Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 9 (2017), pp. 6633-6646

http://www.ripublication.com

Double Layered Complete Fuzzy Graph (DLCFG)

J. Jon Arockiaraj[1] and V. Chandrasekaran[2]

[1]Assistant Professor & Head of the Dept., PG & Research Department of Mathematics, St. Joseph's College of Arts and Science (Autonomous) Cuddalore, Tamil Nadu, India.

[2]Research Scholar, PG & Research Department of Mathematics, St. Joseph's College of Arts and Science (Autonomous), Cuddalore, Tamil Nadu, India.

Abstract

In this paper, a new fuzzy graph named double layered complete fuzzy graph is proposed. The double layered complete fuzzy graph gives a 3-D structure. We have discussed about order, size, degree and μ-complement of fuzzy graph.

Keywords: Order, Size, Vertex Degree, μ-Compliment, Strong Fuzzy Graph, Double Layered Complete Fuzzy Graph.

1. INTRODUCTION

Azriel Rosenfeld introduced fuzzy graph in 1975 [5]. Though introduced recently, it has been growing fast and has numerous applications in various fields. During the same time Yeh and Bang have also introduced various concepts in connectedness with fuzzy graphs [7]. Mordeson and Peng introduced the concept of operations on fuzzy graphs, Sunitha and Vijayakumar discussed about the operations of union, join, Cartesian product and composition on two fuzzy graphs [4]. The degree of a vertex in some fuzzy graphs was discussed by Nagoorgani and Radha [6]. Nagoorgani and Malarvizhi have defined different types of fuzzy graphs and discussed its relationships with isomerism in fuzzy graphs [3].

In this paper we define double layered complete fuzzy graph (DLCFG) or 3 – D Fuzzy graph which gives a 3-D structure in fuzzy graph theory and some of its properties were discussed. Section two contains the basic definitions in fuzzy graphs, in section three we introduce a new fuzzy graph called a double layered complete fuzzy graph, section four presents the theoretical concepts of DLCFG and finally we give conclusion on (DLCFG).

2. PRELIMINARIES

2.1 Definition

A fuzzy graph G is a pair of functions G: (σ, μ) where a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ. The underlying crisp graph of G: (σ, μ) is denoted by G^* :(σ^*, μ^*) [5].

2.2 Definition

Let G: (σ, μ) be a fuzzy graph, the order of G is defined as O (G) = $\sum_{u\in V} \sigma(u)$ [8].

2.3 Definition

Let G: (σ, μ) be a fuzzy graph, the size of G is defined as S (G) = $\sum_{u,v\in V} \mu(u, v)$ [8].

2.4 Definition

Let G: (σ, μ) be a fuzzy graph, the degree of

a vertex u in G is defined as

$d_G(u) = \sum_{\substack{v\neq u \\ v\in V}} \mu(u, v)$ and is denoted as $d_G(u)$ [10].

2.5 Definition

A fuzzy graph G: (σ , μ) is said to be strong fuzzy graph if $\mu(u, v)=\sigma(u) \wedge \sigma(v)$ for all (u , v) in μ^* [9].

2.6 Definition

Let G be a fuzzy graph, the μ –compliment of G is denoted as

G^μ: (σ^μ, μ^μ) where σ^* U $\mu^*$and $\mu^\mu(u, v) = \begin{cases} \sigma(u)\wedge\sigma(v) - \mu(u,v) \text{ if } \mu(u,v) > 0 \\ 0 \text{ if } \mu(u,v) = 0 \end{cases}$ [4].

3. DOUBLE LAYERED COMPLETE FUZZY GRAPH(DLCFG)

3.1 Definition

Let σ_{DL}: V→ [0, 1] be a subset of V and μ_{DL}: V× V→ [0, 1] be a symmetric fuzzy relation on σ_{DL}. Any two vertex of the double layered complete fuzzy graph is adjacent[12]. The vertex set of complete double layered fuzzy graph be σ U μ and it's denoted by $K_{\sigma \cup \mu}$.

Or

Let σ_{DL}: V→ [0, 1] be a fuzzy subset of V then the complete double layered fuzzy graph on σ_{DL} is defined on $K_{\sigma \cup \mu} = (\sigma_{DL}, \mu_{DL})$. Any two vertices of the DLCFG is adjacent.

