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Edge-Odd Graceful Labeling of Sum of K_2 & Null Graph with n Vertices and a Path of n Vertices Merging with n Copies of a Fan with 6 Vertices

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Abstract

A (p, q) connected graph G is edge-odd graceful graph if there exists an injective map f: E(G) → {1, 3, …, 2q-1} so that induced map f_+: V(G) → {0, 1,2, 3, …, (2k-1)}defined by $f_+(x) \equiv \Sigma f(xy) \pmod{2k}$, where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd. In this article, the edge-odd graceful labelings of both $P_2 + N_n$ and $P_n * nF_6$ are obtained.

Keywords: graceful graph, edge -odd graceful labeling, edge -odd graceful graph

INTRODUCTION:

Abhyankar and Bhat-Nayak [2000] found graceful labeling of olive trees. Barrientos [1998] obtained graceful labeling of cyclic snakes, and he also [2007] got graceful labeling for any arbitrary super-subdivisions of graphs related to path, and cycle. Burzio and Ferrarese [1998] proved that the subdivision graph of a graceful tree is a graceful tree. Gao [2007] analyzed odd graceful labeling for certain special cases in terms of union of paths. Kanetkar and Sane [2007] investigated graceful labeling of a family of quasi-stars with paths in arithmetic progressions. Lee et.al. [2005] gave vertex-graceful for (p, p+1)-graphs. Riskin and Weidman [2008] showed that disjoint unions of 2r-regular edge graceful graphs are edge graceful. Sethiraman and Jesintha [2009] verified that all banana trees are graceful. Sethuraman and Selvaraju [2001] invented that one vertex unions of non-isomorphic complete bipartite graphs are graceful.

Section-2: FEW BASIC DEFINITIONS

In this section the edge-odd gracefulness of $K_2 + N_n$ is obtained.

Definition 2.1: Graceful Graph: A function f of a graph G is called a graceful labeling with m edges, if f is an injection from the vertex set of G to the set {0, 1, 2, …, m} such that when each edge uv is assigned the label $|f(u) - f(v)|$ and the resulting edge labels are distinct. Then the graph G is graceful.

Definition 2.2: Edge-odd graceful graph: A (p, q) connected graph is edge-odd graceful graph if there exists an injective map f: E(G) → {1, 3, …, 2q-1} so that induced map f_+: V(G) → {0, 1, 2, …,(2k-1)} defined by $f_+(x) \equiv \Sigma f(x, y) \pmod{2k}$, where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd. Hence the graph G is edge-odd graceful.

Lemma 2.3: The connected graphs $P_2 + N_3$, $P_2 + N_5$, and $P_2 + N_7$ are edge-odd graceful.

Proof: (i). The graph $P_2 + N_3$ is a connected graph with 5 vertices and 7 edges. One of the labeling of edge- odd graceful of the required graph is obtained as follows:

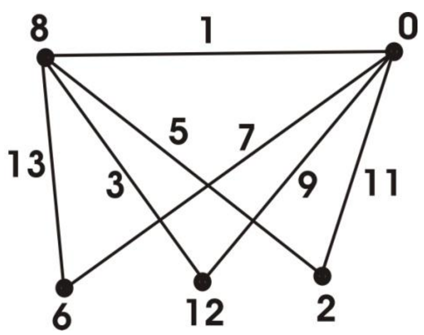

Figure 1: Edge-odd graceful labeling for the graph $P_2 + N_3$

(ii). The graph $P_2 + N_5$ is a connected graph with 7 vertices and 11 edges. One of edge-odd graceful of the required graph is obtained as follows:

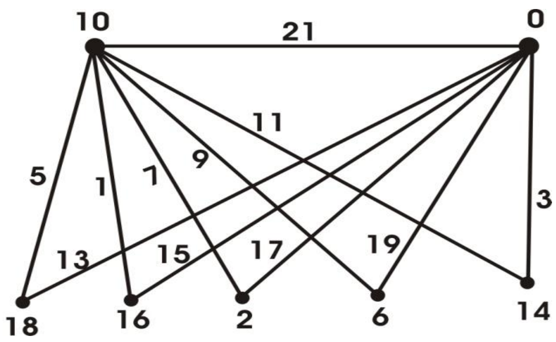

Figure 2: Edge-odd graceful labeling for the graph $P_2 + N_5$

(iii). The graph $P_2 + N_{11}$ is a connected graph with 13 vertices and 23 edges. One of edge- odd graceful of the required graph is obtained as follows:

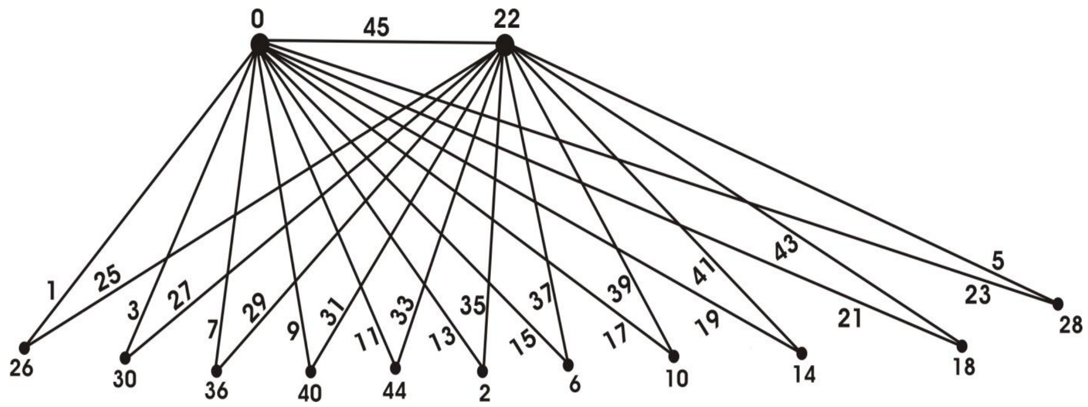

Figure 3: Edge-odd graceful labeling for the graph $P_2 + N_{11}$

Theorem 2.1: The connected graph $P_2 + N_n$ is edge-odd graceful.

Proof: The graph $P_2 + N_n$ is a connected graph with 2n vertices and 2n+1 edges and one of the arbitrary labeling for the edges of $P_2 + N_n$ are mentioned below.

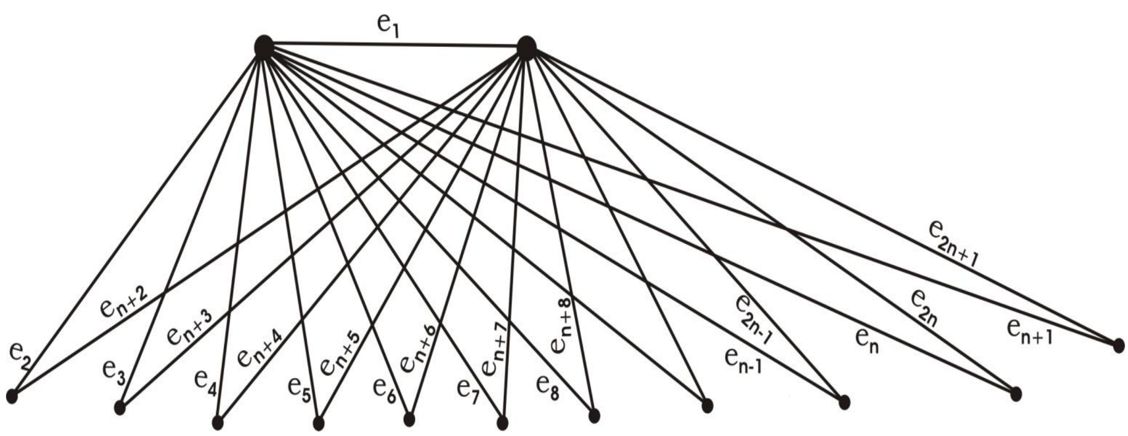

Figure 4: Edge-odd graceful graph $P_2 + N_n$

To find edge-odd graceful, define f: $\mathbf{E\ (P_2 + N_n) \rightarrow \{1, 3, \ldots, 2q-1\}}$ by

n is even or n ≡ 7 (mod 8)

$f(e_i) = 2i-1, \quad i = 1,2,3,\ldots(2n+1)$ … Rule (1).

n is odd

Case i. For n ≡ 1 (mod 8)

$f(e_1) = 4n+1, f(e_2) = 1, f(e_3) = 3, f(e_{2n+1}) = 5.$

$f(e_i) = 2i-1, \quad i = 4, 5, 6, \ldots, 2n.$ Rule (2)

Case ii. For n ≡ 3 (mod 8)

$f(e_1) = 4n-1, f(e_2) = 5, f(e_3) = 1, f(e_{2n}) = 4n+1.$

$f(e_{2n+1}) = 3.$

$f(e_i) = 2i-1, \quad i = 4, 5, 6, \ldots, (2n-1).$ Rule (3)

Case iii. For n ≡ 5 (mod 8)

$f(e_1) = 3$, $f(e_2) = 1$.

