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Perfectly β Generalized Continuous Mappings in Intuitionistic Fuzzy Topological Spaces

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Abstract

In this paper, we introduce intuitionistic fuzzy perfectly β generalized continuous mappings. Furthermore we provide some properties of the same mapping and discuss some fascinating theorems.

Keywords: Intuitionistic fuzzy sets, intuitionistic fuzzy topology, intuitionistic fuzzy β generalized closed sets, intuitionistic fuzzy β generalized continuous mappings, intuitionistic fuzzy perfectly β generalized continuous mappings.

I. INTRODUCTION

Atanassov [1] introduced the idea of intuitionistic fuzzy sets and Coker [2] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. Later this was followed by the introduction of intuitionistic fuzzy β generalized closed sets by Saranya, M and Jayanthi, D [5] in 2016 which was simultaneously followed by the introduction of intuitionistic fuzzy β generalized continuous mappings [7] by the same authors. We now extend our idea towards intuitionistic fuzzy perfectly β generalized continuous mappings and discuss some of their properties.

II. PRELIMINARIES

Definition 2.1 [1]: An ***intuitionistic fuzzy set*** (IFS for short) A is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

Definition 2.2 [1]: Let A and B be two IFSs of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

and

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}.$$

Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$,

(d) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$,

(e) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \langle x, 0, 1\rangle$ and $1_\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3 [2]: An ***intuitionistic fuzzy topology*** (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called ***intuitionistic fuzzy topological space*** (IFTS in short) and any IFS in τ is known as an ***intuitionistic fuzzy open set*** (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an ***intuitionistic fuzzy closed set*** (IFCS in short) in X.

Definition 2.4 [5]: An IFS A in an IFTS (X, τ) is said to be an ***intuitionistic fuzzy β generalized closed set*** (IFβGCS for short) if βcl(A) ⊆ U whenever A ⊆ U and U is an IFβOS in (X, τ).

Definition 2.5 [7]: A mapping f: (X, τ) → (Y, σ) is called an ***intuitionistic fuzzy β generalized continuous*** (IFβG continuous for short) **mapping** if $f^{-1}(V)$ is an IFβGCS in (X, τ) for every IFCS V of (Y, σ).

Definition 2.6 [8] : A mapping f: (X, τ) → (Y, σ) is said to be an ***intuitionistic fuzzy almost β generalized continuous*** (IFaβG continuous for short) **mapping** if $f^{-1}(A)$ is an IFβGCS in X for every IFRCS A in Y.

Definition 2.7 [9]: A mapping f: (X, τ) → (Y, σ) is said to be an ***intuitionistic fuzzy completely β generalized continuous*** (IF completely βG continuous for short) ***mapping*** if $f^{-1}(V)$ is an IFRCS in X for every IFβGCS V in Y.

Definition 2.8 [10] : A mapping f: (X, τ) → (Y, σ) is said to be an ***intuitionistic fuzzy weakly β generalized continuous*** (IFWβG continuous for short) ***mapping*** if $f^{-1}(V) \subseteq \beta gint(f^{-1}(cl(V)))$ for each IFOS V in Y.

Definition 2.9 [4]: A mapping f: (X, τ) → (Y, σ) is said to be an ***perfectly continuous*** if $f^{-1}(V)$ is clopen in X for every open set V ⊂ Y.

III. PERFECTLY β GENERALIZED CONTINUOUS MAPPINGS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

In this section we introduce intuitionistic fuzzy perfectly β generalized continuous mappings and study some of their properties.

Definition 3.1: A mapping f: (X, τ) → (Y, σ) is said to be an ***intuitionistic fuzzy perfectly β generalized continuous*** (IFpβG continuous for short) ***mapping*** if $f^{-1}(A)$ is clopen in (X, τ) for every IFβGCS A of (Y, σ).

Theorem 3.2: Every IFpβG continuous mapping is an IF continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFpβG continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFβGCS [5], A is an IFβGCS in Y. Since f is an IFpβG continuous mapping, $f^{-1}(A)$ is an IF clopen in X. Thus $f^{-1}(A)$ is an IFCS in X. Hence f is an IF continuous mapping.

Example 3.3: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.5_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

Then, IFβC(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1\}$,

IFβO(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, 0 \leq \mu_a+\nu_a \leq 1 \text{ and } 0 \leq \mu_b+\nu_b \leq 1\}$,

IFβC(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1] \,/\, 0 \leq \mu_u+\nu_u \leq 1 \text{ and } 0 \leq \mu_v+\nu_v \leq 1\}$,

IFβO(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1] \,/\, 0 \leq \mu_u+\nu_u \leq 1 \text{ and } 0 \leq \mu_v+\nu_v \leq 1\}$.

