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Solving Transportation Problem with Generalized Hexagonal and Generalized Octagonal Fuzzy Numbers by Ranking Method

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Abstract

Transportation problem plays important role in scheduling time. It also minimizes routine chain. With the help of Fuzzy transportation problem, we can clear the complexity and uncertainty in transportation. One of the goals of transportation problem is schedule the programme in higher degree, which can reduce the expenditure as well as routine time of transportation of goods. It would definitely results in a ranking method for generalized hexagonal and generalized octagonal fuzzy numbers. Fuzzy Transportation Problem has converted into crisp values. By solving numerical examples we get optimal solution.

Keywords: Fuzzy transportation problem, Fuzzy numbers, generalized hexagonal fuzzy numbers, generalized octagonal fuzzy numbers.

MSC: 49K, 90B, 94D

1. INTRODUCTION:

The transportation problem manages to reduce transportation of goods by scheduling less transportation route for a single commodity from given number of sources to given number of destinations. Transportation problem is a study of optimal transportation cost. The standard statement of the transportation problem is a matrix with the rows representing sources and columns representing destinations.

There are different methods which can solve transportation problem in fuzzy environment but all are represented by normal fuzzy numbers. Chen et al. [4] has studied that there is not compulsion for membership function for normal fuzzy numbers. He proposed concept of generalized fuzzy numbers. Amarpreet et al. [1] proposed a generalized fuzzy transportation problem with trapezoidal fuzzy numbers and solved numerical example with ranking function. Ranking of normal fuzzy numbers was firstly, introduced by Jain [8]. Some researchers used ranking method with different manners. Phanibhushan et al. [11] investigate distance method by using ranking of fuzzy numbers with centroid. Rajarajeswari et al. [12] proposed ordering of generalized fuzzy numbers based on area, mode, and divergence, spread and solved with generalized hexagonal fuzzy numbers. Malini et al. [10] introduced a ranking function with octagonal fuzzy numbers. Klir et al. [9] covered a good and advance knowledge of fuzzy set and fuzzy logic.

Chu et al. [5] utilized the area between the centroid based on distance method to rank fuzzy numbers. Wang et al. [15] proposed the revised method of ranking fuzzy numbers with an area between the centroid and original point. Annie et al. [3] proposed the best candidate method and solve transportation problem. They used hexagonal fuzzy numbers by centroid ranking technique. Roseline et al. [13] proposed a ranking technique for generalized trapezoidal fuzzy numbers. He also used a fuzzy Hungarian method to find initial solution. Singh Pushpinder [14] suggested a novel method to rank a generalized fuzzy numbers. Ghadle and Pathade [7] solved octagonal fuzzy numbers by ranking method. They used balanced and unbalanced fuzzy transportation problem.

2. PRELIMINARIES:

2.1 Fuzzy number: A fuzzy set A of the real line R with membership function $\mu_A(x)$: $R \rightarrow [0,1]$ is called fuzzy number if a) A must be normal and convex fuzzy set; b) The support of A, must be bounded; c) $\alpha.A$ must be closed interval for every α in [0,1].

2.2 Fuzzy set: A fuzzy set is characterized by a membership function mapping element of a domain, space, or the universe of discourse X to the unit interval [0,1] i.e. $A = \{(\mu_A(x)\ x \in X)\}$. Here $\mu_A(x)$: $X \rightarrow [0,1]$ is a mapping called the degree of membership value of $x \in X$ in the fuzzy set A. These membership grades are often represented by real numbers ranking from $[0,1]$.

2.3 Generalized fuzzy number: A fuzzy set $A = (a, b, c, d, w)$ is defined on universal set of real numbers$R$, is said to be generalized fuzzy number if its membership function has the following attributes:

(i) $\mu_A(x) : R \rightarrow [0,1]$ is continuous ;

(ii) $\mu_A(x) = 0$ for all $x \in A\ (-\infty, a]\ U\ [\ d, \infty)$;

(iii) $\mu_A(x)$ is strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$;

(iv) $\mu_A(x) = w$ for all $x \in [b, c]$, where $0 < w \leq 1$

2.4 Generalized Hexagonal fuzzy number: If a generalized hexagonal fuzzy number denoted by $A = (a_1, a_2, a_3, a_4, a_5, a_6; w)$ where $a_1, a_2, a_3, a_4, a_5, a_6$ are real numbers and w is its maximum membership degree, its membership degree, its membership function is given below:

$$\mu_A(x) = \begin{cases} \frac{w}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & for\ a_1 \leq x \leq a_2 \\ \frac{w}{2}+\frac{w}{2}\left(\frac{x-a_2}{a_3-a_2}\right), & for\ a_2 \leq x \leq a_3 \\ w, & for\ a_3 \leq x \leq a_4 \\ w - \frac{w}{2}\left(\frac{x-a_4}{a_5-a_4}\right), & for\ a_4 \leq x \leq a_5 \\ \frac{w}{2}\left(\frac{a_6-x}{a_6-a_5}\right), & for\ a_5 \leq x \leq a_6 \\ 0, & otherwise \end{cases}$$

