l\cos t,\ q_1(t) = m\sin t,\ q_2(t) = n\cos t$ and $e(t) = 0.$

Now, if we choose $\eta_1 = 1/4$ and $\eta_2 = 3/4,$ then

$$\bar{Q}(t) = l\cos t + \kappa_1(m\sin t)^{1/4}(n\cos t)^{3/4},$$

where $\kappa_1 = 4^{1/4}\left(\frac{4}{3}\right)^{3/4}$.

Next, by choosing $G(t) = -sin4t$, $a = 0$ and $b = \pi/4$, then it is easy to verify that

$$\int_a^b G^2(t)\bar{Q}(t)dt = 0.359165l + 0.496695m^{1/4}n^{3/4}$$

and

$$\frac{1}{4}\int_a^b \left((1-\alpha) + 2t\frac{|G'(t)|}{|G(t)|}\right)^2 t^{-2\alpha} r(t)G^2(t)dt = 0.941205.$$

If we choose the constants l, m and n such that

$$0.359165l + 0.496695m^{4/9}n^{2/9} > 0.941205 \tag{3.5}$$

then inequality (2.16) will be satisfied for $i = 1, 2$.

In fact, for $l = 2$, $m = 5$ and $n = 4$, inequality (3.5) holds. Thus by Theorem 2.2, equation (3.4) is oscillatory.

Example 3.3. Consider the fractional differential equation

$$T_{1/3}\left(sin^2 t T_{1/3}x(t)\right) + l\,sin t\, x(t) + l_1 cos t\, x^5(t) = 0, \quad t \geq t_0 > 0, \tag{3.6}$$

where l and l_1 are positive constants.

Here $\alpha_1 = 5$, $r(t) = sin^2 t$, $q(t) = l\,sin t$ and $q_1(t) = l_1 cos t$.

Now, if we take $G(t) = sin6t$, $\beta = 1$ and $\rho(t) = 4$,

we have

$$M_1 = \left(\frac{1}{4}\right)^{1/3}\left(\frac{3}{4}\right) \text{and } Q_1(t) = l\,sin\,t - 4^{4/3}M_1(l_1 cos\ t).$$

Next by choosing $a = \pi/6$ and $b = \pi/2$, then it is easy to verify that

$$\int_a^b G^2(t)Q_1(t)dt = 0.436041l - 0.755339l_1$$

and

$$\frac{1}{4}\int_a^b \left((1-\alpha) + 2t\frac{|G'(t)|}{|G(t)|}\right)^2 t^{-2\alpha} r(t)G^2(t)dt = 8.83385.$$

If we choose the constants l and l_1 such that

$$0.436041l - 0.755339l_1 > 8.83385 \tag{3.7}$$

then inequality (2.18) will be satisfied for $i = 1, 2$.

In fact, for $l = 25$ and $l_1 = 2$, inequality (3.7) holds. Thus by Theorem 2.3, equation (3.6) is oscillatory.

Example 3.4. Consider the fractional differential equation

$$T_{1/3}\left(T_{1/3}x(t)\right) + se^t\, x(t) + s_1 e^{t/5}\, x^{1/3}(t) = 0, \quad t \geq t_0 > 0, \tag{3.8}$$

where s and s_1 are positive constants.

Here $\alpha_1 = \frac{1}{3}$, $r(t) = 1$, $q(t) = se^t$ and $q_1(t) = s_1 e^{t/5}$.

Now, if we take $G(t) = sin 8t$, $\alpha_2 = \frac{1}{3}$ and $\rho(t) = 3$, we have

$$M_2 = \frac{1}{4} \text{ and } Q_2(t) = se^t - 9M_2(s_1 e^{t/5}).$$

Next by choosing $a = 0$ and $b = \pi/4$, then it is easy to verify that

$$\int_a^b G^2(t)Q_2(t)dt = 0.594318s - 0.9566s_1$$

and

$$\frac{1}{4}\int_a^b \left((1-\alpha) + 2t\frac{|G'(t)|}{|G(t)|} \right)^2 t^{-2\alpha} r(t)G^2(t)dt = 8.93858.$$

If we choose the constants s and s_1 such that

$$0.594318s - 0.9566s_1 > 8.93858 \quad (3.9)$$

then inequality (2.22) will be satisfied.

In fact, for $s = 25$ and $s_1 = 5$, inequality (3.9) holds. Thus by Theorem 2.4, equation (3.8) is oscillatory.

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