Example: 3.1.1. Consider the complete fuzzy graph with vertex 3 (K_3)

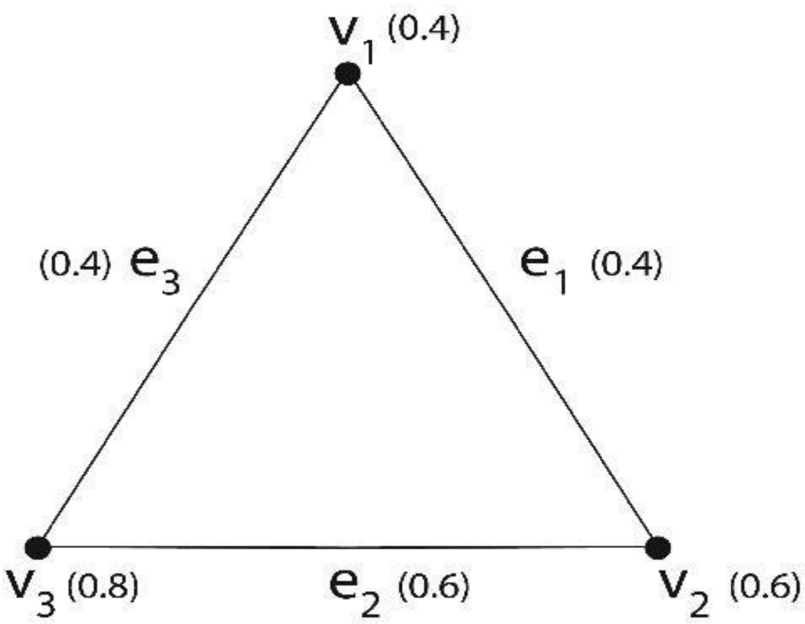

Figure 1. A complete fuzzy graph (K_3)