2.5 Generalized Octagonal Fuzzy number: A fuzzy number A is said to be a generalized octagonal fuzzy number denoted by $A = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; k, w)$ where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8$ are real numbers and its membership function μ_A is given by:

$$\mu_A(x) = \begin{cases} \frac{k(x-a_1)}{(a_2-a_1)}, & for\ a_1 \leq x \leq a_2 \\ k, & for\ a_2 \leq x \leq a_3 \\ k + (w-k)\frac{(x-a_3)}{(a_4-a_3)}, & for\ a_3 \leq x \leq a_4 \\ w, & for\ a_4 \leq x \leq a_5 \\ k + (w-k)\frac{(a_6-x)}{(a_6-a_5)}, & for\ a_5 \leq x \leq a_6 \\ k, & for\ a_6 \leq x \leq a_7 \\ k\frac{(a_8-x)}{(a_8-a_7)}, & for\ a_7 \leq x \leq a_8 \\ 0, & Otherwise \end{cases}$$

3. RANKING OF GENERALIZED HEXAGONAL FUZZY NUMBERS:

For Ranking of fuzzy numbers, we have to rank a more numbers. Ghadle and Pathade [6] solved hexagonal fuzzy numbers with a balanced and unbalanced fuzzy transportation problem by compared both type of numbers and solved by numerical example.

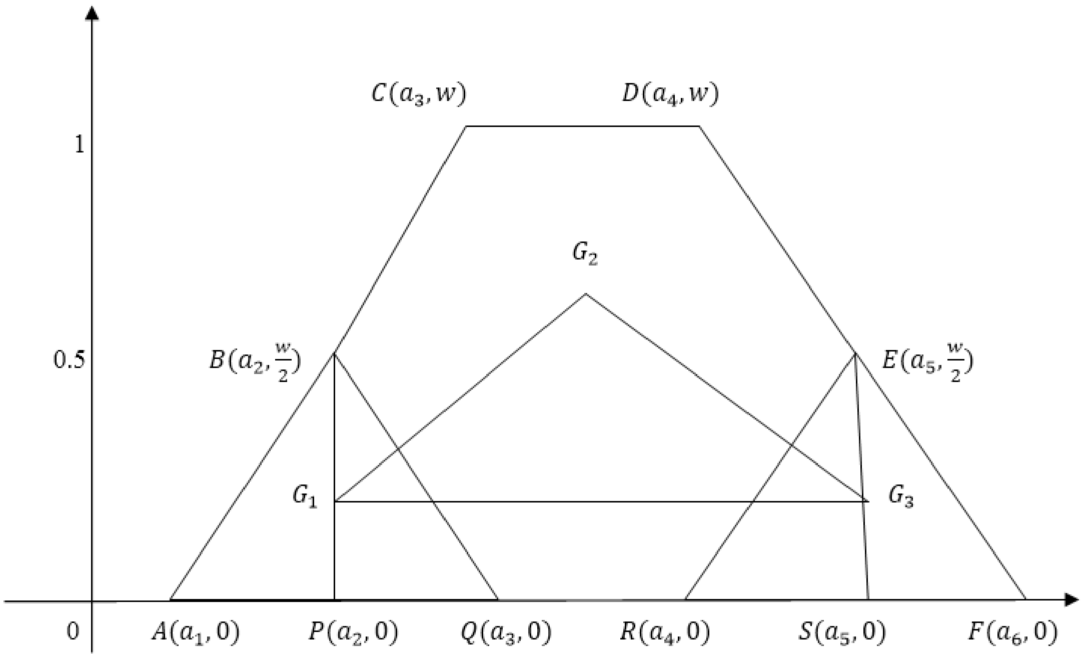

Figure: 1

The centroid of a fuzzy number is considered to be the balancing point of the hexagon. Divide the hexagonal into three plane figures. These three plane figures are a triangle ABQ, Hexagon CDERQB and again a triangle REF respectively. The circumcenter of the centroids of these three plane figures is taken as the point of reference to define the ranking of generalized hexagonal fuzzy numbers. Let the centroid of three plane figures be G_1, G_2, G_3 resply.

The centroid of the three plane figures is

$$G_1 = \left(\frac{a_1+a_2+a_3}{3}, \frac{w}{6}\right) \ , \ G_2 = \left(\frac{a_2+2a_3+2a_4+a_5}{6}, \frac{w}{2}\right) \text{ and } G_3 = \left(\frac{a_4+a_5+a_6}{3}, \frac{w}{6}\right) \text{ resply.}$$

Equation of the line G_1, G_3 is $y = \frac{w}{6}$ and G_2 does not lie on the line G_1, G_3. Thus $G_1, G_2 and\ G_3$ are non collinear and they form a triangle.

