Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 9 (2017), pp. 5307-5315

http://www.ripublication.com

Advanced Approximation Method for Finding an Optimal Solution of Unbalanced Fuzzy Transportation Problems

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Abstract

In this paper, a new method is proposed, namely, Advanced Approximation Method (AAM) for solving an unbalanced fuzzy transportation problem without converting into a balanced one. In this proposed method the transportation costs, demand and supply of the product are represented triangular fuzzy numbers. To illustrate the proposed method, a numerical example is solved and the obtained result is compared with the results of other existing approaches. The proposed method is very easy to understand and it can be applied on real life transportation problems by the decision makers.

Keywords: AAM, Triangular fuzzy numbers, unbalanced fuzzy transportation Problem, Optimization.
Mathematics Subject Classification: 03F55, 03E72, 90B06, 90C08.

1. INTRODUCTION

Transportation problem is an important network structured in linear programming (LP) problem that arises in several contexts and has deservedly received a great deal of attention in the literature. The central concept in the problem is to find the least total transportation cost of a commodity in order to satisfy demands at destinations using available supplies at origins. Transportation problem can be used for a wide variety of situations such as scheduling, production, investment, plant location, inventory control, employment scheduling, and many others. In general, transportation problems are solved with the assumptions that the transportation costs and values of supplies and demands are specified in a precise way i.e., in crisp environment. However, in many cases the decision makers has no crisp information about the coefficients belonging to the transportation problem. In these cases, the corresponding coefficients or elements defining the problem can be formulated by means of fuzzy sets, and the fuzzy transportation problem (FTP) appears in natural way.

The basic transportation problem was originally developed by Hitchcock [9]. An Initial Basic Feasible Solution (IBFS) for the transportation problem can be obtained by using the North- West Corner rule NWCR, Matrix Minima Method MMM, Vogel's Approximation Method. We can find initial basic feasible solution by using Vogel's Approximation Method VAM [13].

Many people worked on this to propose modifications to Vogel's Approximation method for obtaining initial solutions to the unbalanced transportation problem. Shimshak [14] propose a modification (SVAM) which ignores any penalty that involves a dummy row/column. Goyal [8] suggests another modification in (GVAM) where the cost of transporting goods to or from a dummy point is set equal to the highest transportation cost in the problem, rather than to zero. The method proposed by Ramakrishnan [12] consists of four steps of reduction and one step of VAM. Nagaraj Balakrishnan [11] suggests further modification in SVAM. All methods have been established for finding the optimal solution. Among these, some methods directly attain the optimal solution namely zero suffix method [15], ASM–method [1] etc. But these two methods for finding optimal solution of a transportation problem do not reflect optimal solution proved by Mohammed [10]. In general the transportations problems are solved with the assumptions that the coefficients or cost parameters are specified precisely.

In real life, there are many diverse situations due to uncertainty in judgments, lack of evidence etc. Sometimes it is not possible to get relevant precise data for the cost parameter. This type of imprecise data is not always well represented by random variable selected from a probability distribution. Fuzzy number Zedeh [16] may represent the data. Hence fuzzy decision making method is used here. Zimmermann [17] showed that solutions obtained by fuzzy linear programming method and are always efficient. Subsequently, Zimmermann's fuzzy linear programming has developed into several fuzzy optimization methods for solving the transportation problems

Edward Samuel [6,7] showed the unbalanced fuzzy transportation problems without converting into balanced one getting an optimal solution, where the transportation cost, demand and supply are represented by triangular fuzzy number. Edward Samuel

[5] proposed algorithmic approach to unbalanced fuzzy transportation problem, where the transportation cost, demand and supply are represented by triangular fuzzy number.

In this study, the basic idea is to get an optimal solution for an unbalanced fuzzy transportation problem without converting into a balanced one. This paper presents a new method**,** namely, Advanced Approximation Method (AAM), simple and easy to understand technique for solving unbalanced fuzzy transportation problems. The algorithm of the approach is detailed with suitable numerical examples. Further comparative studies of the new technique with other existing algorithms are established by means of sample substitution.

2. PRELIMINARIES

In this section, some basic definitions, arithmetic operations and an existing method for comparing triangular fuzzy numbers are presented.