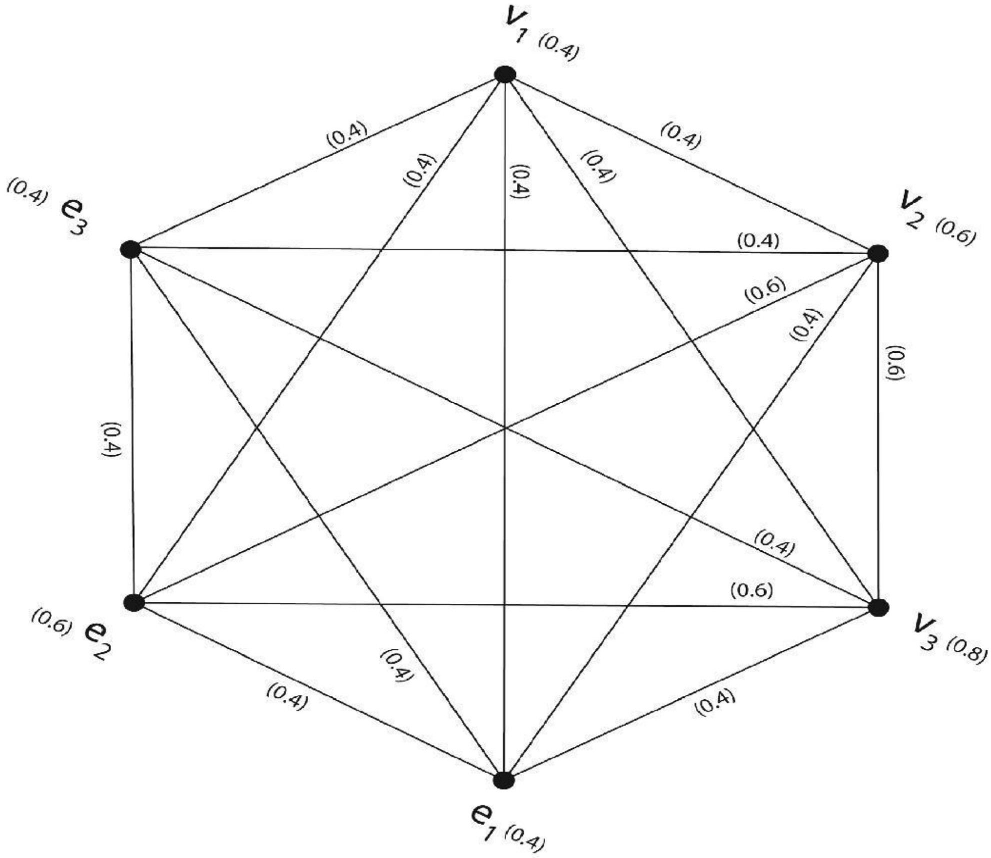

Figure 2. DLCFG of K_3

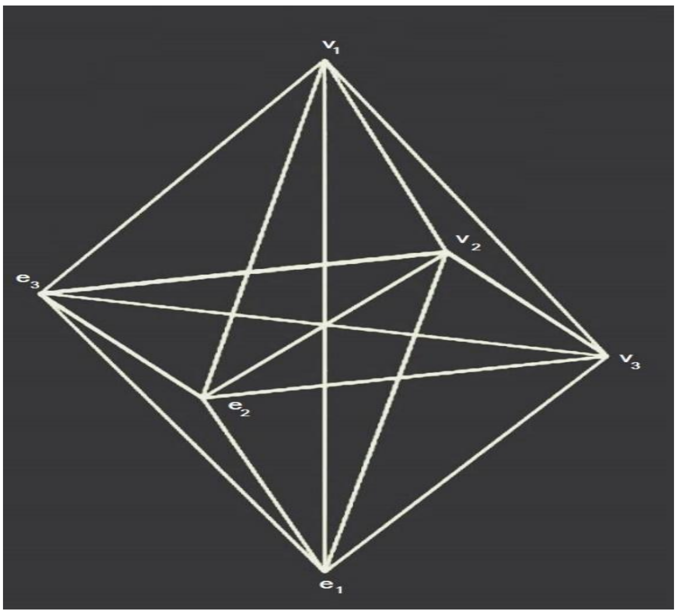

Figure 3. Image of DL (K_3)

Example 3.1.2: Consider the complete fuzzy graph with vertex 4 (K_4).

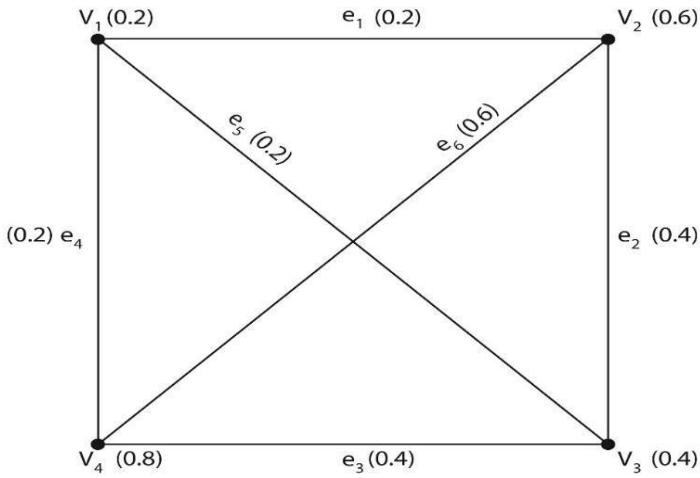

Figure 4. A complete fuzzy graph (K_4)

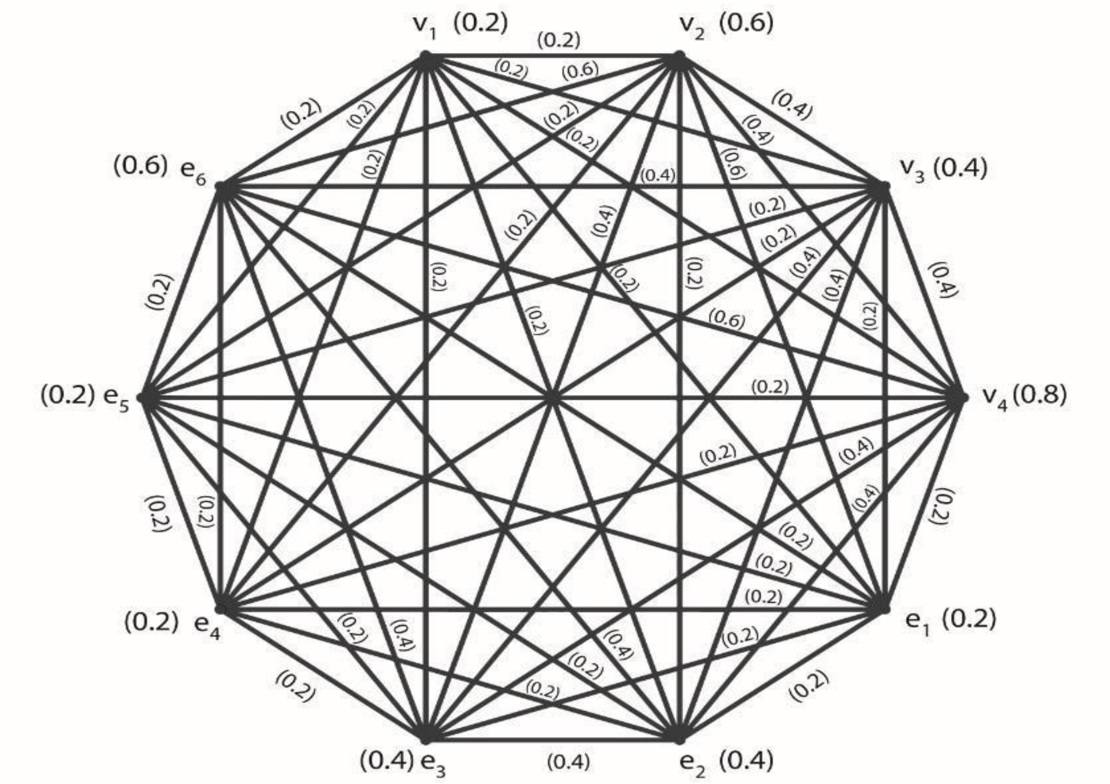

Figure 5. DLCFG of K_4

The figure 6(a) and 6(b) which is given below shows in different projections of Figure 5.

