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# Proof of Beal's Theorem

**Viktor Petrovich Baryshok**

*Cathedra of Chemical Technology. Irkutsk National Research Technical University.*
*Str. Lermontova 83. Irkutsk, 664074. Russia.*

**Abstract**

Identified criteria for an infinite number of numerical solutions of equation $A^x + B^y = C^z$, including case of Beal's Conjecture where A, B, C, x, y and z are positive integers and x, y and z are all greater than 2, (A), (B) and (C) have a common prime factor.

**Keywords**: Beal's Conjecture, common prime factor, eligibility criteria, generalization

Andrew Beal has formulated a Conjecture in number theory [1]: If $A^x + B^y = C^z$, where A, B, C, x, y and z are positive integers and x, y and z are all greater than 2, then A, B and C must have a common prime factor.

In the previous paper shows that the Common Prime Factor in the equation $A^x + B^y = C^z$ for all three numbers A, B and C can be represented as a number in the appropriate degree $P^n$ [2]. Then the equation $A^x + B^y = C^z$ can be written in the following form (*1*):

$$a \cdot P^n + b \cdot P^n = c \cdot P^n, \qquad (1)$$

where $A = (a \cdot P^n)^{1/x}$, $B = (b \cdot P^n)^{1/y}$, $C = (c \cdot P^n)^{1/z}$.

From equation (*1*) follows: $a + b = c$.

Presented in [2] equation (*5-15*) and the examples in the Table given in a recent similar study [3] can be easily converted to expressions of type *1*:

$$3^3 + 6^3 = 3^5 \Rightarrow 3^3 + (2\cdot3)^3 = 3^5 \Rightarrow 1\cdot3^3 + 8\cdot3^3 = 9\cdot3^3 \quad (2)$$

$$7^6 + 7^7 = 98^3 \Rightarrow 7^6 + 7^7 = (2\cdot7^2)^3 \Rightarrow 1\cdot7^6 + 7\cdot7^6 = 8\cdot7^6 \quad (3)$$

$$33^5 + 66^5 = 33^6 \Rightarrow (3\cdot11)^5 + (6\cdot11)^5 = (3\cdot11)^6 \Rightarrow 3\cdot11^5 + 6\cdot11^5 = 9\cdot11^5 \quad (4)$$

$$34^5 + 51^4 = 85^4 \Rightarrow (2\cdot17)^5 + (3\cdot17)^4 = (5\cdot17)^4 \Rightarrow 544\cdot17^4 + 81\cdot17^4 = 625\cdot17^4 \quad (5)$$

$$19^4 + 38^3 = 57^3 \Rightarrow 19^4 + (2\cdot19)^3 = (3\cdot19)^3 \Rightarrow 19\cdot19^3 + 8\cdot19^3 = 27\cdot19^3 \quad (6)$$

$$7^3 + 7^4 = 14^3 \Rightarrow 7^3 + 7\cdot7^3 = (2\cdot7)^3 \Rightarrow 1\cdot7^3 + 7\cdot7^3 = 8\cdot7^3 \quad (7)$$

$$26^3 + 26^4 = 78^3 \Rightarrow 26^3 + 26\cdot26^3 = (3\cdot26)^3 \Rightarrow 1\cdot26^3 + 26\cdot26^3 = 27\cdot26^3 \text{ or}$$

$$8\cdot13^3 + 208\cdot13^3 = 216\cdot13^3 \quad (8)$$

$$255^4 + 255^5 = 1020^4 \Rightarrow 1\cdot255^4 + 255\cdot255^4 = 256\cdot255^4 \quad \text{(in [3] z=3)} \quad (9)$$

$$3124^5 + 3124^6 = 15620^5 \Rightarrow 1\cdot3124^5 + 3124\cdot3124^5 = 3125\cdot3124^5 \quad (10)$$

It is obvious that the exponent $n$ in common prime factor, $P^n$ of equations *1* can take any value if only $a + b = c$ and $A=(a\cdot P^n)^{1/x}$, $B=(b\cdot P^n)^{1/y}$, $C=(c\cdot P^n)^{1/z}$. However, in order for x, y, z correspond to the condition of Beal's Conjecture, the expressions $(a\cdot P^n)^{1/x}$, $(b\cdot P^n)^{1/y}$, $(c\cdot P^n)^{1/z}$ must be a bases of the integer exponents are respectively x, y, z > 2.

Thus, Beal's Conjecture can be formulated as a Theorem:

*If A, B, C, x, y and z are positive integers and x, y and z are all greater than 2, (A), (B) and (C) in equations of type* $A^x + B^y = C^z$ *have a common prime factor.*

A more General Theorem can be represented as:

*There are infinitely many numerical solutions of equation of type* $A^x + B^y = C^z$ *for which* $A=(a\cdot P^n)^{1/x}$, $B=(b\cdot P^n)^{1/y}$, $C=(c\cdot P^n)^{1/z}$ *are the bases of exponents x, y, z respectively, when n is any value if only* $a + b = c$.

Below are examples of obtaining equations $A^x + B^y = C^z$ from the expression (1).

**Example 1.** Let $a = 64$, $b = 64$, $c = 128$ (=$a + b$), $P^n = 2^6$. Then

$$64 \cdot 2^6 + 64 \cdot 2^6 = 128 \cdot 2^6 \Rightarrow 8^4 + 16^3 = 2^{13}. \quad (11)$$

For $P^n = 2^9$ get

$$64 \cdot 2^9 + 64 \cdot 2^9 = 128 \cdot 2^9 \Rightarrow 8^5 + 32^3 = 16^4. \quad (12)$$

**Example 2.** Let $a = 81$, $b = 648$, $c = 729$ (=$a + b$), $P^n = 9^4$. Then

$$81 \cdot 9^4 + 648 \cdot 9^4 = 729 \cdot 9^4 \Rightarrow 27^4 + 162^3 = 9^7. \quad (13)$$

**Example 3.** Let $a = 1$, $b = 8$, $c = 9$ (=$a + b$), $P^n = 3^{3.5}$. Then

$$1 \cdot 3^{3.5} + 8 \cdot 3^{3.5} = 9 \cdot 3^{3.5} \Rightarrow 3^{3.5} + 4^{4.2736} = 3^{3.5}, \quad (14)$$

**Example 4.** Let $a = 81$, $b = 648$, $c = 729$ (=$a + b$), $P^n = 3^{-2}$. Then

$$81 \cdot 9^{-2} + 648 \cdot 9^{-2} = 729 \cdot 9^{-2} \Rightarrow 3^5 + 12^{3.0474} = 3^7. \quad (15)$$

It should be noted the values of A, B or C and x, y or z respectively are uncertain at least for one term in the equation $A^x + B^y = C^z$ in the case of fractional or negative the number $n$. So, in the second summand values B and y for examples 3 and 4 can be any numbers if only preserves the equality $B^y = b \cdot P^n$.

Thus, the equation $A^x + B^y = C^z$, presented in the form $a \cdot P^n + b \cdot P^n = c \cdot P^n$, provided that $a + b = c$ and the expressions $(a \cdot P^n)^{1/x}$, $(b \cdot P^n)^{1/y}$, $(c \cdot P^n)^{1/z}$ are the bases of exponents x, y, z respectively, allows to obtain an infinite number of numerical solutions including ones satisfy the criteria of Andrew Beal's theorem, while A, B, C, x, y, and z are integer all greater than 2.

## REFERENCES

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