The IFS $A = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$ is an IFCS in Y. Then $f^{-1}(A)$ is an IFCS in X. Therefore f is an IF continuous mapping, but not an IFpβG continuous mapping. Since for an IFβGCS $A = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$ in Y, and $f^{-1}(A)$ is not an IF clopen in X, as $cl(f^{-1}(A)) = G_1^{\,c} = f^{-1}(A)$ and $int(f^{-1}(A)) = G_1 \neq f^{-1}(A)$.

Theorem 3.4: Every IFpβG continuous mapping is an IFα continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFpβG continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFβGCS [5], A is an IFβGCS in Y. Since f is an IFpβG continuous mapping, $f^{-1}(A)$ is an IF clopen in X. That is $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFαCS, $f^{-1}(A)$ is an IFαCS in X. Hence f is an IFα continuous mapping.

Example 3.5: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.4_a, 0.3_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle y, (0.8_u, 0.8_v), (0.2_u, 0.2_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

Then, IFβC(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, 0 \le \mu_a+\nu_a \le 1$ and $0 \le \mu_b+\nu_b \le 1\}$,

IFβO(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, 0 \le \mu_a+\nu_a \le 1$ and $0 \le \mu_b+\nu_b \le 1\}$,

IFβC(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1]$ /either $\mu_u < 0.8$ or $\mu_v < 0.8$, $0 \le \mu_u+\nu_u \le 1$ and $0 \le \mu_v+\nu_v \le 1\}$,

IFβO(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1]$ /either $\mu_u > 0.2$ or $\mu_v > 0.2$, $0 \le \mu_u+\nu_u \le 1$ and $0 \le \mu_v+\nu_v \le 1\}$.

Here the mapping f is an IFα continuous mapping, but not an IFpβG continuous mapping. Since for an IFβGCS A = $\langle y, (0.1_u, 0.2_v), (0.9_u, 0.8_v)\rangle$ in Y, $f^{-1}(A)$ is not an IF clopen in X, as $cl(f^{-1}(A)) = G_1^c \neq f^{-1}(A)$ and $int(f^{-1}(A)) = 0_\sim \neq f^{-1}(A)$.

Theorem 3.6: Every IFpβG continuous mapping is an IFS continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFpβG continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFβGCS [5], A is an IFβGCS in Y. Since f is an IFpβG continuous mapping, $f^{-1}(A)$ is an IF clopen in X. That is $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFSCS, $f^{-1}(A)$ is an IFSCS in X. Hence f is an IFS continuous mapping.

Example 3.7: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.6_a, 0.8_b), (0.4_a, 0.2_b)\rangle$, $G_2 = \langle y, (0.6_u, 0.8_v), (0.4_u, 0.2_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

Then, IFβC(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / either $\mu_a < 0.6$ or $\mu_b < 0.8$, $0 \le \mu_a+\nu_a \le 1$ and $0 \le \mu_b+\nu_b \le 1\}$,

IFβO(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / either $\mu_a > 0.4$ or $\mu_b > 0.2$, $0 \le \mu_a+\nu_a \le 1$ and $0 \le \mu_b+\nu_b \le 1\}$,

IFβC(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1]$ / either $\mu_u < 0.6$ or $\mu_v < 0.8$, $0 \le \mu_u+\nu_u \le 1$ and $0 \le \mu_v+\nu_v \le 1\}$,

IFβO(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1]$ / either $\mu_u > 0.4$ or $\mu_v > 0.2$, $0 \le \mu_u+\nu_u \le 1$ and $0 \le \mu_v+\nu_v \le 1\}$.

Here the mapping f is an IFS continuous mapping, but f is not an IFpβG continuous mapping. Since for an IFβGCS A = ⟨y, $(0.1_u, 0.2_v)$, $(0.9_u, 0.8_v)$⟩ in Y, $f^{-1}(A)$ is not an IF clopen in X. As $cl(f^{-1}(A)) = G_1^c \neq f^{-1}(A)$ and $int(f^{-1}(A)) = 0_\sim \neq f^{-1}(A)$.