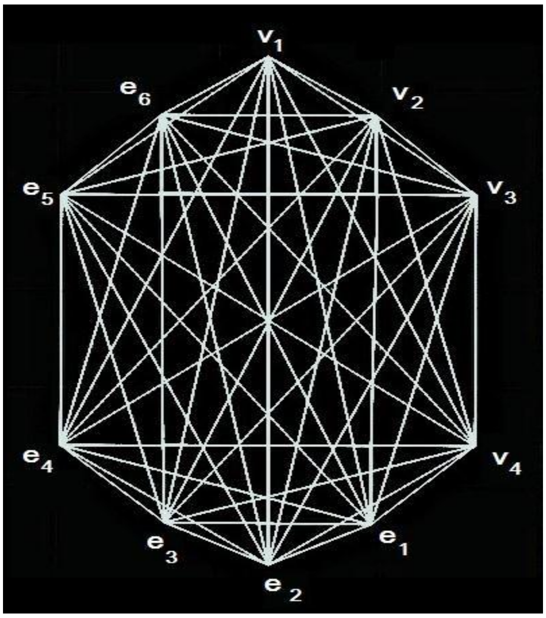

Figure 6(a). Image of DL(K_4)

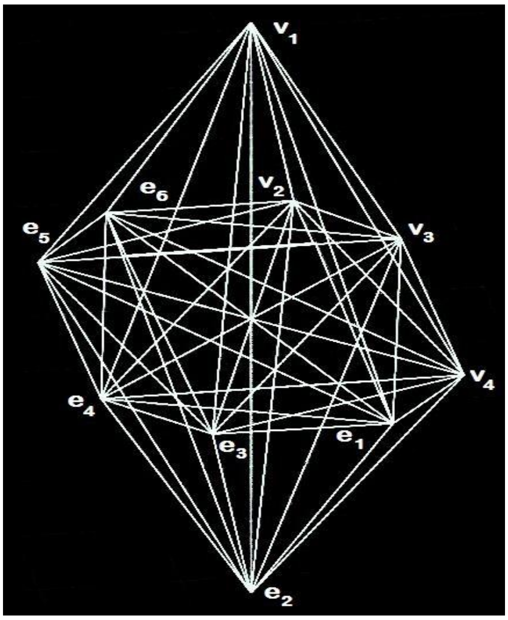

Figure 6(b). Image of DL(K_4)

Example 3.1.3: Consider the complete fuzzy graph with vertex 5 (K_5).

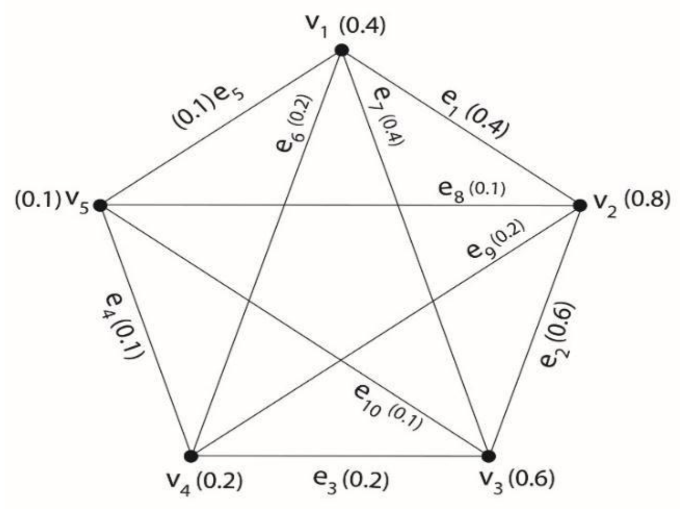

Figure 7. A complete fuzzy graph (K_5)

