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Some Properties on Connected Accurate Domination in Fuzzy Graphs

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Abstract

In this paper, the concept of connected accurate domination in fuzzy graphs discussed. A dominating set D of a fuzzy graph G is said to be an accurate dominating set if $V - D$ has no dominating set of same cardinality $|D|$. Let G be a connected fuzzy graph and an accurate dominating set D of G is said to be a connected accurate dominating set if an induced fuzzy subgraph $\langle D \rangle$ of G is connected. The connected accurate domination number of a fuzzy graph G is the minimum cardinality taken over all connected accurate dominating sets of G, and it is denoted by $\gamma_{fca}(G)$. We prove some results on connected accurate dominating set and exact values of connected accurate domination number, $\gamma_{fca}(G)$, for some standard fuzzy graphs are found.

Keywords: Accurate dominating set, Connected Accurate dominating set, Accurate domination number, Connected Accurate domination number, Strong arc and Strong neighbors.

AMS Subject Classification: 03E72, 05C40, 05C72

1. INTRODUCTION

The study of dominating sets in graphs was started by Ore [12] and Berge [1]. Then, Cockayne and Hedetniemi [3] has introduced the domination number and the independent domination number. An accurate domination and connected accurate domination was introduced by Kulli and Kattimani [4, 5].

The concept of fuzzy relation was introduced by Zadeh [15] in his classical paper in 1965. Rosenfeld [13] introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. Somasundaram and Somasundaram [14] discussed domination in fuzzy graphs using effective edges. Nagoor Gani and Chandrasekaran [8] discussed domination in fuzzy graph using strong arc. Domination, independent domination and irredundance in fuzzy graphs using strong arc was discussed by Nagoor Gani and Vadivel [10, 11].

2. PRELIMINARIES

A *fuzzy graph* $G = \langle \sigma, \mu \rangle$ is a pair of functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$, where for all $x, y \in V$ we have $\mu(x,y) \leq \sigma(x) \wedge \sigma(y)$.

A fuzzy graph $H = \langle \tau, \rho \rangle$ is called a *fuzzy subgraph* of G if $\tau(v_i) \leq \sigma(v_i)$ for all $v_i \in V$ and $\rho(v_i, v_j) \leq \mu(v_i, v_j)$ for all $v_i, v_j \in V$.

The *underlying crisp graph* of a fuzzy graph $G = \langle \sigma, \mu \rangle$ is denoted by $G^* = \langle \sigma^*, \mu^* \rangle$, where $\sigma^* = \{v_i \in V / \sigma(v_i) > 0\}$ and $\mu^* = \{(v_i, v_j) \in V \times V / \mu(v_i, v_j)\ is\ a\ strong\ arc\}$.

A vertex in G is called an *isolated vertex* if it is not adjacent to any vertices of G. An edge in G is called an *isolated edge* if it is not adjacent to any edge in G.

A *path* with n vertices in a fuzzy graph is denoted by P_n. A *cycle* with n vertices in a fuzzy graph is denoted by C_n.

A fuzzy graph $G = \langle \sigma, \mu \rangle$ is said to be a *complete fuzzy graph,* if $\mu(v_i, v_j) = \sigma(v_i) \wedge \sigma(v_j)$ for all $v_i, v_j \in \sigma^*$.

An arc (x, y) in a fuzzy graph $G = \langle \sigma, \mu \rangle$ is said to be *strong* if $\mu^\infty(x,y) = \mu(x,y)$ then x, y are called *strong neighbors*. The *strong neighborhood* of the node *u* is defined as $N_s(u) = \{v \in V: (u,v) is\ a\ strong\ arc\}$.

The *strong neighborhood degree* of a vertex *u* is defined as $dN_s(u) = \sum_{v \in N_s(u)} \sigma(v)$. The *minimum strong neighborhood degree* of a fuzzy graph G is defined as $\delta_{N_s}(G) = \min\{dN_S(u) / u \in V(G)\}$ and the *maximum strong* *neighborhood degree* of a fuzzy graph G is defined as $\Delta_{N_s}(G) = \max \{ dN_S(u)/u \in V(G)\}$.

