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Effective Solution of Large-Scale Optimization Problems Using the Similarity Criteria

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Abstract

While solving of large-scale optimization problems, it is often necessary to obtain the result with incomplete or unreliable information about the research object. To solve such kind of problems it is suggested to use criterial programming method based on the similarity theory techniques, criterial analysis and geometrical programming. The main advantages of the criterial programming are the criteria similarity usage. They allow us to generalize the information of the object under study in the form of criterial mathematic models, find optimal solutions, to extend them for all investigated similar object or phenomena.

Keywords: Similarity criteria, optimization, generalized variables

CRITERIA ANALYSIS

Modern optimization problems in different science and engineering branches require new demands to optimization methods. To solve optimization problems at present a large arsenal of mathematical methods is used (linear, nonlinear, dynamic, statistical and etc.), neural networks, theory of games, theory of fuzzy sets, genetic algorithms and others.

One should underline especially the method of optimization problems solution by means of generalized methods of similarity theory that is called criterion analysis [1].

The advantage of this technique is that it allows to solve some optimization problems with limited initial information. While using criteria analysis method mathematical models of studied object elements and systems are given in the form of generalized canonical posinomes of zero degree difficulty $S = m - n - 1 = 0$ [1]:

$$Z = \sum_{i=1}^{m} A_i \prod_{j=1}^{n} X_j^{\alpha_{ij}}, \qquad (1)$$

where: Z – aimed function, A_i - generalized constants $A_i > 0$, X_j - optimized parameters, α_{ij} - the signs of optimized parameters degree, m – the numbers of added posinomes, n – the number of optimized parameters.

The method of criterial analysis received a wide spread while solving technical-economic problems in power engineering in papers of students and followers of prof. Venikov's scientific school, e.g. while choosing the cross-sections wires in the transmission lines, optimization transformer power due to economic conditions, the choice of operating voltage value in the transmission line and phase design.

Having undeniable advantages criterion analysis technique has same disadvantages that limited their usage range:

- the aimed optimization function must be represented in the form of posinomes;
- it is necessary to fulfil canonic terms;
- it is impossible to solve problems of zero degree difficulty;
- it is difficult to take info account discrete and functional limitations in the form of equalities and inequalities.

Nowadays optimized problems have a great number of optimized parameters, as a result of it the difficulty extent of such problems $S > 0$ and their solution with the help criterion analysis is impossible.

Some of these disadvantages can be eliminated if you get the aimed function with necessary properties. For this purpose different methods of approximation are used while making mathematical models in the form of canonical posinomes.

GEOMETRIC PROGRAMMING

American scientists R. Daffin, E. Peterson, K. Zener, G. Reileitis, A. Reivindran and K. Ragadel developed similar optimization technique – geometric programming [2].

This technique allows to take into account limitations like as equality and inequality. The application of generalizesed geometric programming allows to analyse the models, given in the form of mixinoms, their items can be both negative and positive with the use of so-called σ - functions, taking into account the signs of polynomial items.

The application of geometric programming due to dual variables theory allows to solve the problems of the first difficulty degree, that relaxes requirements for the aimed function and extends the range of solved problems. However, the present method requires a great number of calculating operations and extra change of variables while reducing optimization problem complexity only for unity.

Besides, geometric programming doesn't apply the merits of criterion analysis in the form of similarity criteria.

CRITERION PROGRAMMING ALGORYTHM

The authors are supposed to use the criterion programming method for solution of optimizative problems [3, 4], which is the synthesis of the above-mentioned approaches for optimization. Taking into consideration their merits and leveling demerits.

The new method similarity with criterion analysis consists of application of engineering economic similarity criteria and with geometric programming – in optimization problems solution not only with posinomes but also with negonomes and mixinomes and also the opportunity to take into account the limitations like equality and inequality, imposed on optimized parameters.

Let's consider the suggested algorithm of optimized problems solution in the form of consecutive steps.