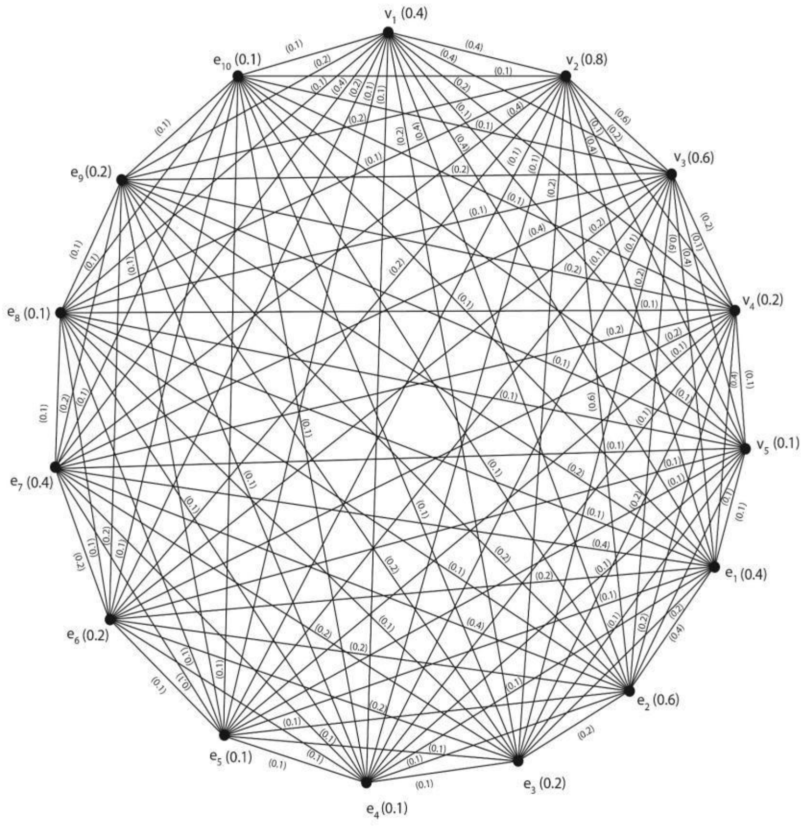

Figure 8. DLCFG of K_5

Similarly we can convert complete fuzzy graph into double layered complete fuzzy graph.

4. THEORITICAL CONCEPTS

4.1 THEOREM

The order of double layered complete fuzzy graph $K_{\sigma U \mu}$ is equal to the sum of the order and size of the complete graph

Proof:

Let σ U μ be a node set of complete double layered fuzzy graph and the fuzzy subset σ_{DL} on

$\sigma^* U \mu^*$ is defined as,

$$\sigma_{DL} = \begin{cases} \sigma(u)\ if\ u \in \sigma * \\ \mu(uv) if\ uv \in \mu * \end{cases}$$

By the definition, order of the double layered fuzzy graph is,

$$O\ (DL\ (G)) = \sum_{u \in \sigma \cup \mu} \sigma_{DL}\ (u)$$

$$= \sum_{u \in \sigma} \sigma_{DL}\ (u) + \sum_{u \in \mu} \sigma_{DL}\ (u)$$

$$= \sum_{u \in \sigma} \sigma\ (u) + \sum_{u \in \mu} \mu\ (u)$$

O (DL (G))=Order (G) + size (G)

4.2 Theorem

Every double layered complete fuzzy graph is a strong fuzzy graph

PROOF

As the node set of DL (G) is $\sigma^* U \mu^*$ and the fuzzy subset σ_{DL} on $\sigma^* U \mu^*$ is defined as,

$$\sigma_{DL} = \begin{cases} \sigma(u)\ if\ u\ \epsilon \sigma * \\ \mu(uv)\ if\ uv\ \epsilon \mu * \end{cases}$$