Theorem 3.8: Every IFpβG continuous mapping is an IFP continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFpβG continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFβGCS [5], A is an IFβGCS in Y. Since f is an IFpβG continuous mapping, $f^{-1}(A)$ is an IF clopen in X. That is $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFPCS, $f^{-1}(A)$ is an IFPCS in X. Hence f is an IFP continuous mapping.

Example 3.9: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, $(0.5_a, 0.2_b)$, $(0.4_a, 0.4_b)$⟩, G_2 = ⟨y, $(0.2_u, 0.6_v)$, $(0.3_u, 0.4_v)$⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

Then, IFβC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $\mu_a < 0.5$ or $\mu_b < 0.2$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$},

IFβO(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ /either $\mu_a > 0.4$ or $\mu_b > 0.4$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$},

IFβC(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0,1]$, $\mu_v \in [0,1]$, $\nu_u \in [0,1]$, $\nu_v \in [0,1]$ /either $\mu_u < 0.2$ or $\mu_v < 0.6$,$0 \leq \mu_u+\nu_u \leq 1$ and $0 \leq \mu_v+\nu_v \leq 1$},

IFβO(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0,1]$, $\mu_v \in [0,1]$, $\nu_u \in [0,1]$, $\nu_v \in [0,1]$ /either $\mu_u > 0.3$ or $\mu_v > 0.4$, $0 \leq \mu_u+\nu_u \leq 1$ and $0 \leq \mu_v+\nu_v \leq 1$}.

Here the mapping f is an IFP continuous mapping, but not an IFpβG continuous mapping. Since for an IFβGCS A = ⟨y, $(0.2_u, 0.2_v)$, $(0.7_u, 0.8_v)$⟩ in Y, $f^{-1}(A)$ is not an IF clopen in X, as $cl(f^{-1}(A)) = G_1^c \neq f^{-1}(A)$ and $int(f^{-1}(A)) = 0_\sim \neq f^{-1}(A)$.

Theorem 3.10: Every IFpβG continuous mapping is an IFβ continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFpβG continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFβGCS [5], A is an IFβGCS in Y. Since f is an IFpβG continuous mapping, $f^{-1}(A)$ is an IF clopen in X. That is $f^{-1}(A)$ is an IFCS in X.

Since every IFCS is an IFβCS, $f^{-1}(A)$ is an IFβCS in X. Hence f is an IFβ continuous mapping.

Example 3.11: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

Then, IFβC(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / 0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$,

IFβO(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / 0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$,

IFβC(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1]$ /either $\mu_u < 0.6$ or $\mu_v < 0.7$, $0 \leq \mu_u+\nu_u \leq 1$ and $0 \leq \mu_v+\nu_v \leq 1\}$,

IFβO(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1]$ /either $\mu_u > 0.4$ or $\mu_v > 0.3$, $0 \leq \mu_u+\nu_u \leq 1$ and $0 \leq \mu_v+\nu_v \leq 1\}$.

Here the mapping f is an IFβ continuous mapping, but not an IFpβG continuous mapping. Since for an IFβGCS $A = \langle y, (0.5_u, 0.2_v), (0.5_u, 0.8_v)\rangle$ in Y, $f^{-1}(A)$ is not an IF clopen in X, as $cl(f^{-1}(A)) = G_1^c \neq f^{-1}(A)$ and $int(f^{-1}(A)) = 0_\sim \neq f^{-1}(A)$.

Theorem 3.12: Every IFpβG continuous mapping is an IFβG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFpβG continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFβGCS [5], A is an IFβGCS in Y. Since f is an IFpβG continuous mapping, $f^{-1}(A)$ is an IF clopen in X. Thus $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFβGCS, $f^{-1}(A)$ is an IFβGCS in X. Hence f is an IFβG continuous mapping.

Example 3.13: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

Then, IFβC(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / 0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$,

IFβO(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / 0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1\}$, IFβC(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0,1]$, $\mu_v \in [0,1]$, $\nu_u \in [0,1]$, $\nu_v \in [0,1]$ /either $\mu_u < 0.6$ or $\mu_v < 0.7$, $0 \leq \mu_u+\nu_u \leq 1$ and $0 \leq \mu_v+\nu_v \leq 1$},

IFβO(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0,1]$, $\mu_v \in [0,1]$, $\nu_u \in [0,1]$, $\nu_v \in [0,1]$ /either $\mu_u > 0.4$ or $\mu_v > 0.3$, $0 \leq \mu_u+\nu_u \leq 1$ and $0 \leq \mu_v+\nu_v \leq 1$}.