Step 1: Optimization problem is formulated where the aimed function is given in the form of arbitrary polynomial and the signs of items are taken into account with the help of σ - (sigma) functions (where $\sigma_i = \pm 1$):

$$Z(x) = \sum_{i=1}^{m} \sigma_i A_i \prod_{j=1}^{n} X_j^{\alpha_{ij}} . \qquad (2)$$

Limitations (if there are) are taken into account:

Type of equality:

$$\sum_{\substack{r=1 \div R \\ i=(m+1)\div I_R}} \sigma_i A_i \prod_{j=1}^{n} X_j^{\alpha_{i,r,j}} = 0, \qquad (3)$$

Type of inequality:

$$\sum_{\substack{s=1 \div S \\ i=(m+I_R+1)\div I_s}} \sigma_i A_i \prod_{j=1}^{n} X_j^{\alpha_{i,s,j}} \geq 0. \qquad (4)$$

where R – limitations quantity of equality type, S - limitations quantity of inequality type.

Step 2: Lagrange function is formed, including the aimed function and limitations of equality type which are taken info account by means of Lagrange indefinite factors λ_r:

$$L(x) = \sum_{i=1}^{m} \sigma_i A_i \prod_{j=1}^{n} X_j^{\alpha_{ij}} + \sum_{r=1}^{R} \lambda_r \sum_{\substack{r=1 \div R \\ i=(m+1)\div I_r}} \sigma_i A_i \prod_{j=1}^{n} X_j^{\alpha_{i,r,j}} . \qquad (5)$$

Application of Lagrange method allows take info account limitations of equality type and increases dimensions of the solved problem.

The limitations of inequality type can be taken into account by the method of penalty function.

Step 3: The matrix of Lagrange function dimensions (5) $\boldsymbol{\alpha}$, is set up, where the line number is equal to the function items and the number of colums per a unity is more than the number of optimized parameters:

$$\boldsymbol{\alpha} = \begin{array}{|c|c|c|c|c|} \hline \alpha_{1,1} & \alpha_{1,2} & \dots & \alpha_{1,n} & \sigma_1 \\ \hline \alpha_{2,1} & \alpha_{2,2} & \dots & \alpha_{2,n} & \sigma_2 \\ \hline \dots & \dots & \dots & \dots & \dots \\ \hline \alpha_{m,1} & \alpha_{m,2} & \dots & \alpha_{m,n} & \sigma_m \\ \hline \hline \dots & \dots & \dots & \dots & \dots \\ \hline \alpha_{M,1} & \alpha_{M,2} & \dots & \alpha_{M,n} & \sigma_M \\ \hline \end{array} \begin{array}{l} \boldsymbol{\alpha}_1 \\ \\ \boldsymbol{\alpha}_2 \end{array}$$

Step 4: Unspecial matrix $\boldsymbol{\alpha}_1$, which rank is equal to $n+1$, is chosen from matrix $\boldsymbol{\alpha}$. If it necessary matrix lines permutation and corresponding regrouping of Lagrange function are used.

Step 5: Auxilary matrix is calculated, its column number for a unity is more than number of optimized parameters $N = n+1$, and the lines number is a unity less than the difference of the item number of Lagrange function and the number of optimized parameters number $K = M - (N+1)$:

$$\mathbf{d} = \boldsymbol{\alpha}_2 \boldsymbol{\alpha}_1^{-1} = \begin{array}{|c|c|c|c|} \hline d_{1,1} & d_{1,2} & \cdots & d_{1,K} \\ \hline d_{2,1} & d_{2,2} & \cdots & d_{2,K} \\ \hline \dots & \dots & \dots & \dots \\ \hline d_{K,1} & d_{K,2} & \cdots & d_{K,K} \\ \hline \end{array}$$

Step 6: Equation system is set up and solve for solution of similarity criteria. This system consist of linear part, that is set up on the basis of transposed dimensions matrix, taking into account σ - function:

$$\underbrace{\begin{array}{|c|c|c|c|}\hline \alpha_{1,1} & \alpha_{2,1} & \dots & \alpha_{m,1} \\ \hline \alpha_{1,2} & \alpha_{2,2} & \dots & \alpha_{m,2} \\ \hline \dots & \dots & \dots & \dots \\ \hline \alpha_{1,n} & \alpha_{2,n} & \dots & \alpha_{m,n} \\ \hline \sigma_1 & \sigma_2 & \dots & \sigma_m \\ \hline \end{array}}_{\boldsymbol{\alpha}_{1t}} \underbrace{\begin{array}{|c|c|}\hline \dots & \alpha_{M,1} \\ \hline \dots & \alpha_{M,2} \\ \hline \dots & \dots \\ \hline \dots & \alpha_{M,n} \\ \hline \dots & \sigma_M \\ \hline \end{array}}_{\boldsymbol{\alpha}_{2t}} \cdot \begin{array}{|c|}\hline \pi_1 \\ \hline \dots \\ \hline \pi_m \\ \hline \hline \dots \\ \hline \pi_M \\ \hline \end{array} \begin{array}{l} \boldsymbol{\pi}_1 \\ \\ \boldsymbol{\pi}_2 \end{array} = \begin{array}{|c|}\hline 0 \\ \hline 0 \\ \hline 0 \\ \hline \dots \\ \hline -1 \\ \hline \end{array} \ \boldsymbol{\beta}$$

