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The Pseudo Adaptive Algorithm of Control over a Dynamic Plant with Limited Uncertainty

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Abstract

A pseudo adaptive algorithm has been developed, which is similar to an nonlinear one, to control a dynamic plant with interval parametric uncertainty. The algorithm ensures the controlled dissipative stability of control processes with ultimate convergence speed and robustness in terms of unmodeled perturbations. A simulation research with MatLab/Simulink software of two-link rigid manipulator dynamics has revealed constantly stable aperiodic dynamics under significant parametric deviations of the dynamic plant. Simulation results show high degree of suppression of manipulator drives' cross-coupling effect. Design, implementation and adjustment of these algorithms does not require significant effort for putting them into operation.

Keywords: Lyapunov method, nonlinear control systems, manipulators, uncertainty, robustness

INTRODUCTION

There are multiple ways to control dynamic plants with uncertainty; and adaptive control has been one of the most popular methods. The Lyapunov functions method and gradient procedures are often used in building adaptive algorithms [1– 6].

Multiple algorithms of this kind can be described as differential equations of type:

$$\dot{k} = \Lambda \Phi(\mathrm{e}, \mathrm{x}),$$

k being matrix/vector of adjustable parameters; right-hand side of an equation is determined by way of synthesized continuous function $\Phi(\mathrm{e}, \mathrm{x})$, Λ being adjustment speed. Apparently, these algorithms when being implemented contain integrators for updating adjustable parameters, what makes them essentially inertial as well as non-robust towards unmodeled perturbations, this may lead to a growing parameter drift [3].

To exclude these phenomena and achieve a robust convergence, algorithms are modified by way of feedback on the adjusted parameter (regularization) in order to receive algorithms of the form [3, 4]:

$$\dot{k} = \Lambda \Phi(\mathrm{e}, \mathrm{x}) - \alpha k,$$

with exponential convergence of adaptive processes or dissipativity [5] (α being parametric feedback depth). However, such a robust method of an algorithm is associated with constantly active transient responses in an adaptive system, i.e. error vector $e(t)$ should not equal zero. Parameter α and adjustment speed Λ should be kept growing until this condition's influence reduces, what is limited by stability for a high-order plant [3].

This work shows synthesis of almost the same algorithm, however, in a static form, i.e.

$$k = \Phi(\mathrm{e})$$

Lyapunov functions method was used to build it as well. Convergence of the processes is practically immediate, it is of exponential nature and is robust in terms of considered perturbations. As for its structure, it looks like a nonlinear law with parametric feedback, however it remains functioning, dynamic plant parameters being significantly (multiply) deviated.

The algorithm here can also be classified as pseudo adaptive (or inertia-free adaptive) algorithm with nonlinear properties. As an example of such algorithms there are signal algorithms [7] and algorithms in finite form or high speed pseudo gradient algorithm as well [8]. The right side of the signal algorithm has the form

$$\Phi(e) = -h \, sign \, P e,$$

$h = const > 0$, P is a matrix/vector. Obviously, it is a relay type algorithm. A relay unit presence may often prevent to realize such control in the multivariate case. However, its application in the adaptive control structure with an adjustable model (adaptive control with an exomodel [9]) makes the problem avoidable. Algorithm in the finite form is close to the linear low or to the relay one:

$$u(t) = -\gamma P e, \quad u(t) = -\gamma\, sign P e,$$

respectively,

$$\gamma = const > 0.$$

In the first case the linear low by its form doesn't refer to the adaptive low. In the second case it is again a signal-type algorithm.

The main feature of algorithm mentioned due to the right side discontinuity of algorithm, similar to the signal ones, consists in the degeneracy (order decreasing) of the equation of motion on the sliding surface (plane) invariant to plant parameters changes in the bounded range of values (adaptive property). To weaken these shortcomings some equivalent control has been developed aimed at replacing disruptive control by continuous one, essentially by the linear low [11]. Such approach is not considered as an expedient one in terms of being applied in practice. It may be estimated as some additional way of discontinuous control system motion.