Let $G = \langle\sigma, \mu\rangle$ be a fuzzy graph and τ be any fuzzy subset of σ,(i.e) $\tau(u) \leq \sigma(u)$ for all u. Then the fuzzy subgraph of G induced strong neighborhood by τ is the maximal fuzzy subgraph of G that has fuzzy node set τ. Evidently, this is just the fuzzy graph $\langle\tau, \rho\rangle$, where $\rho(u, v) = \{(u, v)/(u, v) is\ a\ strong\ arc\}$ for all $u, v \in \tau$.

A subset D of V is called a dominating set of a fuzzy graph G if for every $v \in V - D$, there exist $u \in D$ such that u dominates v. The domination number, $\gamma_f(G)$ of a fuzzy graph G is the minimum cardinality taken over all dominating sets of fuzzy graph G.

A dominating set D is said to be an accurate dominating set of a fuzzy graph G if $V - D$ has no dominating set of same fuzzy cardinality $|D|$. The accurate domination number of a fuzzy graph G, is denoted by $\gamma_{fa}(G)$, is the minimum fuzzy cardinality taken over all accurate dominating sets of a fuzzy graph G.

3. CONNECTED ACCURATE DOMINATION IN FUZZY GRAPHS

In this section, we define connected accurate dominating set and connected accurate domination number of a fuzzy graph with suitable examples. We also discuss some results on connected accurate domination number of the fuzzy graphs.

Definition 3.1:

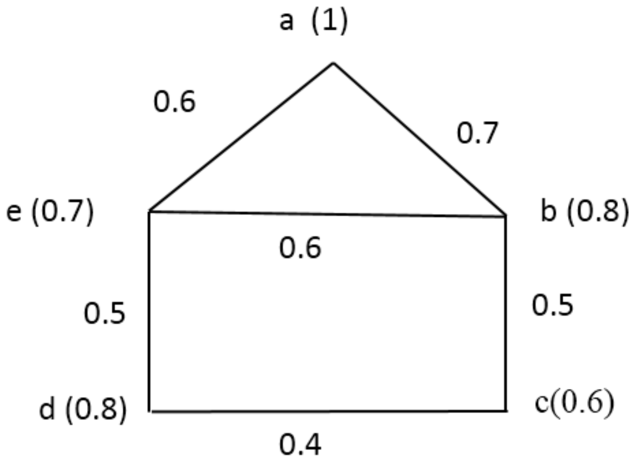
Let G be a fuzzy graph and D be a subset of V. Then, D is said to be an accurate dominating set of a fuzzy graph G, if $V - D$ has no dominating set of cardinality $|D|$. The accurate domination number of a fuzzy graph G, is denoted by $\gamma_{fa}(G)$, is the minimum cardinality taken over all accurate dominating sets of a fuzzy graph G.

Remark: An accurate dominating set of a fuzzy graph G could or could not be a minimal dominating set.

Definition 3.2:

Let G be a connected fuzzy graph and an accurate dominating set D, where D is a subset of V, of a fuzzy graph G is said to be a connected accurate dominating set if a fuzzy induced subgraph of $\langle D\rangle$ is connected. The connected accurate domination number of a fuzzy graph G is the minimum cardinality taken over all connected accurate dominating sets of a fuzzy graph G, and it is denoted by $\gamma_{fca}(G)$.

Example 3.3:



In Fig 3.1, $\{b,d\},\{b,e\},\{c,e\},\{a,c,d\},\{a,b,d\}$ are some of the dominating sets of fuzzy graph G. And $\{b,e\},\{a,c,d\},\{a,b,d\},\{a,b,e\}$ are some accurate dominating sets of G in fig 3.1.

