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Free Vibration of Antisymmetric Angle-Ply Composite Laminated Conical Shell under Classical Theory

A.K. Nor Hafizah [1,2], K.K. Viswanathan [1,2,3,*], Z.A. Aziz [1,2] and J.H. Lee[4]

[1] Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 Skudai, Johor Bahru, Johor, Malaysia.

[2] UTM Centre for Industrial and Applied Mathematics, Ibnu Sina Institute for Scientific & Industrial research, Universiti Teknologi Malaysia, 81310 Skudai, Johor Bahru, Johor, Malaysia.

[3] Kuwait College of Science and Technology, Doha District, Block 4, P.O. Box No. 27235, Safat 13133, Kuwait.

[4]Dept. of Naval Architecture and Ocean Engineering, Inha University, 100 Inharo, Nam-gu, Incheon 22212, South Korea.

**Corresponding author*

[3]Orcid: 0000-0003- 4470-4774

Abstract

This paper focuses on free vibration behaviour of antisymmetric angle-ply laminated conical shell under classical theory. The equations of motion are derived using Love's first approximation thin shell theory. By applying point collocation method together with spline function approximation, the free vibration equations of motion of the composite laminated conical shell are transformed to a set of algebraic equations for the assumed spline coefficients. The eigenvalue problem is solved by employing the eigensolution techniques with eigenvectors as spline coefficients to obtain the required frequencies. This study is conducted for Clamped-Clamped and Simply-Supported boundary conditions. Detailed parametric investigation is carried out to examine the influences of number of laminates, ply orientations, circumferential node number and materials on the free vibration of conical shells.

Keywords: free vibration, laminated conical shells, Love's first approximation thin shell theory, spline function approximation

INTRODUCTION

Thin shells are structural element in which the thickness is small compared to the length and width dimensions. Shell structures made an important contribution to the development of several branches of engineering such as architecture and building, chemical engineering, structural engineering and composite construction. Other examples of the impact of shell structures include water cooling towers for power stations, grain silos, armour, arch dams, tunnels and submarines. The application of laminated composite shell extends the characteristics of shells since it provides higher strength-to-weight ratios, better corrosion resistance, longer fatigue life and also one can design the directional properties. To obtain optimal design of structures in engineering industries, the vibrational behaviour analysis of shell structures is carried out. Knowing more about vibration of structures helps us to control damage and take preventive measures from damages to occur.

Conical shell is one of the revolution of shells that has great demand on industries. Irie et. al [1] and Sankaranarayanan et al. [2] analysed the free vibration of a conical shell with variable thickness by means of transfer matrix approach and Rayleigh-Ritz method respectively. The study of axisymmetric free vibrations of conical shells had been conducted by Tong [3] where a simple and exact solution was obtained directly for the Donnell-type governing equations. Ng et al. [4] used generalized differential quadrature (GDQ) to perform free vibration analysis of rotating composite laminated conical shells where the method was developed to improve the differential quadrature (QD) method. The dynamic analysis of the truncated conical shells subjected to pressure pulse loading was studied by Jankowski and Kubiak [5] using Bubnov-Galerkin analytical-numerical method. Flugge shell theory together with power series method was used by Xie et al. [6] to investigate free and forced vibration of stepped conical shells. Ghasemi et al. [7] applied Galerkin method for studying free vibration of truncated conical composite shells under various boundary conditions. Souza and Saraiva [8] used Rayleigh-Ritz method to analyse free vibration of conical shells on rigidly clamped edge boundary condition. A modified Fourier series solution was adopted by Jin et al. [9] to investigate the vibration of truncated conical shells. Based on first order shear deformation theory, free vibration of rotating graded composite conical shells were studied by Heydarpour et al.

[10] using differential quadrature method (DQM). Viswanathan et al. [11,12] had conducted a few studies on conical shells using spline method.