MAIN RESULT

Assume that a plant has the form of

$$\begin{gathered} \dot{x}(t) = Ax(t) + Bu(t), \;\; x(t_0) = x_0, t \in [t_0, \infty), \\ u(t) = u^S(t) + g(t) \end{gathered} \qquad (1)$$

Being $x(t)$ state vector, $u^s(t)$ being a nonlinear control vector and $g(t)$ being an input vector from admissible domain.

Matrices in description (1) are represented the following way:

$A = \{a_{ij}\}_{n \times n}$ – matrix with interval uncertainty Θ;

$\Theta = \left\{\theta \in R^l : \left|\theta_i - \theta_i^*\right| \le \Delta\theta_i, \quad i = \overline{1,n}\right\}, \quad l = n^2$;

$\theta = [a_{11}, \quad a_{12}, \quad \dots \quad a_{nn}]$;

θ_i^* – nominal value;

$B = \{b_{ij}\}_{n \times m}$ – matrix with known elements.

It is assumed that a plant is fully controllable and state vector components $x(t)$ are measurable.

The algorithm is built on system synthesis with reference model [3].

Assume a reference model equation has the form of

$$\dot{\hat{x}}(t) = A_0\hat{x}(t) + B_0 g(t), \quad \hat{x}(t_0) = \hat{x}_0 \qquad (1)$$

$\hat{x}(t)$ being n-dimensional vector state; the choice of matrices A_0, B_0 is determined by the desired dynamics of the reference model.

Input a control error:

$$e_i(t) = x_i(t) - \hat{x}_i(t), \quad i = \overline{1,n}$$

We receive the following equation from equations (1), (2) after a series of transformations

$$\begin{gathered} \dot{e}(t) = Ae(t) + Bu^S(t) + \delta(t), \quad e(t_0) = e_0 \\ \delta(t) = \left[\left(A - A_0\right)\hat{x}(t) + \left(B - B_0\right)g(t)\right] \end{gathered} \qquad (3)$$

For the case when matrix B is non-invertible we will alter the equation (3) so that

$$\dot{e}(t) = Ae(t) + Bu^S(t) + BB^+\delta(t),$$

where $B^+ = (B^T B)^{-1} B^T$ [6].

The fulfillment of the consistency condition

$$\lim_{t \to \infty}\left(BB^+\delta(t) - \delta(t)\right) = 0 \qquad (4)$$

results in equation (3) and the altered equation being equivalent.

Consider the equation (3) in order to determine the controller structure if $\delta(t) = 0$, being $B = B_0$,

$$\dot{e}(t) = Ae(t) + Bu^S(t), \quad e(t_0) = e_0. \qquad (5)$$

Select the control law in the form of

$$u^S(t) = K(t)e(t), \quad K = \{k_{ij}\}_{m \times n}, \qquad (6)$$

$K(t)$ being the matrix of adjustable parameters.

Define adjustable parameters of the matrix elements in (6). Select Lyapunov function in the form of [6]

$$J(e) = \frac{1}{2}e^T(t)e(t),$$

and determine the derivative

$$\Psi(t) = \frac{dJ}{dt} = e^T(t)\dot{e}(t).$$

To ensure asymptotic stability of the system (5) it is sufficient that

$$\Psi(t) < 0. \qquad (7)$$

From formulas (5) and (6) we receive

$$\dot{e}(t) = \Gamma(t)e(t) \qquad (8)$$

$$\Gamma(t) = A + BK \quad (9)$$

Depict (7) in the form of a sum

$$\Psi(t) = \Psi_1(t) + \Psi_2(t) .$$

Then, by virtue of (8)

$$\Psi(t) = e^T(t)\Gamma(t)e(t) = \Sigma_{i=1}^{n} e_{ii}^2 \gamma_{ii} + \Sigma_{i,j=1, i\neq j}^{n} \gamma_{ij} e_i e_j$$

Set:

$$\Psi_1(t) = \Sigma_{i=1}^{n} \gamma_{ii} e_{ii}^2$$

$$\Psi_2(t) = \Sigma_{i,j=1, i\neq j}^{n} \gamma_{ij} e_i e_j .$$

Lemma

The condition of asymptotic stability (5) is ensured if the following condition is fulfilled

$$\Psi_1(t) < 0, \quad \Psi_2(t) < 0$$

Indeed, let us assume the diagonal elements of the matrix

$$\Gamma(t) = \left\{ \gamma_{ij} \right\}_{i,j=1}^{n}$$

as constant and negative, that is $\gamma_{ii} < 0$, to form $\Psi_2(t) < 0$.