2.1. Definition [4]

A fuzzy number Ã is denoted as a triangular fuzzy number by (a_1, a_2, a_3) and its membership function $\mu_{\tilde{A}}(x)$ is given as:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & if \quad a_1 \le x \le a_2 \\ \dfrac{x-a_3}{a_2-a_3} & if \quad a_2 \le x \le a_3 \\ 0 & , \quad otherwise \end{cases}$$

2.2. Arithmetic Operations [4]. In this section, arithmetic operations between two triangular fuzzy numbers are defined on the universal set of real numbers $\Re$ are presented.

If $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers, then the following is obtained.

(i) $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

(ii) $\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

(iii) $\tilde{A} \times \tilde{B} = (a, b, c)$ where T={a_1b_1,a_1b_3,a_3b_1,a_3b_3}, a=min{T}, b=a_2b_2 and c=max{T}

2.3. Graded mean integration method [3]

The graded mean integration method is used to defuzzify the triangular fuzzy number. The representation of triangular fuzzy number is $\widetilde{A} = (a_1, a_2, a_3)$ and its defuzzified value is obtained by $R(A) = \dfrac{(a_1 + 4a_2 + a_3)}{6}$

3. PROPOSED METHOD

The adopted method of unbalanced fuzzy transportation problems provides us with an efficient way of finding the optimal solution without converting into a balanced one.

The AAM can be summarized in the following steps:

Step 1. (a) Locate the smallest element in each row of the given fuzzy cost table and then subtract that from each element of that row, and

(b) In the reduced matrix obtained from 1(a), locate the smallest element in each column and then subtract that from each element of that column.

(c) Select the smallest fuzzy cost (not equal to zero) in the reduced matrix obtained from 1(b) and then subtract it by selected smallest fuzzy cost only.

Step 2. Verify if Total Demand (TD) exceeds Total Supply (TS) then calculate row penalty else calculate column penalty.

Step 3. Calculate penalties for each row (column) by taking the difference between the smallest and next smallest unit fuzzy transportation cost in the same row (column).

Step 4. Select the row (column) with the largest penalty and allocate as much as possible
in the cell having the least cost in the selected row (column) satisfying the rim
conditions.

Step 5. Adjust the supply and demand and cross out the satisfied the row or column.

Step 6. Repeat step 3 to 5 until the entire demand at various destinations or available supply at various sources is satisfied.

Step 7. Compute total fuzzy transportation cost for the feasible allocation from the original fuzzy cost table.

4. IMPORTANT REMARKS

1. If there is a tie in the minimum fuzzy transportation cost, then select the smallest cell for allocation from the original fuzzy cost table.
2. If there is a tie in the values of penalties then calculate their corresponding the row (column) value and select the one with maximum.

5. NUMERICAL EXAMPLE

5.1. Problem 1

Table 1 gives the availability of the product supply at three sources and their demand at four destinations, and the approximate unit transportation cost, demand and supply of the product from each source to each destination is represented by triangular fuzzy number. Determine the fuzzy optimal transportation of the products such that the total transportation cost is Minimum

Table 1

	D_1	D_2	D_3	D_4	Supply (a_i)
S_1	(5,7,11)	(8,10,14)	(12,14,18)	(5,8,12)	(28,30,35)
S_2	(5,7,12)	(10,12,16)	(10,12,16)	(4,6,8)	(38,40,42)
S_3	(3,5,7)	(5,8,12)	(14,15,18)	(7,9,12)	(47,50,56)
Demand (b_j)	(19,20,21)	(19,20,21)	(23,25,30)	(29,30,31)	

Since $\sum_{i=1}^{3} a_i = (113,120,133) \neq \sum_{j=1}^{4} b_j = (90,95,103)$, so the chosen problem is a unbalanced FTP.