Now $G_2^c = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFCS in Y. We have $f^{-1}(G_2^c) = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is an IFS in X. Now $f^{-1}(G_2^c) \subseteq G_1$. As $\beta cl(f^{-1}(G_2^c)) = f^{-1}(G_2^c) \subseteq G_1$, $f^{-1}(G_2^c)$ is an IFβGCS in X. Thus f is an IFβG continuous mapping. But f is not an IFpβG continuous mapping. Since for an IFβGCS $B = \langle y, (0.5_u, 0.2_v), (0.5_u, 0.8_v)\rangle$ in Y, and $f^{-1}(B)$ is not an IF clopen in X, as $cl(f^{-1}(B)) = G_1^c \neq f^{-1}(B)$ and $int(f^{-1}(B)) = 0_\sim \neq f^{-1}(B)$.

Theorem 3.14: Every IFpβG continuous mapping is an IFaβG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFpβG continuous mapping. Let A be an IFRCS in Y. Since every IFRCS is an IFβGCS [5], A is an IFβGCS in Y. By hypothesis, $f^{-1}(A)$ is an IF clopen in X. Thus $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFβGCS, $f^{-1}(A)$ is an IFβGCS in X. Hence f is an IFaβG continuous mapping.

Example 3.15: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

Then, IFβC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$},

IFβO(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$},

IFβC(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0,1]$, $\mu_v \in [0,1]$, $\nu_u \in [0,1]$, $\nu_v \in [0,1]$ / $0 \leq \mu_u+\nu_u \leq 1$ and $0 \leq \mu_v+\nu_v \leq 1$},

IFβO(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0,1]$, $\mu_v \in [0,1]$, $\nu_u \in [0,1]$, $\nu_v \in [0,1]$ / $0 \leq \mu_u+\nu_u \leq 1$ and $0 \leq \mu_v+\nu_v \leq 1$}.

Now $G_2^c = \langle y, (0.7_u, 0.8_v), (0.3_u, 0.2_v)\rangle$ is an IFRCS in Y, Since $cl(int(G_2^c)) = cl(G_2) = G_2^c$. We have $f^{-1}(G_2^c) = \langle x, (0.7_a, 0.8_b), (0.3_a, 0.2_b)\rangle$ is an IFS in X. Now $f^{-1}(G_2^c) \subseteq 1_\sim$. As $\beta cl(f^{-1}(G_2^c)) = f^{-1}(G_2^c) \subseteq 1_\sim$, $f^{-1}(G_2^c)$ is an IFβGCS in X. Thus f is an IFaβG continuous mapping. But f is not an IFpβG continuous mapping. Since for an IFβGCS B = ⟨y, $(0.5_u, 0.3_v), (0.2_u, 0.1_v)$⟩ in Y and f^{-1}(B) is not an IF clopen in X, as cl(f^{-1}(B)) = $1_\sim \neq f^{-1}$(B) and int(f^{-1}(B)) = $0_\sim \neq f^{-1}$(B).

Theorem 3.16: Every IFpβG continuous mapping is an IFWβG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFpβG continuous mapping. Let A be an IFOS in Y. Then A is an IFβGOS in Y. Since f is an IFpβG continuous mapping, f^{-1}(A) is an IFclopen in X. Now f^{-1}(A) = int(f^{-1}(A)) ⊆ βgint(f^{-1}(A)) = βgint(cl(f^{-1}(A))). Thus f^{-1}(A) = cl(f^{-1}(A)). This implies f^{-1}(A) ⊆ βgint(cl(f^{-1}(A))). Hence f is an IFWβG continuous mapping.

Example 3.17: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, $(0.5_a, 0.4_b), (0.5_a, 0.6_b)$⟩, G_2 = ⟨y, $(0.4_u, 0.3_v), (0.6_u, 0.7_v)$⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

Then, IFβC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$},

IFβO(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$},

IFβC(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0,1]$, $\mu_v \in [0,1]$, $\nu_u \in [0,1]$, $\nu_v \in [0,1]$ / $0 \leq \mu_u+\nu_u \leq 1$ and $0 \leq \mu_v+\nu_v \leq 1$},

IFβO(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0,1]$, $\mu_v \in [0,1]$, $\nu_u \in [0,1]$, $\nu_v \in [0,1]$ / $0 \leq \mu_u+\nu_u \leq 1$ and $0 \leq \mu_v+\nu_v \leq 1$}.