The ranking function of the generalized hexagonal fuzzy number $A_H = (a_1, a_2, a_3, a_4, a_5, a_6;\ w)$ which maps the set of all fuzzy numbers to a set of real numbers.

$$R(A_H) = (x_0)(y_0) = \left(\frac{2a_1+3a_2+ 4a_3+4a_4+ 3a_5+2\ a_6}{18} \times \frac{5w}{18}\right)$$

$$= \ (3.64)(0.63) + (2.15)(2.84) + (2.15)(3.33) + (2.95)(0.42) + (2.82)(3.96)$$

$$= 27.9649$$

Numerical example 1:

Consider the following problem with hexagonal fuzzy numbers:

Table 1

	d_1	d_2	d_3	$supply$
s_1	(3,7,11,15,19,24)	(3,5,7,9,10,12)	(11,14,17,21,25,30)	(7,9,11,13,16,20)
s_2	(3,5,7,9,10,12)	5,7,10,13,17,21)	(7,9,11,14,18,22)	(6,8,11,14,19,25)
s_3	(7,9,11,14,18,22)	(2,3,4,6,7,9)	(5,7,8,11,14,17)	(9,11,13,15,18,20)
$demand$	(6,9,12,15,20,25)	(6,7,9,11,13,16)	(10,12,14,16,20,24)	

Using the above ranking method the given transportation table reduced as follows:

Table.: 2

	d_1	d_2	d_3	$supply$
s_1	3.64	2.15	5.42	3.47
s_2	2.15	3.36	2.95	3.75
s_3	2.95	1.42	2.82	3.96
$demand$	3.96	2.84	4.38	

By applying zero suffix method the allocation are following:

Table: 3

	d_1	d_2	d_3	$supply$
s_1	**0.63** 3.64	**2.84** 2.15	5.42	3.47
s_2	**3.33** 2.15	3.36	**0.42** 2.95	3.75
s_3	2.95	1.42	**3.96** 2.82	3.96
$demand$	3.96	2.84	4.38	

4. RANKING OF GENERALIZED OCTAGONAL FUZZY NUMBERS [2]:

Ranking of fuzzy numbers are very important task to reduce the more numbers. Nowadays number of proposed ranking techniques is available. Ranking methods map fuzzy number directly into the real line i.e. which associate every fuzzy number with a real number.

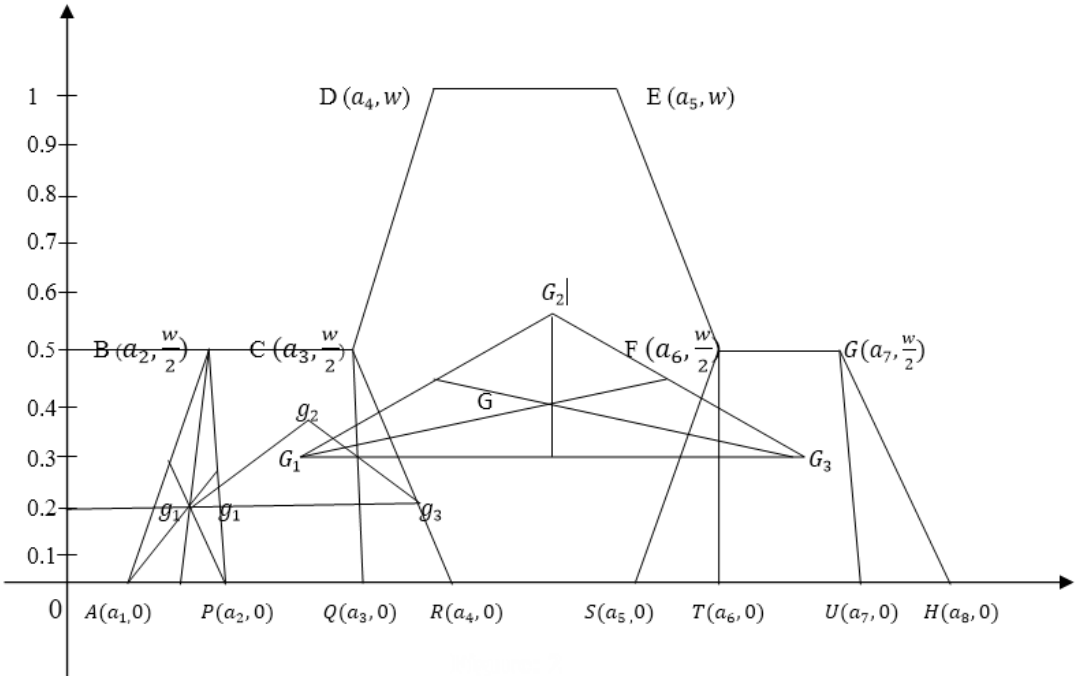

Figure: 2

The centroid point of an octagon is considered to be the balancing point of the trapezoid. Divide the octagon into three plane figures. These three plane figures are a trapezium ABCR, a hexagon RCDEFS, and again a trapezium SFGH respectively. Let the centroid of three plane figures be $G_1, G_2, and\ G_3$ respectively.

