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On Solving a Fabrication Lot-Size and Shipping Frequency Problem with an Outsourcing Policy and Random Scrap

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Abstract

A fabrication lot-size and shipping frequency problem with outsourcing and random scrap was explored by Chiu et al. [1] using mathematical modeling together with differential calculus to decide the best operating policy that minimizes total system cost and relieves in-house capacity loading. This study uses an alternative approach to resolve the problem without the needs of applying neither first- nor second-derivative to system cost function. The objective is to help production planner and controller, who may have insufficient background of calculus, better comprehend and administer such a specific fabrication-shipment system.

Keywords: Operations research, fabrication-shipment problem, outsourcing, random scrap, multiple shipments

INTRODUCTION

The economic production quantity (EPQ) model [2] was proposed to decide best production quantity in a cycle that minimizes total production and inventory holding costs. In contrast to the simple assumptions of EPQ that considers a perfect fabrication process, continuous inventory issuing policy, and pure in-house production, in real manufacturing settings and supply chain environment, imperfection nature of production processes [3-12], discontinuous periodic or multi-shipment end products distribution policies [13-22], and diverse outsourcing plans [23-31] are the routine tasks to managers in the fields of production planning and control.

A partial outsourcing strategy can help production managers resolve capacity constraint and smooth fabrication scheduling. With the purpose of offering information to support production decision making, Chiu et al. [1] investigated a fabrication-shipment problem with outsourcing and random scrap, using mathematical modeling along with differential calculus to determine the best operating policy that minimizes total system cost and relieves in-house capacity loading.

An alternative method which did not use derivatives, was proposed by Grubbstrom and Erdem [32] to determine the economic order quantity (EOQ) problem with backlogging. The same or similar approaches have been employed to solve a vendor-buyer integrated system with rework and a specific multi-delivery policy [33] and to deal with a multi-item EPQ model with scrap, rework, and multi-delivery [34]. This study extends such a simplified method to resolve the problem in [1] and demonstrates that the optimal fabrication-shipment policy can be derived without the needs of applying neither first- nor second-derivative to system cost function in the solution procedure.

PROBLEM DESCRIPTION AND THE PROPOSED APPROACH

This study reexamines a fabrication lot-size and shipping frequency problem with outsourcing and random scrap [1]. Description of the problem is as follows: consider annual demand and fabrication rates of a specific product are λ and P units, respectively, and the fabrication process may randomly produce x percentage of scrap. No shortages are allowed, so P must be greater than the sum of λ and the production rate of scrap d (where $d = Px$). In each replenishment cycle there is a π portion (where $0 < \pi < 1$) of batch size Q that is outsourced, and the other $(1 - \pi)$ portion of Q is fabricated in-house. Outsourcing items are requested to be of perfect quality and received in the end of in-house uptime (see Figure 1). Fixed quantity n installments of the entire batch are distributed to buyer, at a fixed interval of time during delivery time $t_{2\pi}$. in the proposed model (in green) as compared to that in a model without outsourcing (in black) [1]

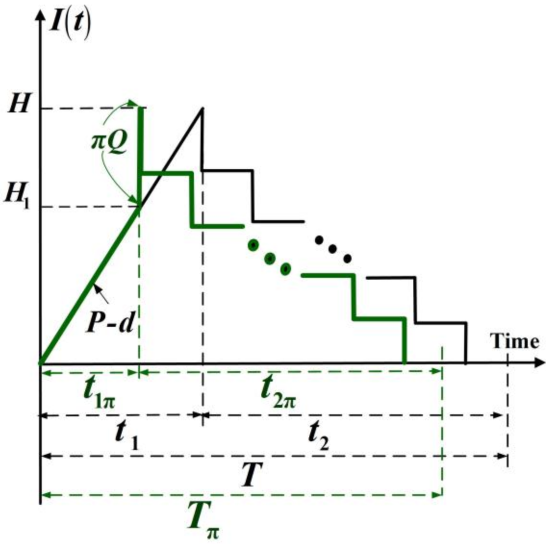

Figure 1: The on-hand inventory level of perfect quality items

To relieve comparison efforts for readers, the same notation as in [1] is used in this study (see Appendix A). Total fixed and variable costs for outsourcing products in a replenishment cycle are