Then, $\{b,e\},\{a,b,e\},\{b,d,e\},\{b,c,e\}$ are connected accurate dominating sets of G in fig 3.1. and its connected accurate domination number is $\gamma_{fca}(G)$.

$\gamma_{fca}(G) = |\{b,e\}| = 0.8 + 0.7 = 1.5$

$\Rightarrow \gamma_{fca}(G) = 1.5$

Theorem 3.4:

If G be a connected fuzzy graph, where $P \geq 3$ nodes, then $\gamma_f(G) \leq \gamma_{fa}(G) \leq \gamma_{fca}(G)$.

Proof:

Let G be a connected fuzzy graph, with $P \geq 3$ nodes.

A subset D of V, be a minimum dominating set of a fuzzy graph G and its domination number is $\gamma_f(G) = |D|$.

Case (i): $\gamma_f(G) \leq \gamma_{fa}(G)$

Let D be a dominating set of fuzzy graph G. Suppose that, $V - D$ has no dominating set with same cardinality $|D|$, then D will be an accurate dominating set of fuzzy graph G, (i.e.) $\gamma_f(G) = |D| = \gamma_{fa}(G)$.

We know that, every accurate dominating set of a fuzzy graph G is also a dominating set of G.

$$\therefore\ \gamma_f(G) \leq \gamma_{fa}(G) \quad \longrightarrow \quad (3.1)$$

Case (ii): $\gamma_{fa}(G) \leq \gamma_{fca}(G)$

Similarly by case (i), Let G be a connected fuzzy graph and D be an accurate dominating set of a fuzzy graph G.

If an induced subgraph $\langle D \rangle$ is connected, then D be a connected accurate dominating set of a fuzzy graph G.

$$\therefore\ \gamma_{fa}(G) \leq \gamma_{fca}(G) \quad \longrightarrow \quad (3.2)$$

From equations (3.1) & (3.2), we get, $\gamma_f(G) \leq \gamma_{fa}(G) \leq \gamma_{fca}(G)$.

Theorem 3.5:

If G be a connected fuzzy graph, with $P \geq 3$ nodes, then $\gamma_{fc}(G) \leq \gamma_{fca}(G)$.

Proof:

Let G be any connected fuzzy graph, with $P \geq 3$ nodes, and let D be a minimum connected dominating set of a fuzzy graph G, then the connected domination number of G is denoted by $\gamma_{fc}(G) = |D|$.

Suppose, $V - D$ has no connected dominating set with fuzzy cardinality $|D|$, then D is said to be a connected accurate dominating set of a fuzzy graph G.

We also know that, every connected accurate dominating set of a fuzzy graph G is also a connected dominating set of G.

Therefore, $\gamma_{fc}(G) \leq \gamma_{fca}(G)$.

Theorem 3.6:

Let G be any connected fuzzy graph, with $P \geq 5$ nodes, then, $\frac{P}{\Delta_{N_s}+1} \leq \gamma_{fca}(G) \leq P - 2$.

Proof:

Let G be a connected fuzzy graph, with $P \geq 5$ nodes. A subset D of V, be a dominating set of a fuzzy graph G, and $v \in D$, be a vertex of fuzzy graph G whose, $|N_s(v)| = P - 1 = \Delta_{N_s}(G)$.

Then, $D = \{v\}$ dominates all $(P-1)$ vertices of G and also dominated by itself.

$$\therefore\ \gamma_f(G) \geq \frac{P}{\Delta_{N_s}+1}$$

$$\geq \frac{P}{P-1+1}$$

$\gamma_f(G) \geq 1.$

By theorem 3.4, we know that, for any connected fuzzy graph G, with $P \geq 5$, $\gamma_f(G) \leq \gamma_{fa}(G) \leq \gamma_{fca}(G)$.

$$\therefore \frac{P}{\Delta_{N_S}+1} \leq \gamma_f(G)$$

$$\frac{P}{\Delta_{N_S}+1} \leq \gamma_{fca}(G).$$

Now, let us prove that, $\gamma_{fca}(G) \leq P - 2$.