The centroid of these centroids $G_1, G_2, and\ G_3$ is taken as a point of reference to define the ranking of generalized octagon fuzzy numbers. The reason for selecting this point as a point of reference is that each centroid point G_1 of a trapezoid ABCR, G_2 of a hexagon RCDEFS and G_3 of a trapezoid SFGH are balancing point of each individual plane figure and the centroid o these centroids points is much more balancing point for a general octagonal fuzzy number.

Consider a generalized octagonal fuzzy number A = $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; w)$. The centroid of these plane figures is

$$G_1 = \left(\frac{a_1+2a_2}{3} + \frac{a_2+a_3}{2} + \frac{2a_{3+}a_4}{3},\ \frac{w}{6} + \frac{w}{4} + \frac{w}{6}\right)$$

$$= \left(\frac{2a_1+7a_2+7a_3+2a_4}{18}, \frac{7w}{36}\right)$$

Similarly,

$$G_3 = \left(\frac{2a_5+7a_6+7a_7+2a_8}{18}, \frac{7w}{36}\right)$$

$$G_2 = \left(\frac{a_1+2a_4+2a_5+a_6}{6}, \frac{w}{2}\right)$$

The equation of the line G_1, G_3 is $y = \frac{w}{5}$, and G_2 does not lie on the line G_1 and G_3. Thus G_1 , G_2 *and* G_3 are non-collinear and they form a triangle.

The ranking function of the generalized hexagonal fuzzy number $A_0 = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; w)$ which maps the set of all fuzzy numbers to a set of real numbers is defined as;

$$R(A_0) = G_A(x_0, y_0) = \left(\frac{2a_1+7a_2+10a_3+8a_4+8a_5+10a_6+7a_7+2a_8}{54}, \frac{8w}{27}\right)$$

$$\left(\frac{2a_1 + 7a_2 + 10a_3 + 8a_4 + 8a_5 + 10a_6 + 7a_7 + 2a_8}{54} \times \frac{8w}{27}\right)$$

$$= (1.04)\,(2.13) + (2.79)\,(1.04) + (1.63)\,(0.09) + (2.55)\,(0.5) + (1.93)\,(1.04)$$

$$= 7.3705$$

Numerical Example 2:

Consider the following fuzzy transportation problem:

Table: 1

	d_1	d_2	d_3	$supply$
s_1	(0,1,2,3, 4,5,6,7)	(8,9,10,11, 12,13,14,15)	(4,5,6,7, 8,9,10,11)	(3,4,5,7, 8,9,10,12)
s_2	(2,4,5,6, 7,8,9,11)	(5,6,8,9, 10,11,12,15)	(0,1,2,3, 4,5,6,7)	(0,1,2,3, 4,5,6,7)
s_3	(2,3,4,5, 6,7,8,9)	(3,6,7,8,9, 10,12,13)	(2,4,5,6, 7,8,9,11)	(2,3,4,5, 6,7,8,9)
$demand$	(4,5,6,7, 8,9,10,1)	(1,2,3,5, 6,7,8,10)	(0,1,2,3, 4,5,6,7)	(0,1,2,3, 4,5,6,7)

Solution:

Using the ranking method the above problem can be reduced as follows:

Table: 2

	d_1	d_2	d_3	$supply$
s_1	1.04	3.4	2.22	2.13
s_2	1.93	2.79	1.04	1.04
s_3	1.63	2.55	1.93	1.63
$demand$	2.22	1.54	1.04	

By applying the zero suffix method allocations are obtained as follows:

Table: 3

	d_1	d_2	d_3	$supply$
s_1	**2.13** 1.04	3.4	2.22	2.13
s_2	1.93	**1.04** 2.79	**1.04** 1.04	1.04
s_3	**0.09** 1.63	**1.54** 2.55	1.93	1.63
$demand$	2.22	1.54	1.04	

CONCLUSION:

In this paper a fuzzy transportation problem were introduced by using ranking of fuzzy numbers. We solved numerical examples and found satisfactory result. Here generalized ranking method is used to solve generalized hexagonal and generalized octagonal fuzzy numbers with centroid ranking technique. However, the result obtained in this paper is that generalized octagonal fuzzy number has less optimal solution as compared to the generalized hexagonal fuzzy number.

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