$$K_\pi + C_\pi(\pi Q) \tag{1}$$

Total in-house fabrication costs including setup cost, variable manufacturing cost, disposal cost for scrap, and holding cost for both finished and scrap products, are as follows:

$$K + C\left[(1-\pi)Q\right] + C_S x\left[(1-\pi)Q\right] + h\left[\frac{H_1 + dt_{1\pi}}{2}(t_{1\pi}) + \left(\frac{n-1}{2n}\right)Ht_{2\pi}\right] \tag{2}$$

Total costs of fixed and variable transportation costs for finished lot and stock holding cost at buyer's side are as follows [1]:

$$nK_1 + C_T H + \frac{h_2}{2}\left[\frac{H}{n}t_{2\pi} + T_\pi\left(H - \lambda t_{2\pi}\right)\right] \tag{3}$$

Hence, total production-inventory-delivery costs per cycle, $TC(Q, n)$ is

$$\begin{aligned} TC(Q,\ n) = {} & K_\pi + C_\pi(\pi Q) + K + C\left[(1-\pi)Q\right] + C_S x\left[(1-\pi)Q\right] + nK_1 + C_T H \\ & + h\left[\frac{H_1 + dt_{1\pi}}{2}(t_{1\pi}) + \left(\frac{n-1}{2n}\right)Ht_{2\pi}\right] + \frac{h_2}{2}\left[\frac{H}{n}t_{2\pi} + T_\pi\left(H - \lambda t_{2\pi}\right)\right] \end{aligned} \tag{4}$$

Substituting K_π and C_π in Eq. (4) and using the expected values of x in the cost analysis to cope with randomness of scrap rate and with extra derivations, the long-run average system costs per unit time $E[TCU(Q, n)]$ becomes as follows [1]:

$$\begin{aligned} E\left[TCU(Q,\ n)\right] = \frac{E\left[TC(Q,\ n)\right]}{E[T]} = {} & \frac{\lambda\left[(2+\beta_1)K + nK_1\right]}{\left(1-E[x](1-\pi)\right)Q} + \frac{(1+\beta_2)\pi C\lambda}{\left(1-E[x](1-\pi)\right)} + \frac{(1-\pi)\lambda\left[C + C_S E[x]\right]}{\left(1-E[x](1-\pi)\right)} \\ & + C_T\lambda + \frac{h(1-\pi)\lambda Q}{2\left(1-E[x](1-\pi)\right)}\left\{\frac{E[x](1-\pi)-\pi}{P}\right\} + \frac{hQ\left[1-E[x](1-\pi)\right]}{2} + \frac{h_2\lambda Q}{2}\left[\frac{1-\pi}{P}\right] \\ & + \frac{\lambda Q(h_2-h)}{2}\left(\frac{1}{n}\right)\left[\frac{1-E[x](1-\pi)}{\lambda} - \frac{1-\pi}{P}\right] \end{aligned} \tag{5}$$

The Proposed Algebraic Approach

Phase 1: deriving number of shipments

It can be seen that the decision variables Q and n in E[$TCU(Q, n)$] exhibits four different forms, namely, Q, Q^{-1}, nQ^{-1}, and $n^{-1}Q$. Suppose we let z_1, z_2, z_3, z_4, and z_5 represent the following:

$$z_1 = \frac{(1+\beta_2)\pi C\lambda}{(1-E[x](1-\pi))} + \frac{(1-\pi)\lambda[C+C_S E[x]]}{(1-E[x](1-\pi))} + C_T\lambda \tag{6}$$

$$z_2 = \frac{\lambda(2+\beta_1)K}{1-E[x](1-\pi)} \tag{7}$$

$$z_3 = \frac{\lambda K_1}{1-E[x](1-\pi)} \tag{8}$$

$$z_4 = \frac{h(1-\pi)\lambda}{2(1-E[x](1-\pi))}\left\{\frac{E[x](1-\pi)-\pi}{P}\right\} + \frac{h[1-E[x](1-\pi)]}{2} + \frac{h_2\lambda}{2}\left[\frac{1-\pi}{P}\right] \tag{9}$$

$$z_5 = \frac{\lambda(h_2-h)}{2}\left[\frac{1-E[x](1-\pi)}{\lambda} - \frac{1-\pi}{P}\right] \tag{10}$$