$f(e_i) = 2i-1$, $i = 3, 4, 5, 6, \ldots, (2n+1)$. Rule (4)

Define $f_+: V(G) \to \{0, 1, 2, \ldots, (2k-1)\}$ by

$f_+(v) \equiv \Sigma f(uv) \bmod (2k)$,where this sum run over all edges through v …Rule (5).

Hence the induced map f_+ provides the distinct labels for vertices and also the edge labeling is distinct. Hence the connected graph $P_2 + N_n$ is edge-odd graceful.

Example 2.2: The connected graph $P_2 + N_6$ is edge-odd graceful.

Proof: The graph $P_2 + N_6$ is a connected graph with 8 vertices and 13 edges, where n is even or n ≡ 7 (mod 8).

Due to the rules (1) & (5) in (2.1), edge-odd graceful labeling of the required graph is obtained as follows:

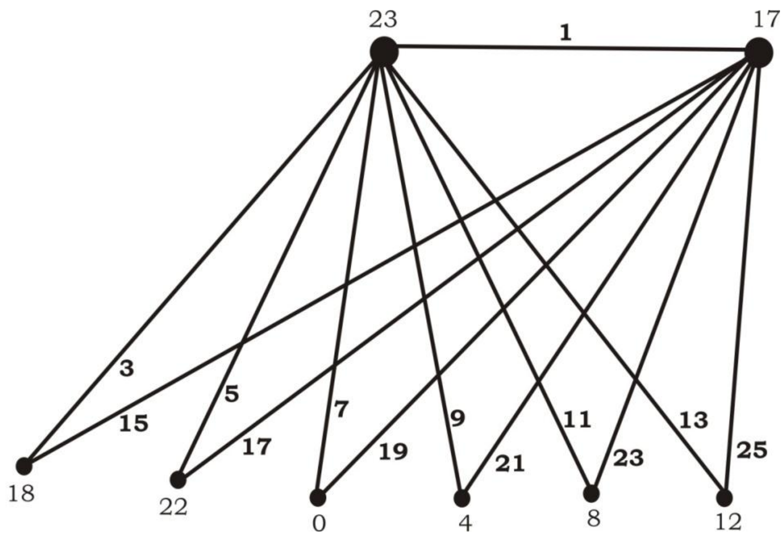

Figure 5: Edge-odd graceful graph $P_2 + N_6$

Example 2.3: The connected graph $P_2 + N_9$ is edge-odd graceful.

Proof: The graph $P_2 + N_9$ is a connected graph with 11 vertices and 19 edges, where n ≡ 1 (mod 8). Due to the rules (2) & (5) in (2.1), edge-odd graceful labeling of the required graph is obtained as follows:

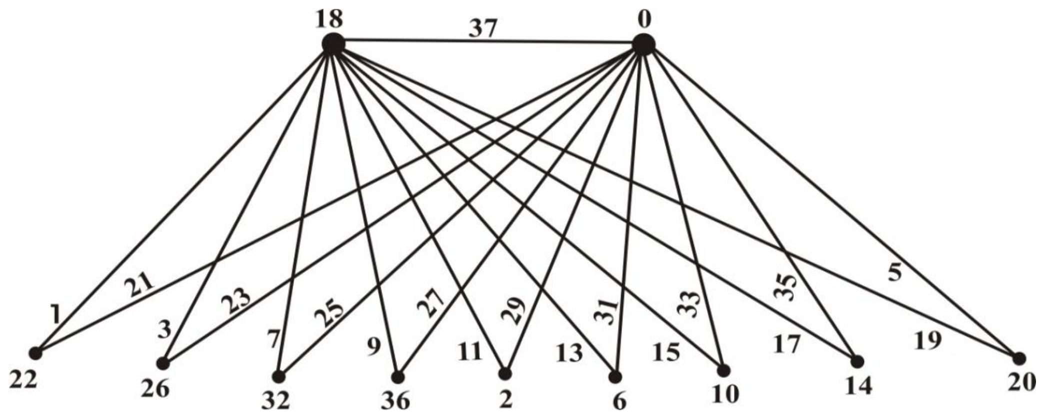

Figure 6: Edge-odd graceful graph $P_2 + N_9$

Example 2.4: The connected graph $P_2 + N_{19}$ is edge-odd graceful.