Matrix equation is given as :

$$\boldsymbol{\pi}_1 = \boldsymbol{\alpha}_{1t}^{-1}\left(\boldsymbol{\beta} - \boldsymbol{\alpha}_2\boldsymbol{\pi}_2\right) \qquad (6)$$

The equation (6) shows the dependence of the main similarity criteria $\boldsymbol{\pi}_1$ on additional similarity criteria $\boldsymbol{\pi}_2$.

The second part of linear equation system is set up as the result of variable logarithming and shows the dependence of additional criteria on the main ones:

$$\begin{bmatrix} \ln\frac{\pi_{n+2}}{A_{n+2}} \\ \ldots \\ \ln\frac{\pi_K}{A_K} \end{bmatrix} = \begin{bmatrix} d_{1,1} & \ldots & d_{1,K} \\ \ldots & \ldots & \ldots \\ d_{K,1} & \ldots & d_{K,K} \end{bmatrix} \cdot \begin{bmatrix} \ln\frac{\pi_1}{A_1} \\ \ldots \\ \ln\frac{\pi_{n+1}}{A_{n+1}} \end{bmatrix}$$

Step 7: Equation systems for calculating of optimal parameter values and corresponding value of aimed function:

$$\begin{bmatrix} \ln x_1 \\ \ldots \\ \ln x_n \\ \ln 3 \end{bmatrix} = \begin{bmatrix} \alpha_{1,1} & \ldots & \alpha_{n,1} & \alpha_{m,1} \\ \ldots & \ldots & \ldots & \ldots \\ \alpha_{1,n} & \ldots & \alpha_{n,n} & \alpha_{m,n} \\ \alpha_{1,m} & \ldots & \alpha_{n,m} & \alpha_{m,m} \end{bmatrix} \cdot \begin{bmatrix} \ln\frac{\pi_1}{A_1} \\ \ldots \\ \ln\frac{\pi_n}{A_n} \\ \ln\frac{\pi_m}{A_m} \end{bmatrix}$$

Step 8: Carrying out of limitations of inequality type is checked. These limitation can be taken into account: by active set method or penalty functions method.

Step 9: Aimed function in the criteria form is written:

$$Z_* = \sum_{i=1}^{m} \pi_i \prod_{j=1}^{n} X_{*j}^{\alpha_{ij}} \qquad (7)$$

where Z_* - relative meaning of the objective function, X_{*j} - the relative meaning of the optimized parameters, π_i - criteria of engineering-economic similarity.

Step 10: Aimed function stability for deviation of parameters from optimal meaning, using equation (7) is studied.

Similarity criteria are divided into 2 groups: main and additional.

The main similarity criteria correspond to the canonic part of Lagrange function (5) and are independent on generalized constants and their signs, but additional similarity criteria are dependent of generalized constant.

The division of criteria into main and additional depend in the way of aimed function creation and optimized parameters.

It is suggested to include parameters which information isn't full or isn't well-founded.

In (4) an opportunity is shown to use similarity criteria by definite method of criteria programming as generalized variables while solving problems by iterative methods. So, in Newton's method, the procedure of getting the 1st and the 2nd derivative at each iteration can be changed by multiplication operation info corresponding similarity criteria that speeds up and simplifies iterative procedure.

CONCLUSION

The proposed technique of optimization problems solution has the following merits:

- criteria similarity usage allows to study optimized function without detailed information about the studied object;
- it is possible to solve problems, which difficulty degree is more than unity;
- optimization aimed function can be represented in arbitrary noncanonic form;
- limitation of different kinds are taken info account;
- the process of optimization problems solution is speeded up and simplified by iterative methods.

The methods showed the effectiveness while solving different optimized problems in the field of power engineering and can be used in other branches.

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