Since, G is a connected fuzzy graph, then there exists a strong path between each pair of vertices of the fuzzy graph G.

Now, let v_1 and v_n be the initial and terminal vertices of a strong path P_n.

Then, the connected dominating set of P_n is $D = V - \{v_1, v_n\}$.

That is the connected domination number of a fuzzy graph G is

$$\gamma_{fca}(G) = |D|$$

$$\gamma_{fca}(G) \leq |V - \{v_1, v_n\}|$$

$$\gamma_{fca}(G) \leq P - 2.$$

Theorem 3.7:

If G be a connected fuzzy graph and S be a set of all strong arcs in G, then $\gamma_{fca}(G) \leq 2S - P$.

Proof:

Let G be a connected fuzzy graph.

Let S be a set of all strong arcs in G.

Since, G is a connected fuzzy graph.

$$|S| \geq P - 1 \quad \longrightarrow \quad (3.3)$$

By theorem 3.6,

$$\gamma_{fca}(G) \leq P - 2$$

$$\leq 2P - P - 2$$

$$\leq 2(P - 1) - P$$

$$\leq 2S - P \qquad \text{(By equation (3.3))}$$

$$\therefore \gamma_{fca}(G) \leq 2S - P.$$

Theorem 3.8:

Let G be a connected fuzzy graph and T be a spanning fuzzy tree of G, then $\gamma_{fca}(G) = P - \mathcal{E}_T$ where, $\mathcal{E}_T$ be a number of pendent vertices of a fuzzy graph G.

Proof:

Let G be a connected fuzzy graph and T be a spanning fuzzy tree of a fuzzy graph G and also T is connected.

Let $\mathcal{E}_T$ be a number of pendent vertices of a spanning fuzzy tree T.

(ie) $\mathcal{E}_T = \{v \in V \mid |N_s(v)| = 1\}$.

Since, every $u \in \mathcal{E}_T$ has a neighborhood degree atmost one, (ie) $dN_s(u) = 1$, and the dominating set D is a connected accurate dominating set of G, therefore, there exist $v \in D$ for every $u \in \mathcal{E}_T$.

Then, the connected accurate dominating set $D = V - \{v \in V \mid |N_s(v)| = 1\}$.

∴ The connected accurate domination number of a fuzzy graph G is

$$\gamma_{fca}(G) = |D|$$

$$\leq |V - \{v \in V \mid |N_s(v)| = 1\}|$$

$$\gamma_{fca}(G) \leq P - \mathcal{E}_T.$$

Corollary:

In a connected fuzzy graph G, with $P \geq 3$ nodes, let H be any connected spanning fuzzy subgraph then, $\gamma_{fca}(G) \leq \gamma_{fca}(H)$.

Proof:

Since, H is a spanning fuzzy subgraph of a connected fuzzy graph G.

It is evident that every connected accurate dominating set of a spanning fuzzy subgraph H is also a connected accurate dominating set of a connected fuzzy graph G. Hence $\gamma_{fca}(G) \leq \gamma_{fca}(H)$.

4. CONNECTED ACCURATE DOMINATION NUMBER OF SOME STANDARD CONNECTED FUZZY GRAPHS

In this section we discuss about the connected accurate dominating set and connected accurate domination number of some standard fuzzy graphs.

Observation 4.1:

Let G be a complete fuzzy graph, $G^* = K_P$, with $P \geq 3$, then $\gamma_{fca}(K_P) \leq \left\lfloor \frac{P}{2} \right\rfloor + 1$.

Proof:

Let G be a complete fuzzy graph and D, subset of V, be a connected accurate dominating set of G, therefore $V - D$ should not have any connected dominating set with same fuzzy cardinality $|D|$.

Since, D is a connected accurate dominating set of G, then D must have at most $\left\lfloor \frac{P}{2} \right\rfloor + 1$ vertices and the induced fuzzy subgraph $\langle D \rangle$ is connected.

