Time Dependence of the EoS Parameter in Brans-Dicke Framework

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Abstract

In the framework of Brans-Dicke (BD) theory, the time dependence of equation-of-state (EoS) parameter has been determined, using BD field equations and the wave equation for the scalar field. The characteristics of the EoS parameter has been studied for a homogeneous and isotropic space of zero curvature, unlike other recent studies in this regard. This study is based on a model where a linear combination of BD field equations has been taken. It leads to a generalized expression of the EoS parameter which is a function of the ratio of the constant coefficients corresponding to that linear combination. This ratio has been varied to see its effect on the time variation of the EoS parameter. The value of the EoS parameter, at the present epoch, is found to depend solely on a constant parameter that governs the dependence of the scalar field upon time. This parameter and the ratio of constant coefficients have been varied to show the time dependence of the EoS parameter graphically. In most cases it is found that, the EoS parameter starts rising from a negative value, at a point of time prior to the present epoch, at a gradually decreasing rate, to reach a negative saturation value.

Keywords: Cosmology, Equation-of-State Parameter, Brans-Dicke Theory, Scalar Field, Brans-Dicke Parameter

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1.   INTRODUCTION

Interpretation of data obtained from high precision astrophysical observations has established beyond doubt that the universe is expanding with acceleration [1]. An exotic entity, known as dark energy (DE), is believed to generate and govern this accelerated expansion. The exact nature of this DE is yet to be determined. A parameter, known as cosmological constant (Λ) in General Relativity (GR), has very often been used to represent dark energy in theoretical calculations. It has its own shortcomings, although it has explained several experimental observations satisfactorily [2]. To account for gravitational observations, various alternative theoretical models have been proposed. One may find ample information regarding the strengths and weaknesses of these models in scientific literature [3]. Among the non-minimally coupled scalar field theories, the Brans-Dicke (BD) theory of gravity has been found to be highly useful in explaining the observations of accelerated expansion [4]. In the prediction of observational findings, the dimensionless parameter ω in BD theory plays a very significant role [5, 6]. BD theory has several models where a small value of ω, typically of the order of unity, leads to accelerated expansion[4-6]. The Brans-Dicke scalar field alone has been found to predict an accelerated expansion, in the present matter dominated era of the universe, without taking into account the presence of any quintessence matter or any interaction between the dark matter and the BD field [7]. Bergman and Wagoner proposed a generalized version of Brans-Dicke theory [8]. A more useful form of this theory, proposed by Nordtvedt, can predict these observations [9, 10]. The dimensionless BD parameter (ω) is regarded as a function of the scalar field (φ) in generalized BD theory, and thereby, it should be regarded as a function of time [10].

Observations on large scale structure and cosmic microwave background radiation demonstrate that the universe is highly homogeneous and isotropic on large scales [11, 12]. In the recent years, the idea of an accelerating universe has emerged, on the basis of observational results [13]. One of the major aims of research in this regard is to find the true nature of a strange type of repulsive force, driving the accelerated expansion, which is said to be generated by an entity named dark energy (DE). One does not have a concrete understanding about the true nature of DE which is known to have a constant or a slightly changing energy density as the universe expands [2]. Dark energy has been conventionally characterized by the equation of state (EoS) parameter ($\gamma \equiv P/\rho$), which should not be regarded as a constant. We find $-1.67 < \gamma < -0.62$ from observational results obtained from SN Ia data[14]. One should not essentially treat γ as a constant. On account of insufficient observational evidence to estimate the time variation of γ, the EoS parameter has been regarded as a constant in many theoretical studies, with values -1, 0, $1/3$ and $+1$ for vacuum fluid, dust fluid, radiation and stiff fluid dominated universe respectively [5]. In general, the EoS parameter is a function of time or redshift [15]. A number of models on the time

dependence of γ have emerged in recent years[16]. Variable EoS parameter, based on generalized dark energy models, has been studied byYadav *et al* and Pradhan [17, 18].