The IFS G_2 = ⟨y, $(0.4_u, 0.3_v), (0.6_u, 0.7_v)$⟩ is an IFOS in Y. Now G_2^c = ⟨y, $(0.6_u, 0.7_v), (0.4_u, 0.3_v)$⟩ is an IFCS in Y.

We have βgint(f^{-1}(cl(G_2))) = ⟨x, $(0.6_a, 0.7_b), (0.4_a, 0.3_b)$⟩. Hence f^{-1}(G_2) ⊆ βgint(f^{-1}(cl(G_2))). Therefore f is an IFWβG continuous mapping. But f is not an IFpβG continuous mapping. Since the IFS B = ⟨y, $(0.5_u, 0.3_v), (0.2_u, 0.1_v)$⟩ is an IFβGCS in Y, but f^{-1}(B) is not an IF clopen in X. As cl(f^{-1}(B)) = $1_\sim \neq f^{-1}$(B) and int(f^{-1}(B)) = $0_\sim \neq f^{-1}$(B).

Theorem 3.18: Every IFpβG continuous mapping is an IF completely βG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFpβG continuous mapping. Let A be an IFβGCS in Y. By hypothesis, f^{-1}(A) is an intuitionistic fuzzy clopen in X. Therefore cl(f^{-1} (A)) = f^{-1} (A) and int(f^{-1} (A)) = f^{-1} (A). Now cl(int(f^{-1} (A))) = cl(f^{-1} (A)) = f^{-1} (A). Therefore f^{-1} (A) is an IFRCS in X. Hence f is an IF completely βG continuous mapping.

Theorem 3.19: A mapping f: (X, τ) → (Y, σ) is an IFpβG continuous mapping if and only if the inverse image of each IFβGOS [6] in Y is an intuitionistic fuzzy clopen in X.

Proof: Necessity: Let a mapping f: (X, τ) → (Y, σ) be IFpβG continuous mapping. Let A be an IFβGOS in Y. Then A^c is an IFβGCS in Y. Since f is an IFpβG continuous mapping, f^{-1} (A^c) is IF clopen in X. As f^{-1} (A^c) = (f^{-1} (A))^c, we have f^{-1} (A) is IF clopen in X.

Sufficiency: Let B be an IFβGCS in Y. Then B^c is an IFβGOS in Y. By hypothesis, f^{-1} (B^c) is IF clopen in X. Which implies f^{-1} (B) is IF clopen in X, as f^{-1} (B^c) = (f^{-1} (B))^c . Therefore f is an IFpβG continuous mapping.

Theorem 3.20: Let f: (X, τ) → (Y, σ) be an IF continuous mapping and g: (Y, σ) → (Z, δ) is IFpβG continuous mapping, then g ∘ f : (X, τ) → (Z, δ) is an IFpβG continuous mapping.

Proof: Let A be an IFβGCS in Z. Since g is an IFpβG continuous mapping, g^{-1} (A) is an IF clopen in Y. Since f is an IF continuous mapping, f^{-1} (g^{-1} (A)) is an IFCS in X, as well as IFOS in X. Hence g∘f is an IFpβG continuous mapping.

Theorem 3.21: The composition of two IFpβG continuous mapping is an IFpβG continuous mapping in general.

Proof: Let f: X → Y and g: Y → Z be any two IFpβG continuous mappings. Let A be an IFβGCS in Z. By hypothesis, g^{-1}(A) is IF clopen in Y and hence an IFCS in Y. Since every IFCS is an IFβGCS, g^{-1}(A) is an IFβGCS in Y. Further, since f is an IFpβG continuous mapping, f^{-1} (g^{-1}(A)) = (g∘f)$^{-1}$ (A) is IF clopen in X. Hence g∘f is an IFpβG continuous mapping.

Theorem 3.22: Let f: (X, τ) → (Y, σ) be an IFpβG continuous mapping and g: (Y, σ) → (Z, δ) is an IFβG irresolute mapping [9], then g∘ f : (X, τ) → (Z, δ) is an IFpβG continuous mapping.

Proof: Let A be an IFβGCS in Z. By hypothesis, g^{-1} (A) is an IFβGCS in Y. Since f is an IFpβG continuous mapping, f^{-1}(g^{-1}(A)) = (g ∘ f)$^{-1}$ (A) is IF clopen in X. Hence g∘f is an IFpβG continuous mapping.

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