Iteration 1. Using step 1(a) & 1(b), we get

	D_1	D_2	D_3	D_4	Supply (a_i)
S_1	(-12,0,12)	(-12,0,12)	(-11,1,11)	(-10,1,11)	(28,30,35)
S_2	(-9,1,14)	(-7,3,15)	(-14,0,14)	(-8,0,8)	(38,40,42)
S_3	(-10,0,10)	(-11,0,12)	(-5,4,13)	(-4,4,13)	(47,50,56)
Demand (b_j)	(19,20,21)	(19,20,21)	(23,25,30)	(29,30,31)	

Iteration 2. Using step 1(c), we get

	D_1	D_2	D_3	D_4	Supply (a_i)
S_1	(-12,0,12)	(-12,0,12)	(-11,1,11)	(-10,1,11)	(28,30,35)
S_2	(-9,1,14)	(-7,3,15)	(-14,0,14)	(-8,0,8)	(38,40,42)
S_3	(-10,0,10)	(-23,0,23)	(-5,4,13)	(-4,4,13)	(47,50,56)
Demand (b_j)	(19,20,21)	(19,20,21)	(23,25,30)	(29,30,31)	

Iteration 3. Using step 2**(i.e., TS > TD)** & step 3, we get

	D_1	D_2	D_3	D_4	Supply (a_i)
S_1	(-12,0,12)	(-12,0,12)	(-11,1,11)	(-10,1,11)	(28,30,35)
S_2	(-9,1,14)	(-7,3,15)	(-14,0,14)	(-8,0,8)	(38,40,42)
S_3	(-10,0,10)	(-23,0,23)	(-5,4,13)	(-4,4,13)	(47,50,56)
Demand (b_j)	(19,20,21)	(19,20,21)	(23,25,30)	(29,30,31)	
Column penalty	**(-22,0,22)**	**(-35,0,35)**	**(-25,1,25)**	**(-18,1,19)**	

Iteration 4. Using step 4 to step 5, we get

	D_1	D_2	D_3	D_4	Supply (a_i)
S_1	(-12,0,12)	(-12,0,12)	(-11,1,11)	*	(28,30,35)
S_2	(-9,1,14)	(-7,3,15)	(-14,0,14)	(-8,0,8) **(29,30,31)**	(7,10,13)
S_3	(-10,0,10)	(-23,0,23)	(-5,4,13)	*	(47,50,56)
Demand (b_j)	(19,20,21)	(19,20,21)	(23,25,30)	*	
Column penalty	(-22,0,22)	(-35,0,35)	(-25,1,25)	**(-18,1,19)**	

Iteration 5. Using step 6, we get.

	D_1	D_2	D_3	D_4	Supply (a_i)
S_1	(-12,0,12)	(-12,0,12)	(-11,1,11) **(10,15,23)**	(-10,1,11)	(28,30,35)
S_2	(-9,1,14)	(-7,3,15)	(-14,0,14) **(7,10,13)**	(-8,0,8) **(29,30,31)**	(38,40,42)
S_3	(-10,0,10) **(19,20,21)**	(-23,0,23) **(19,20,21)**	(-5,4,13)	(-4,4,13)	(47,50,56)
Demand (b_j)	(19,20,21)	(19,20,21)	(23,25,30)	(29,30,31)	

Using step 7, we get.

	D_1	D_2	D_3	D_4	Supply (a_i)
S_1	(5,7,11)	(8,10,14)	(12,14,18) **(10,15,23)**	(5,8,12)	(28,30,35)
S_2	(5,7,12)	(10,12,16)	(10,12,16) **(7,10,13)**	(4,6,8) **(29,30,31)**	(38,40,42)
S_3	(3,5,7) **(19,20,21)**	(5,8,12) **(19,20,21)**	(14,15,18)	(7,9,12)	(47,50,56)
Demand (b_j)	(19,20,21)	(19,20,21)	(23,25,30)	(29,30,31)	

The minimum fuzzy transportation cost is
= **(10,15,23)**(12,14,18) + **(7,10,13)**(10,12,16) + **(29,30,31)** (4,6,8) + **(19,20,21)** (3,5,7) + **(19,20,21)**(5,8,12)