Therefore, $E[TCU(Q, n)]$ can be rearranged as follows:

$$E[TCU(Q,\ n)] = z_1 + z_2Q^{-1} + z_3Q^{-1}n + z_4Q + z_5Qn^{-1} \tag{11}$$

or

$$E[TCU(Q,\ n)] = z_1 + \left(\sqrt{z_2} - \sqrt{z_4}Q\right)^2 Q^{-1} + \left(\sqrt{z_3} - \sqrt{z_5}Qn^{-1}\right)^2 \left(Q^{-1}n\right) + 2\sqrt{z_2z_4} + 2\sqrt{z_3z_5} \tag{12}$$

It can be seen that if the second and third terms on the right-hand side of Eq. (12) equal to zeros then $E[TCU(Q, n)]$ is minimized. That is

$$Q = \sqrt{\frac{z_2}{z_4}} \quad \text{and} \quad n = Q\sqrt{\frac{z_5}{z_3}} = \sqrt{\frac{z_2z_5}{z_4z_3}} \tag{13}$$

Substitute z_2, z_3, z_4, and z_5 in equation (13), n can be obtained as follows:

$$n = \sqrt{\frac{(2+\beta_1)K(h_2-h)[1-E[x](1-\pi)]\left\{[1-E[x](1-\pi)] - \frac{\lambda(1-\pi)}{P}\right\}}{K_1\left\{+h\left\{\frac{\lambda(1-\pi)}{P}[E[x](1-\pi)-\pi] + [1-E[x](1-\pi)]^2\right\} + \frac{h_2\lambda(1-\pi)[1-E[x](1-\pi)]}{P}\right\}}} \tag{14}$$

Since the number of shipment n can only be integer, let n^+ be the smallest integer greater than or equal to n and n^- denote the largest integer less than or equal to n. Hence, optimal n^* is either n^+ or n^-.

Phase 2: deriving lot-size

Upon obtaining n, the long-run average system costs per unit time $E[TCU(Q, n)]$ can now be considered as a cost function with one unknown variable Q, as follows:

$$E[TCU(Q,\ n)] = z_1 + (z_2 + z_3n)Q^{-1} + (z_4 + z_5n^{-1})Q \tag{15}$$

or

$$E\left[TCU\left(Q,\ n\right)\right]=z_1+\left(\sqrt{z_2+z_3n}-Q\sqrt{z_4+z_5n^{-1}}\right)^2Q^{-1}+2\sqrt{z_2+z_3n}\sqrt{z_4+z_5n^{-1}} \qquad (16)$$

It can be seen that if the second term on the right-hand side of Eq. (16) equals to zero, then $E[TCU(Q, n)]$ is minimized. That is

$$Q^*=\sqrt{\frac{z_2+z_3n}{z_4+z_5n^{-1}}} \qquad (17)$$

Substitute z_2, z_3, z_4, and z_5 in equation (17), Q^* can be derived as follows:

$$Q^*=\sqrt{\frac{2\left[\left(2+\beta_1\right)K+nK_1\right]\lambda}{\left\{\begin{array}{l}+h\left\{\left(1-\pi\right)\lambda\left[\dfrac{E\left[x\right]\left(1-\pi\right)-\pi}{P}\right]+\left[1-E\left[x\right]\left(1-\pi\right)\right]^2\right\}+\dfrac{h_2\lambda\left(1-\pi\right)\left[1-E\left[x\right]\left(1-\pi\right)\right]}{P}\\+\dfrac{\left(h_2-h\right)\left[1-E\left[x\right]\left(1-\pi\right)\right]}{n}\left\{\left[1-E\left[x\right]\left(1-\pi\right)\right]-\dfrac{\lambda\left(1-\pi\right)}{P}\right\}\end{array}\right\}}} \qquad (18)$$

It is noted that Eq. (18) is identical to what was derived in [1]. Applying n^+ or n^- to Eqs. (18) and then to Eq. (11), both optimal (Q^*, n^*) policy and minimum expected system cost $E[TCU(Q, n)]$ for this fabrication-shipment problem with outsourcing and random scrap can be determined.