Time dependence of the equation of state parameter (γ) has been explored in the present study, from the field equations and also the wave equation for the scalar field, in the framework of Brans-Dicke theory of gravity. For this purpose, empirical expressions for the scale factor, scalar field and the BD parameter have been used. The scale factor has been chosen in a manner such that it generates a time dependent deceleration parameter which changes sign with time, from positive to negative, indicating a transition of the expanding universe from a phase of deceleration to acceleration, as per several recent observations [4, 10]. As a step towards generalization, a linear combination of the two field equation has been taken and the ratio of the constant coefficients has been varied to see its effect on the time dependence of the EoS parameter. The present value of the EoS parameter (γ_0) has been found to depend, almost solely, on a constant parameter which controls the variation of the scalar field as a function of scale factor. The time dependence of the EoS parameter has been shown graphically for different values of these parameters. The nature of dependence of γ upon time, obtained from BD theory of gravity, is found to be similar to the results of studies based on dark energy models [17, 18], assuming anisotropy of space, in the framework of Einstein's theory of gravity.

2.   SOLUTION OF FIELD EQUATIONS

For a space of curvature k, the field equations of generalized Brans-Dicke theory, obtained by using FRW space-time, are expressed as,

$$3\frac{\dot{a}^2+k}{a^2} + 3\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi} - \frac{\omega(\varphi)}{2}\frac{\dot{\varphi}^2}{\varphi^2} = \frac{\rho}{\varphi} \tag{01}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2+k}{a^2} + \frac{\omega(\varphi)}{2}\frac{\dot{\varphi}^2}{\varphi^2} + 2\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi} + \frac{\ddot{\varphi}}{\varphi} = -\frac{P}{\varphi} \tag{02}$$

The wave equation for the scalar field (φ) is expressed as,

$$\ddot{\varphi} + 3\frac{\dot{a}\dot{\varphi}}{a} = \frac{\rho - 3P}{2\omega+3} - \frac{\dot{\omega}\dot{\varphi}}{2\omega+3} \tag{03}$$

Combining equations (1), (2) and (3), one obtains,

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0 \tag{04}$$

The equation of state of the cosmic fluid is $P = \gamma\rho$, where P and ρ are the pressure

and the density of matter (dark + baryonic) respectively and γ is the equation of state (EoS) parameter. Solution of equation (4), for a constant value of γ, is given by,

$$\rho = \rho_0 a^{-3(1+\gamma)} \tag{05}$$

A new differential equation can be formed by taking a linear combination of the equations (1) and (2). Taking a sum of these two equations, after multiplying them by two constants, x and y respectively, one gets the following expression of the EoS parameter (γ).

$$\gamma = \frac{1}{y}\left[x - \frac{\varphi}{\rho}\left\{2y\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}(3x+y) + \frac{\omega\dot{\varphi}^2}{2\varphi^2}(y-x) + \frac{\dot{a}\dot{\varphi}}{a\varphi}(3x+2y) + y\frac{\ddot{\varphi}}{\varphi}\right\}\right] \tag{06}$$

Using the relation $P = \gamma\rho$ in equation (3) one gets,

$$\frac{\varphi}{\rho} = \frac{1-3\gamma}{2\omega+3}\left(\frac{\ddot{\varphi}}{\varphi} + 3\frac{\dot{a}\dot{\varphi}}{a\varphi} + \frac{\dot{\omega}}{2\omega+3}\frac{\dot{\varphi}}{\varphi}\right)^{-1} \tag{07}$$