By the definition of double layered complete fuzzy graph

μ (u, v) =σ (u) ∧ σ (v) ----------------------①

And also by the definition of strong fuzzy graph

μ (u, v) =min (σ (u), σ (v)) ---------------- ②

From equation① &②; we get

Every double layered complete fuzzy graph is a strong fuzzy graph

Example 4.2.1

We choose DL (G) of K3 graph,

v_1=0.4; v_2=0.6; v_3=0.8 and e_1=0.4; e_2=0.6; e_3=0.4

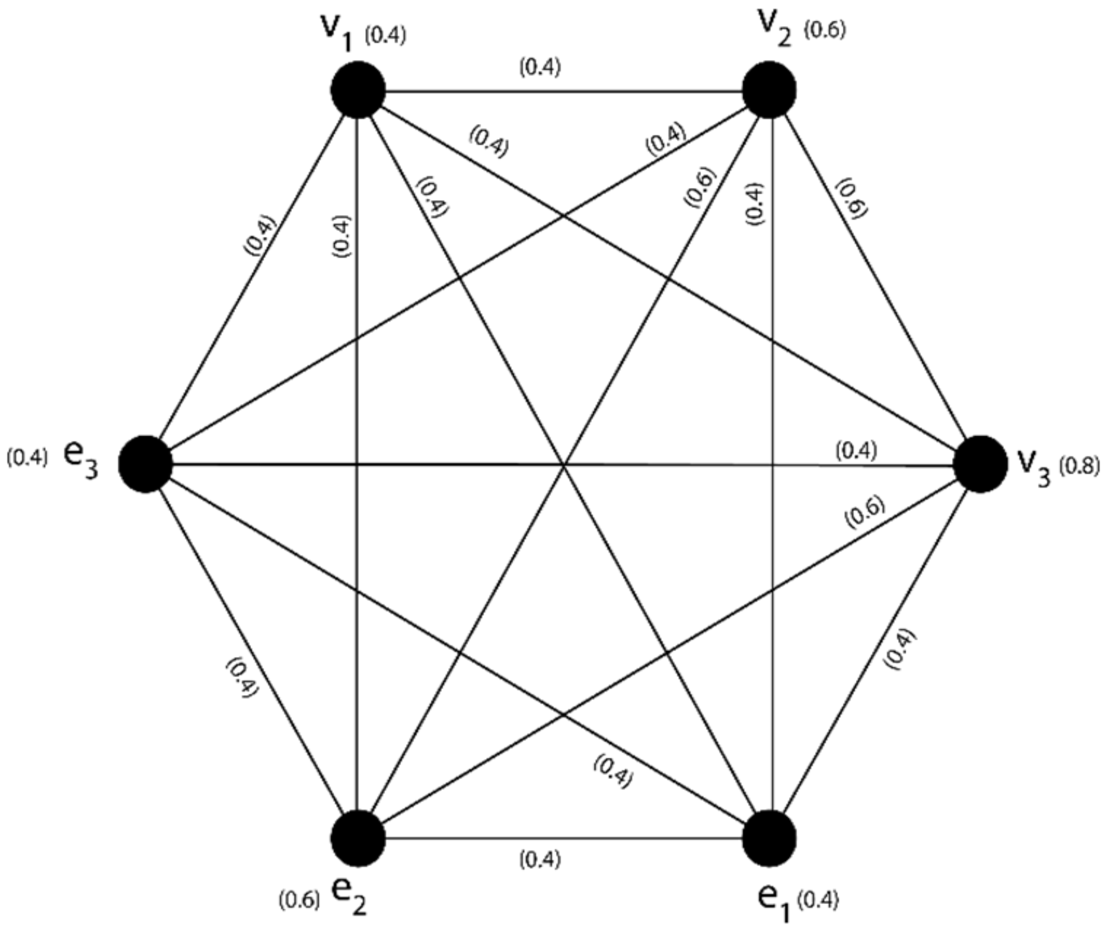

Figure 9. DLCFG of K_3

(i)$\mu\ (v_1, v_2) = \sigma\ (v_1) \wedge\ \sigma\ (v_2)$

$=0.4 \wedge\ 0.6$

$=0.4$

(ii)$\mu\ (e_1, e_2) = \sigma\ (e_1) \wedge\ \sigma\ (e_2)$

$=0.4 \wedge\ 0.6$

$=0.4$

(iii)$\mu\ (v_1, e_1) = \sigma\ (v_1) \wedge\ \sigma\ (e_1)$

$=0.4 \wedge\ 0.4$

$=0.4$

Every double layered fuzzy graph is a strong fuzzy graph

4.3 THEOREM

If G is a strong fuzzy graph then the μ-complement of DL (G) is isolated vertices

Proof

Let G be a strong fuzzy graph by the previous theorem,

Every double layered complete graph is strong fuzzy graph

$\mu\ (u, v) = \sigma\ (u) \wedge\ \sigma\ (v) \rightarrow$ ①

And by the definition of μ-complement,

$\mu^{\mu}\ (u, v) = \sigma\ (u) \wedge\ \sigma\ (v) - \mu\ (u, v)$

$= \mu\ (u, v) - \mu\ (u, v)$

$=0$

$\mu^{\mu}\ (u, v) = 0$ for all u, v in $\sigma^{*} U \mu^{*}$

$d_{DL}\ (u)=0$ for all u in $\sigma^{*} U \mu^{*}$

Every vertices of complement of DL (G) have isolated vertices.