= **(458, 770, 1269)**

R (A) = **801.17**

5.2. Problem 2

	D_1	D_2	D_3	Supply (a_i)
S_1	(1,2,6)	(6,7,11)	(12,14,19)	(3,5,10)
S_2	(2,3,5)	(2,3,5)	(0,1,3)	(7,8,12)
S_3	(3,5,8)	(3,4,7)	(6,7,11)	(6,7,11)
S_4	(0,1,3)	(4,6,9)	(1,2,6)	(13,15,20)
Demand (b_j)	(6,7,11)	(7,9,14)	(16,18,23)	

To illustrate the comparatiave efficiency, the solutions obtained by unbalanced transportation problems converting into balanced one **VAM [13], SVAM [14], GVAM[8], BVAM[11], RVAM[12], ASM[1], ZSM[15], MODI[2]** and the proposed method with fifteen randomly generated unbalanced fuzzy transportation problems are shown in the following table. A complete set of results of these fifteen problems are not shown here due to space consideration, and can be made available from the author.

SOLUTIONS OBTAINDED BY ALL PROCEDURES

S.NO	ROW	COLUMN	VAM	SVAM	GVAM	BVAM	RVAM	ASM	ZSM	MODI	AAM
1.	3	4	821.50	**801.17**	**801.17**	855.53	821.17	**801.17**	856.67	**808.17**	**801.17**
2.	4	3	117.50	129.67	119.13	118.67	118.67	125.00	125.83	**116.50**	**113.17**
3.	3	5	1192.50	**971.33**	985.67	1192.33	1058.17	**971.33**	**971.33**	**1011.17**	**971.33**
4.	**4**	**3**	93.00	106.67	88.67	88.33	102.33	102.33	88.33	**87.00**	**85.00**
5.	**3**	**3**	**2450.00**	**2450.00**	2779.33	2471.33	**2450.00**	2471.33	**2450.00**	**2450.00**	**2450.00**
6.	**3**	**2**	2616.67	**2500.00**	**2500.00**	3150.00	**2500.00**	**2500.00**	**2500.00**	**2500.00**	**2500.00**
7.	**3**	**3**	1785.00	1760.00	1760.00	**1685.00**	**1678.33**	**1678.33**	**1678.33**	**1711.67**	**1678.33**
8.	**3**	**5**	**9400.00**	**9400.00**	**9400.00**	10066.67	**9400.00**	**9400.00**	**9400.00**	**9400.00**	**9400.00**
9.	**3**	**3**	**543.33**	**543.33**	**543.33**	**543.33**	**543.33**	**543.33**	**543.33**	**543.33**	**543.33**
10.	**3**	**3**	154.33	**153.00**	**153.00**	**153.00**	**153.00**	154.33	**153.00**	**153.00**	**153.00**
11.	**2**	**4**	62.00	59.33	**52.67**	62.00	**52.67**	59.33	59.33	**55.33**	**52.67**
12.	**3**	**5**	1193.33	**980.00**	986.87	1220.00	1033.33	**980.00**	**980.00**	**1020.33**	**980.00**
13.	**3**	**2**	**501.33**	**501.33**	**501.33**	745.33	**501.33**	**501.33**	**501.33**	**501.33**	**501.33**
14.	**3**	**4**	825.00	**801.67**	**801.67**	853.33	931.67	**801.67**	811.67	**801.67**	**801.67**
15.	**3**	**2**	**95.33**	**95.33**	**95.33**	119.00	**95.33**	**95.33**	**95.33**	**95.33**	**95.33**

6. RESULTS AND DISCUSSION

From the investigations and the results given above**,** it is clear that the **AAM** is better than **VAM [13], SVAM [14], GVAM [8], BVAM [11], RVAM [12], ASM [1] and ZSM [15],** for solving unbalanced fuzzy transportation problems and it has its merit that it produces an optimal solution, without converting into a balanced one.

7. CONCLUSION

In this paper, a new method is proposed, namely, AAM is solving unbalanced transportation problem without converting into a balanced one. The AAM is better than the other existing methods. The algorithm of the approach is detailed with suitable numerical examples. Further comparative studies of the new technique with other existing fuzzy transportation algorithms are established by means of sample problems. By the proposed method an unbalanced fuzzy transportation problem can be solved without converting into a balanced one, which gives an optimal solution and can be applied for solving transportation problems occurring in real life situations.

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