Substituting for $\frac{\varphi}{\rho}$ in equation (6) from equation (7) one gets,

$$\gamma = \frac{\frac{x}{y} - \frac{1}{y(2\omega+3)}\frac{f_1}{f_2}}{1 - \frac{3}{y(2\omega+3)}\frac{f_1}{f_2}} \tag{08}$$

where,

$$f_1 = 2y\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}(3x+y) + \frac{\omega\dot{\varphi}^2}{2\varphi^2}(y-x) + \frac{\dot{a}\dot{\varphi}}{a\varphi}(3x+2y) + y\frac{\ddot{\varphi}}{\varphi}$$

$$f_2 = \frac{\ddot{\varphi}}{\varphi} + 3\frac{\dot{a}\dot{\varphi}}{a\varphi} + \frac{\dot{\omega}}{2\omega+3}\frac{\dot{\varphi}}{\varphi}$$

Equation (8) may be regarded as the general expression of the EoS parameter (γ) in BD theory. To determine its time dependence, following empirical expressions have been used.

$$a = a_0(t/t_0)^\varepsilon Exp[\mu(t-t_0)] \tag{09}$$

$$\varphi = \varphi_0\left(\frac{a}{a_0}\right)^n \tag{10}$$

$$\omega = \omega_0\left(\frac{\varphi}{\varphi_0}\right)^m \tag{11}$$

The scale factor (eqn. 9) has been chosen to ensure a change of sign of the deceleration parameter with time, from positive to negative, in accordance with recent observations indicating a change of phase of the expanding universe from decelerated expansion to accelerated expansion [4]. In the calculations of the present study we have taken $a_0 = 1$.

Here $\varepsilon, \mu > 0$ to ensure increase of scale factor with time. The Hubble parameter (H) and the deceleration parameter (q), calculated from this scale factor, are written below.

$$H = \mu + \frac{\varepsilon}{t} \tag{12}$$

$$q = -1 + \frac{\varepsilon}{(\varepsilon + \mu t)^2} \tag{13}$$

For $0 < \varepsilon < 1$, we get $q > 0$ at $t = 0$ and, $q \to -1$ as $t \to \infty$.
Using the conditions that, $H = H_0$ and $q = q_0$ at $t = t_0$, one obtains,

$$\varepsilon = (H_0 t_0)^2 (q_0 + 1) \tag{14}$$

$$\mu = H_0 - H_0{}^2 t_0 (q_0 + 1) \tag{15}$$

The scalar field (eqn. 10) has been chosen on the basis of some previous studies on Brans-Dicke theory of gravitation [4, 10]. The empirical expression of BD parameter (eqn. 11) has been chosen according to the generalized Brans-Dicke theory, where ω is regarded as a function of the scalar field (φ) [4]. The values of ω_0 and m have to be determined from the field equations.

Using equation (10) along with the relation $G = 1/\varphi$ one obtains,

$$n = -\frac{1}{H_0} \left(\frac{\dot{G}}{G}\right)_{t=t_0} \tag{16}$$

With the help of experimental observations regarding H_0 and $\left(\frac{\dot{G}}{G}\right)_{t=t_0}$, the parameter n can be determined from equation (16). Experimental observations regarding $\left(\frac{\dot{G}}{G}\right)_{t=t_0}$, as obtained by many researchers, have been found to be both positive and negative [19]. According to S. Weinberg, we must have, $\left|\left(\frac{\dot{G}}{G}\right)_{t=t_0}\right| \leq 4 \times 10^{-10} Yr^{-1}$ [20]. Using equation (16), this requirement can be expressed as,

$$|n| \leq \frac{4 \times 10^{-10} Yr^{-1}}{H_0} \quad \text{or,} \quad |n| \leq 5.44 \text{ (taking } H_0 = 7.348 \times 10^{-11} Yr^{-1}) \tag{17}$$

An expression of ω_0, determined from equation (1), taking $k = 0$, is given by,

$$\omega_0 = \frac{6}{n^2}\left\{1 + n - \frac{\rho_0}{3\varphi_0 H_0{}^2}\right\} \tag{18}$$

Combining equations (2), (3), (10) and (11) and writing all values for $t = t_0$ one gets,

$$m = \frac{\rho_0/\varphi_0 + H_0{}^2\{q_0(2n\omega_0 - 6) + 3(1 - n) - 0.5\omega_0 n^2 - 4n\omega_0\}}{\omega_0 n^2 H_0{}^2} \tag{19}$$

The values of n and ω_0 in equation (19) can be obtained from equations (16) and (18).