Example 4.3.1

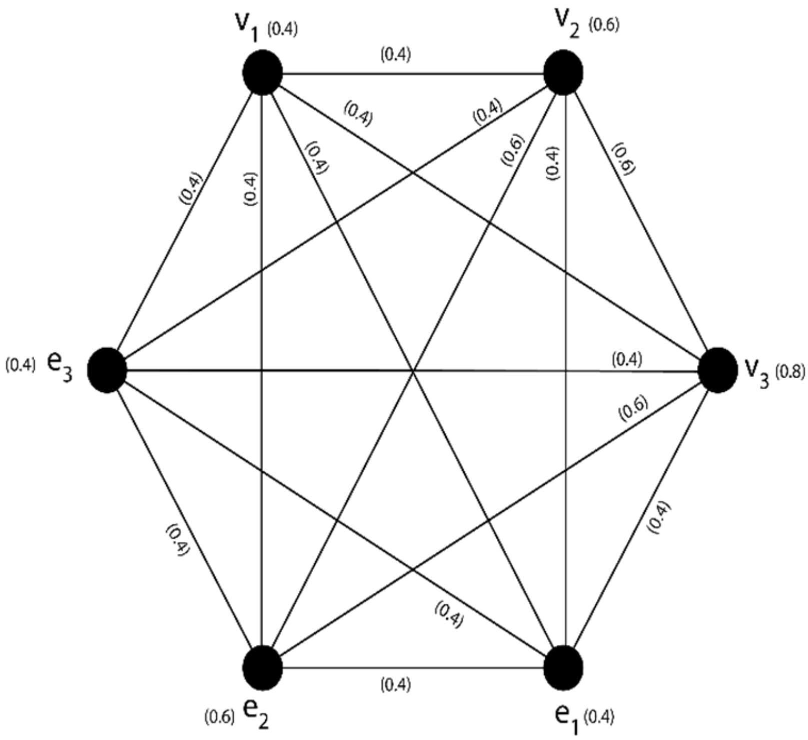

Figure 10. DLCFG of K_3

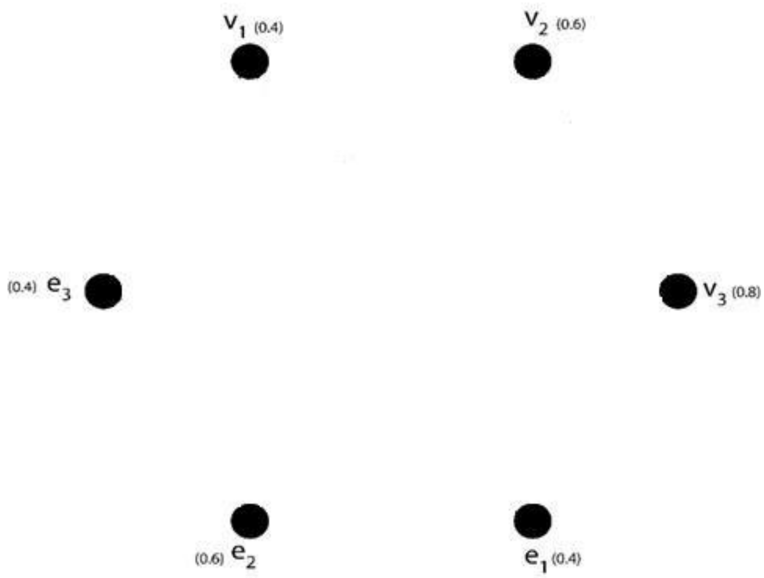

Figure 11. μ-complement DLCFG of K_3

$$\text{DLCFG}(K_n) = K_n + \text{D LCFG}(K_{n-1})$$

Example 4.3.1

(i) $\text{DLCFG}(K_4) = K_4 + \text{DLCFG}(K_3)$

$= K_4 + K_6$

$= K_{10}$

$\text{DLCFG}(K_4) = \text{CFG}(K_{10})$

(i) $\text{DLCFG}(K_5) = K_5 + \text{DLCFG}(K_4)$

$= K_5 + K_{10}$

$= K_{15}$

$\text{DLCFG}(K_5) = \text{CFG}(K_{15})$

Table 1: Relation between complete fuzzy graph and Double layered complete fuzzy graph.