Proof: The graph $P_2 + N_{19}$ is a connected graph with 21 vertices and 39 edges, where n ≡ 3 (mod 8).

Due to the rules (3) & (5) in (2.1), edge-odd graceful labeling of the required graph is obtained as follows:

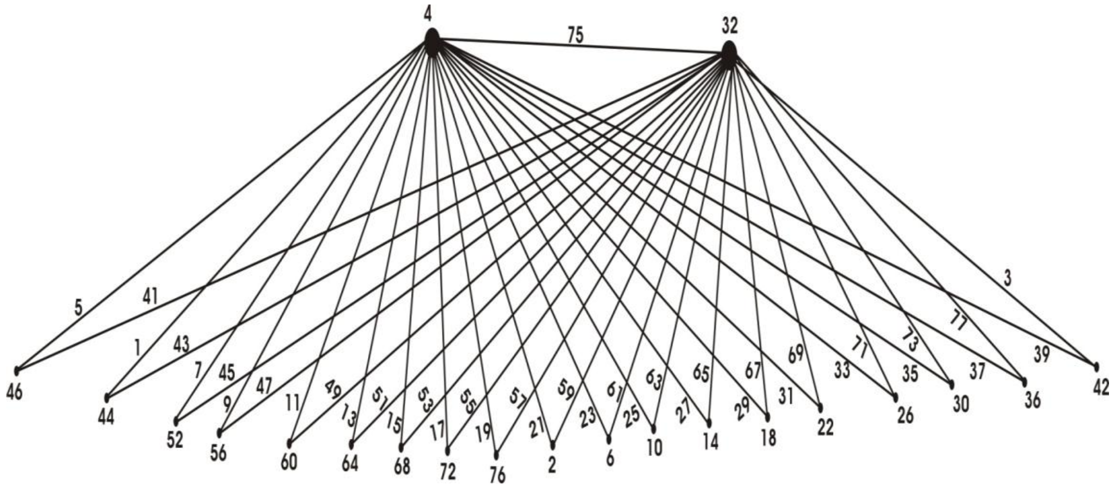

Figure 7: Edge-odd graceful graph $P_2 + N_{19}$

Example 2.5: The connected graph $P_2 + N_{13}$ is edge-odd graceful.

Proof: The graph $P_2 + N_{13}$ is a connected graph with 15 vertices and 27 edges, where $n \equiv 5 \pmod 8$.

Due to the rules (4) & (5) in (2.1), edge-odd graceful labelings of the required graph is obtained as follows:

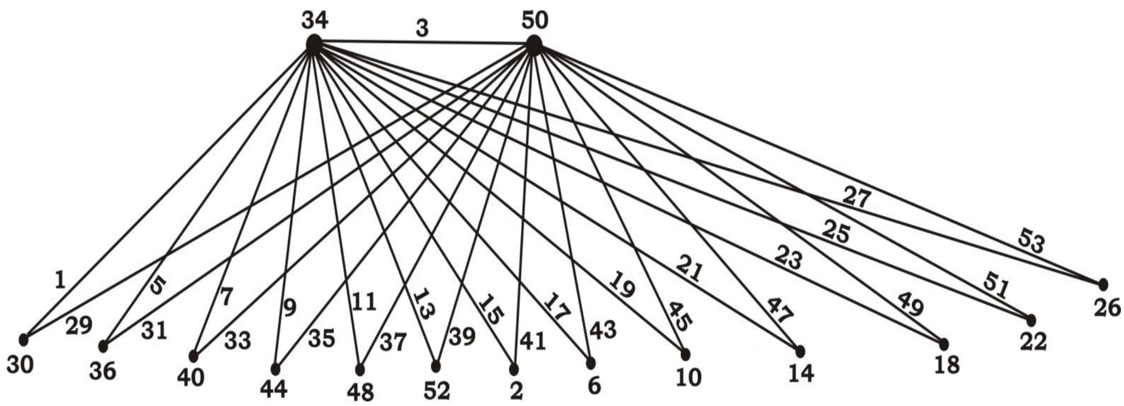

Figure 8: Edge-odd graceful graph $P_2 + N_{13}$

Definition 2.6: $P_n * nF_6$ is a connected graph whose vertex set is $\{v_1, v_2, \ldots, v_{7n}\}$, and edge set is $\{v_i v_{i+1} : i = 1 \text{ to } 6n \,;\, i \neq 0 \pmod 6\} \cup \{ v_{6n+i}\, v_{(i-i)6+j} : i = 1 \text{ to } n \,;\, j = 1 \text{ to } 6\} \cup \{ v_{6n+i}\, v_{6n+i+1} : i = 1 \text{ to } (n-1) \}$.