Combining equations (9), (10) and (11) the time dependence of BD parameter is obtained as,

$$\omega = \omega_0(t/t_0)^{mn\varepsilon} Exp[mn\mu(t - t_0)] \tag{20}$$

The values of n, ω_0 and m in equation (20) can be obtained from equations (16), (18) and (19) respectively.

Combining equations (9) and (10), φ can be written as,

$$\varphi = \varphi_0(t/t_0)^{n\varepsilon} Exp[n\mu(t - t_0)] \tag{21}$$

In equations (20) and (21), ε and μ are obtained from equations (14), (15) respectively.

Using equations (10) and (11) in the generalized γ expression (eqn. 8), one obtains,

$$\gamma = \frac{r - \frac{f}{2\omega + 3}}{1 - \frac{3f}{2\omega + 3}} \tag{22}$$

where, $r = \frac{x}{y}$ and $f = \dfrac{-2q + 3r + 1 + 0.5\omega n^2(1 - r) + n(3r + 2) + n(n - q - 1)}{n^2 - nq + 2n + \frac{m\omega n^2}{2\omega + 3}}$

In equation (22), the values of q, n, m and ω should be taken from equations (13), (16), (19) and (20) respectively. Thus, in the present model, the time dependence of the EoS parameter (γ) can be studied from equation (22).The value of γ at $t = t_0$, from equation (22), is,

$$\gamma_0 = \frac{r - \frac{f_0}{2\omega_0+3}}{1 - \frac{3f_0}{2\omega_0+3}} \tag{23}$$

where, $r = \dfrac{x}{y}$ and $f_0 = \dfrac{-2q_0+3r+1+0.5\omega_0 n^2(1-r)+n(3r+2)+n(n-q_0-1)}{n^2-nq_0+2n+\frac{m\omega_0 n^2}{2\omega_0+3}}$

The values of different cosmological parameters used for this study are given below.

$$H_0 = \frac{72\frac{Km}{s}}{Mpc} = 2.33 \times 10^{-18} sec^{-1}, \ q_0 = -0.55, \ \rho_0 = 2.83 \times 10^{-27} Kgm^{-3}$$

$$\varphi_0 = \frac{1}{G_0} = 1.498 \times 10^{10} Kg^2 m^{-2} N^{-1}, \ t_0 = 4.36 \times 10^{17} s$$

Using the values of these cosmological parameters, the expression of γ_0, from equation (23), can be written as,

$$\gamma_0 = \frac{-1.6884 + 5.0652\,r + n\{-1.51667+4.55\,r + n(r-1/3)\}}{0.0115996-0.0347987\,r} \quad \text{where,} \ r = \frac{x}{y} \tag{24}$$

As per equation (24), γ_0 is determined mainly by the value of the parameter n. The dependence of γ_0 upon the value of the ratio r is extremely small. The time variation of γ, obtained from equation (22) is controlled by both n and r.