Therefore, connected accurate domination number of G is

$$|D| \leq \left\lfloor \frac{P}{2} \right\rfloor + 1,$$

$$\gamma_{fca}(K_P) \leq \left\lfloor \frac{P}{2} \right\rfloor + 1.$$

Observation 4.2:

For any connected fuzzy graph G and $G^* = P_n$, is a path P_n, with $n \geq 3$ nodes,

$$\gamma_{fca}(P_n) \leq n - 2.$$

Proof:

Let G be a connected fuzzy graph and the underlying crisp graph G^* of a fuzzy graph G is a path with $n \geq 3$ vertices, P_n.

Since, D is a connected accurate dominating set of a fuzzy graph G and G is a path, then D must have exactly $n - 2$ vertices, i.e., $D = V - \{v_1, v_n\}$, where, v_1 and v_n are the initial and terminal vertices of a path P_n.

Therefore, connected accurate domination number of a fuzzy graph P_n is

$$\gamma_{fca}(P_n) = |D|$$

$$\leq |V - \{v_1, v_n\}| = n - 2$$

ie) $\gamma_{fca}(P_n) \leq n - 2$.

Observation 4.3:

For any connected fuzzy graph G, and $G^* = C_n$, with $n \geq 5$ vertices, $\gamma_{fca}(C_n) \leq n - 2$.

Proof:

Let G be a connected fuzzy graph and the underlying crisp graph G^* of a fuzzy graph G is a cycle C_n, with $n \geq 5$ vertices.

By observation 4.2, since, D is a connected accurate dominating set of a fuzzy graph G and G is a cycle, then D must have exactly $n-2$ vertices, i.e., $D = V - \{u, v\}$, where, u and v are the strong neighbors to each other in a fuzzy graph C_n.

Therefore, each v_i vertex will dominate two vertices v_{i-1} and v_{i+1} of a fuzzy graph C_n.

Hence, the connected accurate domination number of a fuzzy graph C_n is

$$\gamma_{fca}(C_n) = |D|$$

$$\leq |V - \{u, v\}| = n - 2$$

ie) $\gamma_{fca}(C_n) \leq n - 2$.

Observation 4.4:

For any connected fuzzy graph G and $G^* = K_{m,n}$, where $m \leq n$, $\gamma_{fca}\left(K_{m,n}\right) \leq m + 1$.

Proof:

Let G be a connected fuzzy graph and its underlying crisp graph $G^* = K_{m,n}$ where $m \leq n$.

Since, $K_{m,n}$ is a complete bipartite fuzzy graph then, the vertex set is partitioned into two sets V_1 and V_2 where, $|V_1| = m$ and $|V_2| = n$.

Since, D be a connected accurate dominating set, then D must have at most $m + 1$vertices, i.e., $D = V_1 \cup \{v\}$ where $v \in V_2$.

Therefore, the connected accurate domination number of a complete bipartite fuzzy graph is

$$\gamma_{fca}\left(K_{m,n}\right) = |D|$$

$$\leq |V_1 \cup \{v\}|$$

$$\leq m + 1.$$

Observation 4.5:

For any a connected fuzzy graph G and G^* is a star,S_n, with $n + 1$ nodes, $n \geq 2$, then $\gamma_{fca}(G) \leq 2$.

Proof:

Let G be a connected fuzzy graph and its underlying crisp graph G^* is a star,S_n, with $n+1$ nodes, $n \geq 2$.

We know that, fuzzy graph S_n is also a complete bipartite fuzzy graph $K_{1,n}$

Therefore, by observation 4.4, $\gamma_{fca}(G) = |D| \leq |\{u\} \cup \{v\}| \leq 2$.

5. CONCLUSION:

Some important results of connected accurate domination of a fuzzy graph discussed. The exact values of accurate domination number, $\gamma_{fca}(G)$, for some standard fuzzy graphs are found. Further works are to find the relation between connected accurate domination numbers with other accurate domination parameters of fuzzy graphs.

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