Theorem 2.7: The graph $P_n * nF_6$ is edge-odd graceful where n is a positive integer.

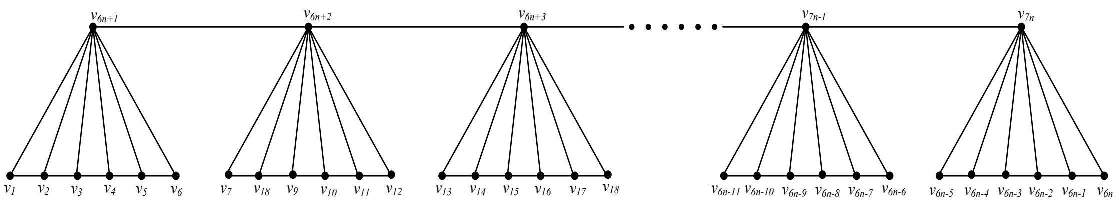

Figure 9: The connected graph $P_n * nF_6$

Its vertex set is $\{v_1, v_2, \ldots . v_{7n}\}$, and edge set is $\{ v_i v_{i+1} : i = 1 \text{ to } 6n ; i \neq 0 (\text{mod } 6)\} \cup \{ v_{6n+i}\, v_{(i-i)6+j} : i = 1 \text{ to } n ; j = 1 \text{ to } 6\} \cup \{ v_{6n+i}\, v_{6n+i+1} : i = 1 \text{ to } (n-1) \}$.

Define a map **$f : E(P_n * nF_6) \to \{1, 3, 5, \ldots, (2q-1)\}$** by

$f(v_i v_{i+1}) = 12k+1$, $i = 6k+1$, $i = 1$ to $5n$, $i \neq 0$ (mod5), $i = 1$(mod5) ; k is even

$= 12k+3$, for $i = 6k+2$

$= 12k+5$, for $i = 6k+3$

$= 12k+7$, for $i = 6k+4$

$= 12k+9$, for $i = 6k+5$, $i = 1$ to $6n$; $i \neq 0$ (mod 6) ; $i \neq (6n-5)$

$f(v_{6n+i}\, v_{(i-1)6+j}) = 24(i-1)+(9+2j)$, $j = 2, 3, 4, 5$ where $i = 1$ to n

n is odd

$f(v_{6n+i}\, v_{6(i-1)+1}) = 24(i-1)+11$; $i = 1$ to n

$f(v_{6n+i}\, v_{6n+i+1}) = 24(i-1)+21$; $i = 1$ to $(n-1)$

$f(v_{6n+i}\, v_{6(i-1)+6}) = 24(i-1)+23$; $i = 1$ to $(n-1)$

$f(v_{6n-5}\, v_{6n-4}) = (2q-1)$; $f(v_{6n}\, v_{7n}) = 2q-1-48$

n is even

$f(v_{6n+i}\, v_{6(i-1)+1}) = 24(i-1)+21$; $i = 1$ to n

$f(v_{6n+i}\, v_{6(i-1)+1}) = 24(i-1)+23$; $i = 1$ to $(n-1)$

$f(v_{6n+i}\, v_{6n+i+1}) = 24(i-1)+11$; $i = 1$ to $(n-1)$

$f(v_{6n-5}\, v_{6n-4}) = 2q-1-48$

$f(v_{6n}v_{7n}) = (2q - 1)$

Define $\mathbf{f^+ : V(P_n * nF_6) \rightarrow \{0, 1, 2, ..., q\}}$ by $f^+(U) = \sum_{V \in G} f(UV)$ (mod 2q) where this sum run over all edges through the vertex U. Hence the map f and the induced map f_+ provide labels as distinct odd numbers for edges and also the labelings for vertex set have distinct values in {0, 1, 2,…., (2k-1)}. Hence the graph $P_n * nF_6$ is edge-odd graceful.

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