Combining equations (2) and (10), and also using the relation $P = \gamma\rho$, the expression of matter density (ρ) (dark + baryonic), as a function of EoS parameter (γ), is given by,
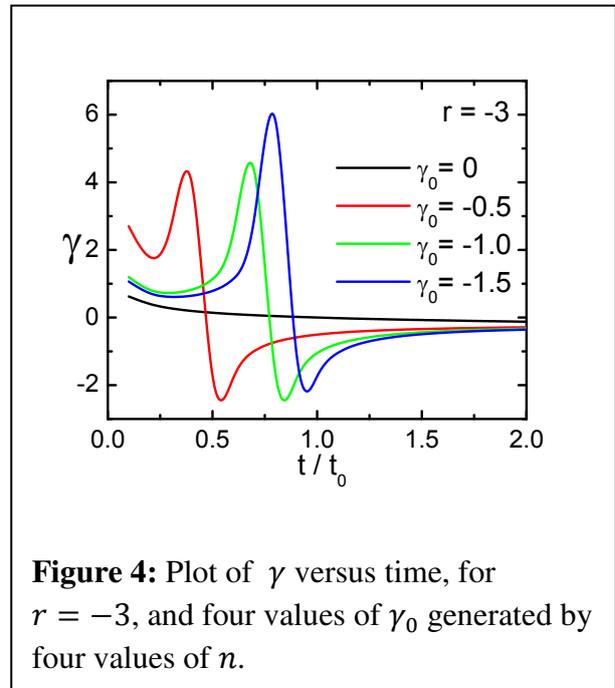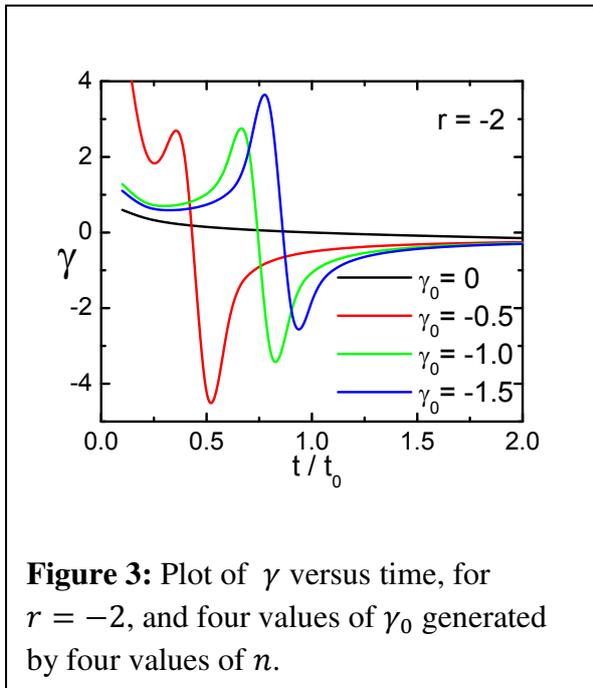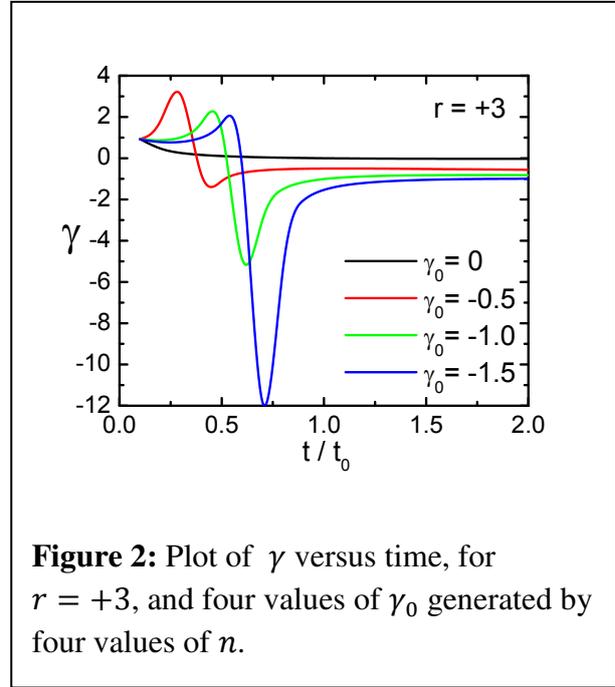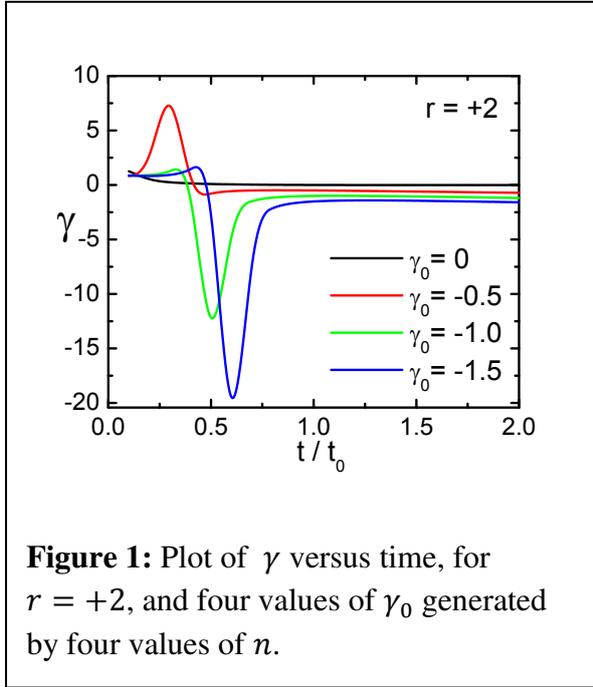
$$\rho = \frac{\varphi H^2}{2\gamma}[2q(2+n) - 2(1+n+n^2) - \omega n^2] \tag{25}$$