NUMERICAL EXAMPLE

To relieve comparison efforts for readers and also be able to demonstrate the obtained results from the proposed approach, the same numerical example as in [1] is used in this section. The values of parameters include: λ = 4000, P = 20,000, K = \$5,000, C = \$100, π = 0.4, K_π = \$30 (or β_1 = −0.7), C_π = \$130 (or β_2 = 0.3), x follows the uniform distribution over the range of [0, 0.2], h = \$30, C_S = \$20, K_1 = \$800, and C_T = \$0.5.

Applying equations (14), (18) and (11), the optimal replenishment lot-size and shipment policy is found to be (Q^* = 1,229, n^* = 3), and the long-run average system cost per unit time $E[TCU(Q^*, n^*)]$ = \$545,344. The behavior of the long-run average system cost per unit time with respect to the different lot-size Q is depicted in Figure 2.

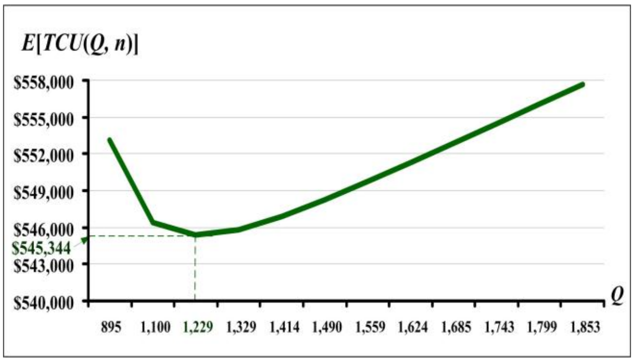

Figure 2 Behavior of E[$TCU(Q, n)$] with respect to lot-size Q

CONCLUSIONS

Chiu [1] used mathematical modeling together with differential calculus to determine the best operating policy for a fabrication-shipment problem with outsourcing and random scrap. This study proposes an alternative two-phase algebraic method to resolve the problem without the needs of applying derivatives in the solution procedure. The objective of this study is to help production planner and controller, who may have insufficient background of calculus, better comprehend and administer such a specific fabrication-shipment system.

APPENDIX

K	=	in-house fabrication setup cost,
C	=	unit fabrication cost,
H	=	unit holding cost,
C_S	=	unit disposal cost,
K_π	=	fixed outsourcing cost per cycle,
C_π	=	unit outsourcing cost,
K_1	=	fixed transportation cost per shipment,
C_T	=	unit transportation cost,
h_2	=	unit stock holding cost per unit time at the buyer's side,
T_π	=	cycle time,
n	=	number of fixed quantity installments in a replenishment cycle,
t_n	=	the fixed interval of time between

		two consecutive shipments in $t_{2\pi}$,
β_1	=	relating parameter between K_π and K (where $K_\pi = (1 + \beta_1)K$ and $\beta_1 \leq -1$),
β_2	=	relating parameter between C_π and C (where $C_\pi = (1 + \beta_2)C$ and $\beta_2 > 0$),
H_1	=	inventory level at the time in-house uptime ends,
H	=	maximum inventory level upon receipt of outsourcing products,
$t_{1\pi}$	=	fabrication uptime,
$t_{2\pi}$	=	delivery time,
t_1	=	uptime of the same model without outsourcing option,
t_2	=	delivery time of the same model without outsourcing option,
$I(t)$	=	on-hand inventory of perfect quality items at time t,
$TC(Q, n)$	=.	total production-inventory-delivery cost per cycle,
$E[TCU(Q, n)]$	=.	the long-run average costs per unit time for the proposed system

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