COMPLETE FUZZY GRAPH	DOUBLE LAYERED COMPLETE FUZZY GRAPH
K_3	$\text{DLCFG}(K_3)=K_6$
K_4	$\text{DLCFG}(K_4)=K_{10}$
K_5	$\text{DLCFG}(K_5)=K_{15}$
K_6	$\text{DLCFG}(K_6)=K_{21}$
K_7	$\text{DLCFG}(K_7)=K_{28}$
K_8	$\text{DLCFG}(K_8)=K_{36}$
K_9	$\text{DLCFG}(K_9)=K_{45}$
K_{10}	$\text{DLCFG}(K_{10})=K_{55}$
K_{11}	$\text{DLCFG}(K_{11})=K_{66}$
K_{12}	$\text{DLCFG}(K_{12})=K_{78}$

K_{13}	**DLCFG(K_{13})=K_{91}**
K_{14}	**DLCFG(K_{14})=K_{105}**
K_{15}	**DLCFG(K_{15})=K_{120}**
K_{16}	**DLCFG(K_{16})=K_{136}**
K_{17}	**DLCFG(K_{17})=K_{154}**
K_{18}	**DLCFG(K_{18})=K_{173}**
K_{19}	**DLCFG(K_{19})=K_{192}**
K_{20}	**DLCFG(K_{20})=K_{212}**
K_{21}	**DLCFG(K_{21})=K_{233}**
K_{22}	**DLCFG(K_{22})=K_{255}**

Remark:

The edge relation between complete fuzzy graph and Double layered complete fuzzy graph is,

DLCFG (K_n) = K_n + DLCFG (K_{n-1})

$$Number\ of\ edges\ (E_{DL}) = \frac{n_{DL}(n_{DL}-1)}{2}$$

$$n_{DL}\ represents\ number\ of\ vertices\ in\ DLCFG$$

CONCLUSION

In this paper we have find a double layered complete fuzzy graph and illustrated with some examples. Further structures can be developed by increasing number of cycles. These structural patterns with the cycles gives to prove further into different patterns in networking models.

REFERENCES

[1] A.Nagoorgani and J.Malarvizhi, "Some aspects of total fuzzy graph", Proceedings of International Conference on Mathematical Methods and Computation, Tiruchirappalli, (2009) 168 – 179.

[2] A.Nagoorgani and J.Malarvizhi, "Some aspects of neighbourhood fuzzy

graph", Inter. Journal of Bulletin Pure and Applied Sciences, 29E (2010) 327 – 333.

[3] A.Nagoorgani and J.Malarvizhi, "Properties of μ - complement of a fuzzy graph", Inter. Journal of Algorithms, Computing and Mathematics, 2(3) (2009) 73 - 83.

[4] M.S.Sunitha and A.Vijayakumar, "Complement of a fuzzy graph", Indian Journal of Pure and Applied mathematics, 33(9) (2002) 1451-1464.

[5] A.Rosenfeld, "Fuzzy graphs, in: L.A. Zadeh, K.S. Fu, K. Tanaka and M. Shimura,(editors), Fuzzy sets and its application to cognitive and decision process", Academic press, New York (1975) 77 – 95.

[6] A.Nagoorgani and K.Radha, "The degree of a vertex in some fuzzy graphs", Inter. Journal of Algorithms, Computing and Mathematics, 2(3) (2009) 107 - 116.

[7] R.T.Yeh and S.Y.Bang, "Fuzzy relations, fuzzy graphs and their applications to clustering analysis, in: L.A. Zadeh, K.S. Fu, K. Tanaka and M. Shimura, (editors), Fuzzy sets and its application to cognitive and decision process", Academic press, New York (1975) 125 – 149.

[8] A.Nagoorgani and M.Basheed Ahamed, "Order and size in fuzzy graphs", Bulletin of Pure and Applied Sciences, 22E (1) (2003) 145 – 148.

[9] J.N.Mordeson, "Fuzzy line graphs", Pattern Recognition Letter, 14(1993) 381– 384.

[10] J.N.Mordeson and P.S.Nair, "Fuzzy graphs and Fuzzy Hypergraphs", Physica Verlag Publication, Heidelbserg, Second edition 2001.

[11] T.Pathinathan and J.Jesintha Rosline, "Characterization of Fuzzy graphs into different categories using arcs in Fuzzy Graphs", Journal of Fuzzy Set valued Analysis (accepted for publication).

[12] Pathinathan T. and J. Jesintha Rosline, 2014. "Double layered fuzzy graph", Annals of Pure and Applied Mathematics, 8(1) 135-143.

[13] S.Samanta and M.Pal, "Fuzzy tolerance graphs", International Journal of Latest Trends in Mathematics, 1(2) (2011) 57-67.

[14] S.Samanta and M.Pal, "Irregular bipolar fuzzy graphs", International Journal of Applications of Fuzzy Sets, 2 (2012) 91-102.

[15] H.Rashmanlou and M.Pal, "Isometry on interval-valued fuzzy graphs", International Journal of Fuzzy Mathematical Archive, 3 (2013) 28-35.

[16] S.Samanta and M.Pal, "Some more results on bipolar fuzzy sets and bipolar fuzzy intersection graphs", The Journal of Fuzzy Mathematics, 22(2) (2014) 253-262.