In equation (25), the values of H, q, n, ω, φ and γ can be obtained from equations (12), (13), (16), (20), (21) and (22) respectively.

3. RESULTS

In several studies, the present value of the EoS parameter (γ_0) for the matter dominated universe has been taken to be zero [5, 6]. Equation (24) of the present model shows that, $\gamma_0 = 1.58 \times 10^{-9}$ for $n = -1.942701081$. This value of γ_0 can be effectively taken to be zero for all practical purposes. It has been found that the value of γ_0 is approximately equal to -0.5, -1.0 and -1.5 for $n = -1.9174$, -1.8939

and -1.8716 respectively. We have plotted the EoS parameter (γ) as a function of time for four values of n for which γ_0 is approximately equal to 0, -0.5, -1.0 and -1.5, in view of the range of variation of γ_0 obtained from observations [17, 18]. For positive values of n, equation (24) generates large negative values of γ_0, contrary to observations according to which γ_0 should have a small negative value close to -1 [17, 18]. So we need to choose the value of n in the approximate range from -1.94 to -1.87. Figures 1, 2, 3 and 4 show the γ versus time plots for four different values of the ratio $r(\equiv x/y)$, which are $+2, +3, -2, -3$ respectively. In each of these four figures (Figs. 1 - 4), we have shown four graphs for four different values of the parameter n, leading to four values of γ_0, which are 0, -0.5, -1.0 and -1.5. The plots with $\gamma_0 = 0$, in figures 1-4, show that the EoS parameter (γ) decreases monotonically with time with a slope which also decreases with time. The time dependence of γ, in these curves, are similar to that shown in figure 2 of an article by A. Pradhan, based on Einstein's theory of gravity [18]. The plots with other three values of γ_0, i.e. $\gamma_0 = -0.5$, -1.0 and -1.5, in figures 1-4, show that, after going through some initial crests and troughs, the EoS parameter starts rising before the present epoch ($t = t_0$), at a gradually smaller rate to reach a saturation level. This saturation level for γ is found to be different for different values of γ_0, caused by different values of the parameter n, which controls the nature of time dependence of the scalar field (φ) according to equation (10). The nature of the curves for $\gamma_0 = -0.5, -1, -1.5$, are found to be similar to figure 6 of A. Pradhan's study on dark energy models with anisotropic fluid [18]. The time dependence of γ, shown by the curves with these three values of γ_0, are also similar to that obtained in a study of dark energy models by Yadav et al. [17]. A comparison between the figures 1 and 2 shows that, a smaller positive value of the parameter r causes the curves to attain the saturation value faster. A comparison between the figures 3 and 4 shows that, the plots with $\gamma_0 = 0$ have almost the same nature of time dependence, i.e. γ decreases monotonically with time, gradually attaining saturation at a negative value. Each curve with $\gamma_0 = -0.5, -1, -1.5$ passes through a minimum before reaching the present epoch ($t = t_0$). This minimum is at a greater depth for each value of γ_0 in figure 3, in comparison to figure 4. The peaks, preceding these minima, have greater heights in figure 4 than in figure 3. For positive values of the parameter r, we see a faster journey of the EoS parameter to its saturation value, compared with the negative values of r. All curves for $\gamma_0 \neq 0$, in figures 3 and 4, show a transition of the universe from a state of phantom energy ($\gamma < -1$) to quintessence ($\gamma > -1$), passing through an era dominated by vacuum fluid ($\gamma_0 = 0$). All curves for $\gamma_0 \neq 0$, in figures 1-4, show a transition from quintessence to phantom energy, before reaching the minimum, preceding the present time ($t = t_0$), from where they begin to rise to reach their saturation values.