The condition to fulfill the inequality $\Psi_2(t) < 0$ is found through a substitution [12]:

$$\gamma_{ij}(t) = \alpha_{ij}^{-1} e_i(t) e_j(t), \quad i \neq j ,$$

consequently,

$$\Psi_2(t) = \Sigma_{i,j=1, i\neq j}^{n} \gamma_{ij} e_i e_j = \Sigma_{i,j=1, i\neq j}^{n} \alpha_{ij} \gamma_{ij}^2(t),$$

if $\alpha_{ij} < 0$ the function $\Psi_2(t) < 0$.

Now consider equation (3) with $\delta(t) \neq 0$. Then the asymptotic stability condition (3) is not ensured, but only the dissipativity with controlled radius of the limit set.

Indeed, we write equation (3) with regard to (6) in the form

$$\dot{e}(t) = \Gamma(t)e(t) + \delta(t) .$$

Then we get

$$\Psi(t) = e^T(t)\Gamma(t)e(t) + e^T(t)\delta(t) .$$

We introduce the estimate

$$e^T(t)\Gamma(t)e(t) \leq -\beta \left\| e(t) \right\|^2 ,$$

where $-\beta = \max_{i,j}(\gamma_{ii}, \alpha_{ij})$. As a result, we turn to inequality

$$\dot{J}(e) \leq -\beta \left\| e(t) \right\|^2 + \left\| e(t) \right\| \left\| \delta(t) \right\| .$$

Introducing substitutions

$$J(e) = \frac{1}{2} e^T(t)e(t) = \rho^2, \quad \dot{J}(e) = 2\rho\dot{\rho}$$

and carrying out the calculations for solving the differential inequality, we obtain finally

$$\left\| e(t) \right\| \geq \beta^{-1} \left\| \delta(t) \right\| .$$

The parameter β (free in the choice) determines the radius of the limit set (the region of dissipativity).

Receive from formula (9) a matrix of adjustable parameters

$$K(t) = B^{-1}\left(\Gamma - A\right) \quad (10)$$

in case $n = m$ and matrix *B* allows invertability; in case $n \neq m$ matrix B^{-1} is substituted with a pseudo inverse one B^{+}. To calculate matrix $K(t)$, the “nominal” values of θ_i^* elements of matrix A are used.

Theorem

System (1) involves dissipative stability of processes with respect to $e(t)$ with control law,

$$u^S(e) = Ke(t),$$
$$K(t) = B^{-1}\left(\Gamma - A\right)$$
$$K(t) = B^{+}\left(\Gamma - A\right)$$

in case consistency condition (4) is fulfilled;

ALGORITHM APPLICATION EXAMPLE

Improving dynamic precision of a robotic system, e.g. performing assembly operations, is possible on the basis of adaptive (robust) and decoupling control. These are the functions of this type of control:

- stabilizing dynamics of drives ensuring degrees of freedom and suppression (decoupling) of mutual influence effect among subsystems;
- synthesis for the robust and decoupling laws of control has been performed for a two-degree-of-freedom manipulator (Fig. 1), motion equations of which being represented in the form of the second type of Lagrange equations, in axes decomposition for rigid motion.

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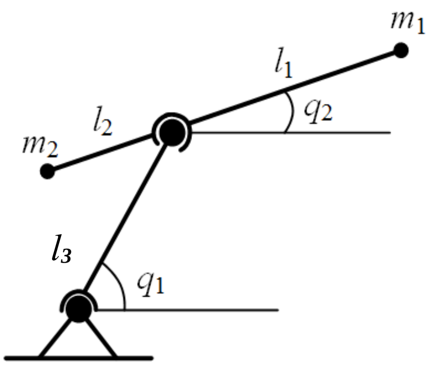

Figure. 1. Two-degree-of-freedom manipulator

Manipulator model :

$$J_1 \ddot{q}_1 + J_3 \cos(q_2 - q_1)\ddot{q}_2 - \\ - J_3 \sin(q_2 - q_1)\dot{q}_2^2 + (\frac{J_3}{l_3} g \cos q_1) = \tau_1, \quad (11)$$

$$J_2 \ddot{q}_2 + J_3 \cos(q_2 - q_1)\ddot{q}_1 - \\ - J_3 \sin(q_2 - q_1)\dot{q}_1^2 + (\frac{J_2}{l_3} g \cos q_2) = \tau_2, \quad (12)$$

q_1, q_2 being generalized (positional) coordinates; τ_1, τ_2 being torques; $l_1 - l_3$ being linear sizes of joint and arm; m_1, m_2 being balance mass and load 0 ÷ 25 kg.

$$\begin{aligned} J_1 &= m_1 l_1^2 + m_2 (l - l_1)^2, \\ J_2 &= m l_3^2, \\ J_3 &= (m_2 l - m l_1) l_3, \\ l &= l_1 + l_2, \end{aligned} \quad (13)$$

Manipulator parameters are represented in the Table.

Table. Manipulator parameters

Parameters	Manipulator model	Average value
l_1, m	1.4	
l_2, m	0.57	
l_3, m	0.7	
l, m	1.97	
m_1, kg	83.5÷108.5	96
m_2, kg	130	
$m = m_1 + m_2$, kg	213.5 ÷ 238.5	226
J_1, kg·sq m	206 ÷ 255	230
J_2, kg·sq m	104.6÷ 117	111
J_3, kg·sq m	– (30 ÷54.5)	– 42.2

Reference model has been chosen taking into account average values of mass and inertia properties and introduction of feedback on angular displacement together with their velocities for each drive, so that transient aperiodic processes of displacement in a linear domain for stepped response around 0.2 s should be ensured. The amplitude of input control signals corresponds to changes of angular displacement up to $\pi/4$ rad ≈ 0.8 rad (assumed as 1 rad).

Reference model motion equations have the form of

$$\ddot{\hat{q}}_1 + 77\dot{\hat{q}}_1 + 1490\hat{q}_1 = 1490 u_1^0$$

$$\ddot{\hat{q}}_2 + 65\dot{\hat{q}}_2 + 1100\hat{q}_2 = 1100 u_1^0$$

Pseudo adaptive control law (algorithm) :

In accordance with (10) – (13) for the first mechanical subsystem the result is:

$$K^{(1)} = \left(B_1^T B_1\right)^{-1} B_1^T \left(\begin{bmatrix} \gamma_{11} & \alpha_{12}^{(1)} e_1 e_2 \\ \alpha_{21}^{(1)} e_1 e_2 & \gamma_{22} \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -a_{21} & -a_{22} \end{bmatrix} \right) =$$

$$= \begin{bmatrix} \frac{\alpha_{21}^{(1)} e_1 e_2}{1490} + 1 & \frac{\gamma_{22}^{(1)} + 77}{1490} \end{bmatrix}$$

$$u_1^s = K^{(1)} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \frac{1}{1490} \left[\alpha_{21}^{(1)} e_1^2 e_2 + 1490 e_1 + (\gamma_{22}^{(1)} + 77) e_2 \right] \quad (14)$$

Similarly, for the second mechanical subsystem it is:

$$u_2^s = K^{(2)} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \frac{1}{1100} \left[\alpha_{21}^{(2)} e_1^2 e_2 + 1100 e_1 + (\gamma_{22}^{(2)} + 65) e_2 \right] \quad (15)$$

$$\gamma_{22}^{(1)} = \gamma_{22}^{(2)} = -50, \quad \alpha_{21}^{(1)} = \alpha_{21}^{(2)} = -1$$