Figure 1: Plot of γ versus time, for $r = +2$, and four values of γ_0 generated by four values of n.

Figure 2: Plot of γ versus time, for $r = +3$, and four values of γ_0 generated by four values of n.

Figure 3: Plot of γ versus time, for $r = -2$, and four values of γ_0 generated by four values of n.

Figure 4: Plot of γ versus time, for $r = -3$, and four values of γ_0 generated by four values of n.

4. CONCLUSIONS

In the present study we have taken a linear combination of equations (1) and (2) where x and y are the constant coefficients. Equation (6) is a differential equation formed by their linear combination. It has the same mathematical validity as the equations (1), (2) and (3), for any choice of values for x and y, in the particular sense that it is supposed to be satisfied by the scale factor (a), scalar field (φ) and their time derivatives, like those equations. This linear combination of two field equations of BD theory may be regarded as a first step towards a generalization to form an effective field equation. At the same time, this method makes it difficult for us to determine the correct combination of values of the variables x and y to get the correct behaviour of the EoS parameter (γ). Luckily, these variables are found everywhere in the form of a ratio, denoted by r ($\equiv \frac{x}{y}$). So, we don't have to vary them separately. We need to observe the behaviour of γ as a function of the ratio r. It has been found from this formulation that γ_0 is determined, almost solely, by the parameter n which determines the dependence of the scalar field upon the scale factor, thereby governing the time dependence of the scalar field, although the nature of time dependence of γ is controlled by both n and r. Equation (24) shows that, the larger the value of r ($\equiv \frac{x}{y}$), smaller would be the dependence of γ_0 upon r. According to a study by Banerjee and Pavon, the approximate range of variation of ω_0 is $-1.5 < \omega_0 < 0$ [7]. It is found from equation (18) that ω_0 is always positive for positive values of n. Therefore, only the negative values of n are likely to predict the cosmic expansion characteristics correctly. We have to choose only those values of n for which γ_0 has a small negative value (close to -1), as per experimental observations [14]. Therefore, we must choose n such that $-1.942701081 \leq n < 0$. This range of n also satisfies the condition expressed by equation (17). One needs more astrophysical observations to determine the value of r ($\equiv \frac{x}{y}$), which is found to control the rapidity with which γ approaches its saturation value. One may formulate a better model by taking a linear combination of equations (1), (2) and (3), which can be regarded as a generalization of the present formulation. One may improve this model by using a scale factor which is a solution of the field equations (eqns. 1 and 2), obtained from them by using the empirical expression of the scalar field (eqn. 10). This further generalization would pose a much greater challenge of determining the values of three constant coefficients. An important aspect of the present study is that, unlike other recent studies [17, 18], the time dependent behaviour of the EoS parameter has been determined, from Brans-Dicke field equations and the wave equation for the scalar field, assuming the isotropy and homogeneity of space and without considering the role played by the entity named dark energy in cosmic expansion.

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