In the general case, when control forces and cross-coupling effects are engaged, equations for torques of the mechanical subsystems have the form of

$$\tau_i = f_i (H_i x_i + \tilde{H}_i x_j + h_i u_i^0 + \tilde{h}_i u_j^0 + u_i^s), \quad (16)$$

u_i^0 being input signals (program control), i, j =1, 2 being indexes of mechanical subsystems (index j in the index line i points at impact by an j-th subsystem and vice versa);

$$x_1 = \begin{bmatrix} q_1 & \dot{q}_1 \end{bmatrix}^T,$$
$$x_2 = \begin{bmatrix} q_2 & \dot{q}_2 \end{bmatrix}^T,$$

H_i, h_i being adjustable parameters of mechanical subsystems regulators per own variables; $\tilde{H}_i$, $\tilde{h}_i$ being adjustable parameters of cross-coupling effects for decoupling (compensation) of mechanical systems and decoupling input cross-coupling signals correspondingly.

We note that in formula (16) $f_1(\ldots)$, $f_2(\ldots)$ being "force" controller operators and directly creating torques $\tau_1(t)$, $\tau_2(t)$ for mechanical subsystem drives are chosen in the form of constant dimension factors.

Comments

The research supposed no input cross-coupling signals therefore summand $\tilde{h}_i u_j^0$ in (16) is not taken into account; links $h_i u_i^0$ were chosen as the ones with constant matrices, their values were taken into account while choosing a reference model and therefore they are not used in adjusting either. Links $H_i x_i$ are set through proposed algorithm parameters k_{ij}. Links $\tilde{H}_i x_j$ represent subsystems mutual influence effect as functional relationship: matrix $\tilde{H}_i$ consists of elements in the form of trigonometric functions. The influence of these relations are difficult to decouple using only parameters adjusting, therefore the latter envisages no decoupling here. Proposed algorithm suppresses this cross-coupling effect as an external perturbation.

Simulation results:

Mathematical simulation in MatLab/Simulink has been carried out to research algorithm efficiency in terms of stabilizing dynamics under variable values (of masses) and mutual influence effect of degrees of freedom. The research has been performed while testing a manipulator positioning command, namely a square periodic wave signal with 1 rad amplitude and 5 sec. period. For mutual influence of degrees of freedom, test signal being sent to the lowest degree of freedom. The signal was not sent to the high degree of freedom.

Simulation of nonlinear algorithm was performed in comparison with adaptive algorithm of parametric adjustment in the form of comparison system [3]

$$\dot{k} = \Lambda\Phi(e,q),$$

$$\dot{k}_{ij} = -\lambda_{ij}\left(\sum_{i,j=1}^{n} p_{ij} e_j\right) q_j^c \qquad (17)$$

for the same manipulator.

The researched structure is shown in Fig.2.

The upper block represents a controllable mechanical system with a reference model and nonlinear controller (14), (15). The lower block is a comparison system with a similar structure but with a different (adaptive) controller (17) used for nominal controller parameters rearranging in the feedback channels. Thus the figure shows the examined nonlinear system and the adaptive system. As the proposed system with nonlinear controller has adaptive properties, it should be compared with an adaptive system. As is clear from the figure 2 both systems have parallel connection with each other, i.e., they have common input (test) signal u^0.

Let us note here that u0 is a two-dimensional vector, i.e., u0i, $i = 1, 2$ in accordance with the degree of freedom number. In researches, as mentioned in 3.4, it is assumed $u_2^0 = 0$ for the second degree of mobility in order to simplify the problem and be confined to influence examination of the first degree dynamics over the second degree of mobility. The comparison of adaptive properties of both systems is observed for the first degree of mobility.

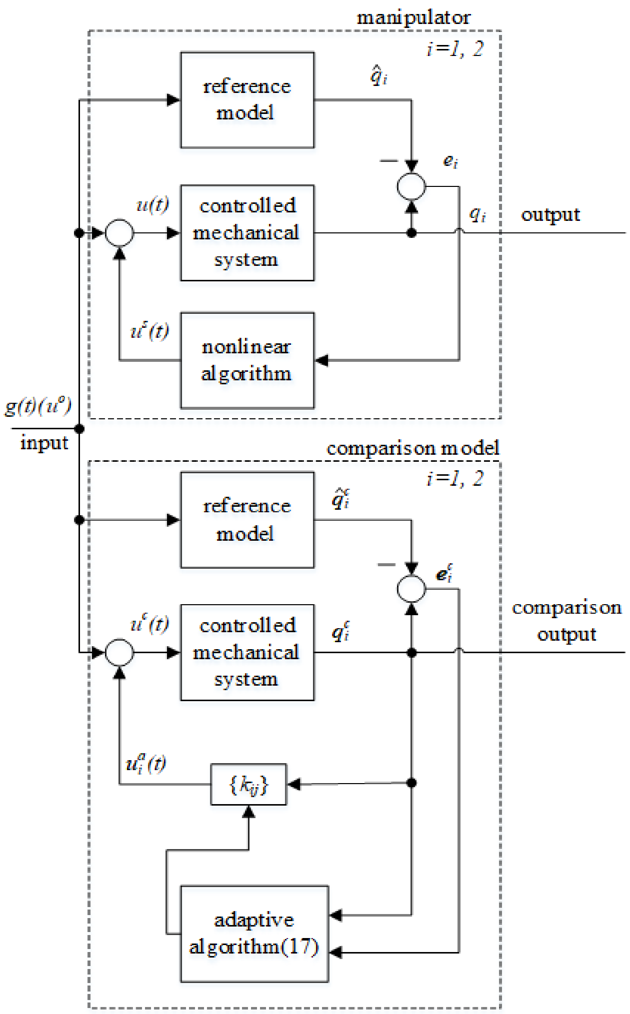

Figure 2. Researched structure

Research program:

1. A parameter mismatching is introduced to weaken the manipulator model parameters up to 100 times. Test signal (meander: amplitude 1 rad, period 5 sec.) is supplied to the common system input but only to the drive of the first degree mobility, i.e., $u_1^0 = 1$ rad.

2. Transient adaptive processes are estimated by coordinates q_1, $\hat{q}_1^c$ (fig. 3a). For nonlinear system these variables coincide completely within a single period.

3. For an adaptive camparison system the corresponding coordinates q_1, q_1^c (fig. 4a) converge in 12 sec. The beginning

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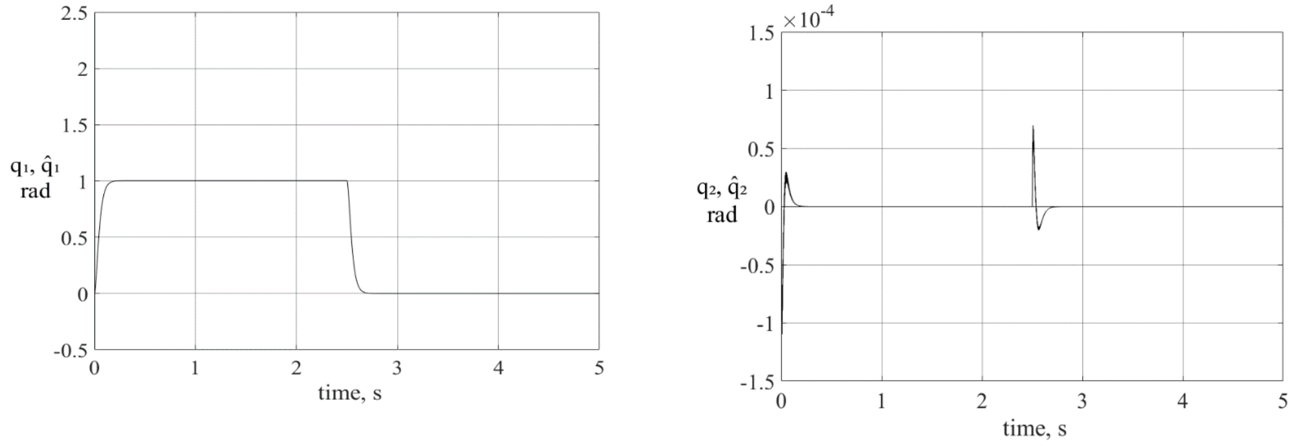
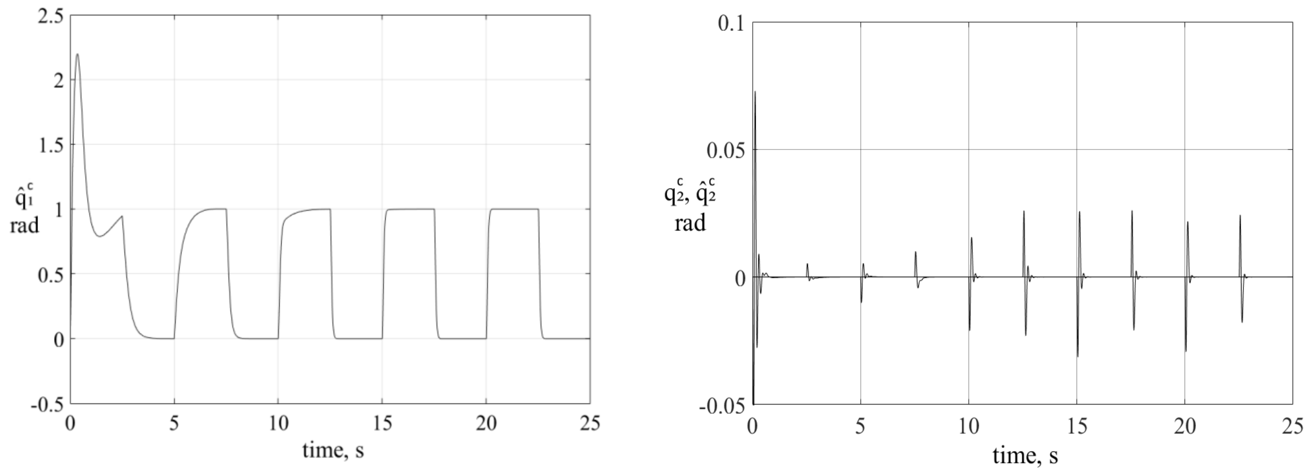
of transient process in the mechanical system with parameters mismatching (an impulse) can be seen. In the course of adjusting the system processes and the reference model ones converge.

4. Mutual influence over the second degree of mobility can be estimated by the coordinates q_2, $\hat{q}_2$ and q_2, $\hat{q}_2^c$ where q_2, $\hat{q}_2^c$ = 0 (fig. 3b, fig. 4b) by the maximum deviations of coordinates q_2, q_2^c relative to zero value.



a b

Figure 3. System response with the proposed control algorithm



a b

Figure 4. Comparison system response with the adaptive control algorithm

Comparative researches related with the response of mass changes have revealed time-free convergence of proposed algorithms processes; adaptive algorithm convergence time was 10 sec. Fig. 3 and Fig. 4 (at the left) illustrate the processes of mobility degree mutual influence for proposed and adaptive algorithms respectively in the mode of test signal response only by the lower degree of mobility.

The signal was not supplied to the upper degree of mobility. Thus, the figures at the bottom illustrate the upper drive response to the lower one through interrelations. Response amplitude for proposed algorithm is 11·105 rad. and 2.5·10–2 rad. for the adaptive one. Proposed algorithm comparative effectiveness ≈220.

CONCLUSION

The obtained pseudo adaptive algorithm for a dynamic plant control with limited parametric uncertainty is time-free, asymptotically (dissipative) stable and robust relatively unconsidered perturbations. Simulation with a MatLab/Simulink software of a two-link manipulator has shown that during simulated test modes motion process under significant parametric deviations of a dynamic plant is of constantly stable aperiodic nature and is distinguished by high degree of suppression of manipulator drives interrelation which can be regarded as robustness relatively unconsidered perturbations. Design and implementation of these algorithms are simple enough and do not require fine-tuning.

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DISCLOSURE STATEMENT

No potential conflict of interest was reported by the authors.

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