From the literature survey above, free vibration analysis of laminated composite antisymmetric angle-ply conical shells using Bickley-type spline still has not been investigated. The aim of this paper is to investigate this problem. The advantage of this spline is that it has low-order approximation and yield better accuracy compared to a global higher order approximation. Love's first approximation thin shell theory is used to formulate the theoretical models of the conical problem. Bickley-type spline together with collocation method is used to approximate the solution into a system of algebraic equation. The solution equations are then considered as eigenvalue problem which is solved using eigensolution technique. The results are obtained for Clamped-Clamped and Simply-Supported boundary conditions. The accuracy and reliability of the current solution are validated by comparing with the results available in the literature. The effects of circumferential node number, number of layers, length ratio, cone angle and material composition on the frequency of conical shells are further investigated.

MATHEMATICAL FORMULATION

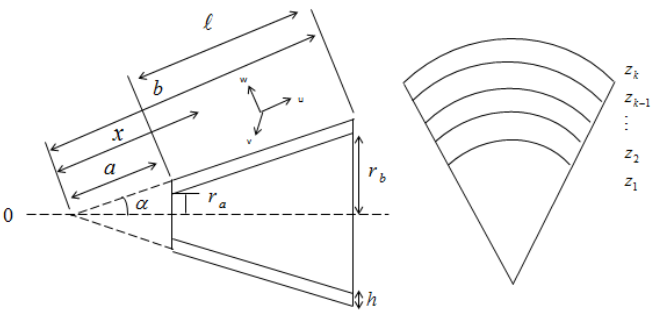

Figure 1. Geometry of layered conical shell

A laminated composite conical shell is considered as depicted in Figure 1. The conical shell is referred to a coordinate system (x,θ,z) which is fixed in the middle surface. The radius of the cone, the radius of the cone at the small end and radius of large end of the cone are denoted by $r = x\sin\alpha$, $r_a = a\sin\alpha$ and $r_b = b\sin\alpha$ respectively. α is the semi-vertical angle of the cone, ℓ is the length and h represents the thickness of the conical shell.

The equation of motion for conical shell in terms of stress and moment resultants are can be defined by [13]

$$\begin{aligned}
&\frac{\partial N_x}{\partial x}+\frac{1}{x}\left(N_x-N_\theta\right)+\frac{1}{x\sin\alpha}\frac{\partial N_{x\theta}}{\partial\theta}=\rho h\frac{\partial^2 u}{\partial t^2},\\
&\frac{\partial N_{x\theta}}{\partial x}+\frac{2}{x}N_{x\theta}+\frac{1}{x\sin\alpha}\frac{\partial N_\theta}{\partial\theta}+\\
&\frac{1}{x\tan\alpha}\left(\frac{\partial M_{x\theta}}{\partial x}+\frac{2}{x}M_{x\theta}+\frac{1}{x\sin\alpha}\frac{\partial M_\theta}{\partial\theta}\right)=\rho h\frac{\partial^2 v}{\partial t^2},\\
&\frac{\partial^2 M_x}{\partial x^2}+\frac{2}{x}\frac{\partial M_x}{\partial x}+\frac{2}{x\sin\alpha}\frac{\partial^2 M_{x\theta}}{\partial x\partial\theta}-\frac{1}{x}\frac{\partial M_\theta}{\partial x}+\\
&\frac{2}{x^2\sin\alpha}\frac{\partial M_{x\theta}}{\partial\theta}\ \frac{2}{x^2\sin^2\alpha}\frac{\partial^2 M_\theta}{\partial\theta}-\\
&\frac{1}{x\tan\alpha}N_\theta=\rho h\frac{\partial^2 w}{\partial t^2}
\end{aligned}\tag{1}$$

where ρ is the density. The stress resultants N_{ij} and moment resultants M_{ij} are written as

$$\begin{aligned}
\left(N_x,N_\theta,N_{x\theta}\right)&=\int_z\left(\sigma_x,\sigma_\theta,\sigma_{x\theta}\right)dz\\
\left(M_x,M_\theta,M_{x\theta}\right)&=\int_z\left(\sigma_x,\sigma_\theta,\sigma_{x\theta}\right)z\,dz
\end{aligned}\tag{2}$$

The constitutive equations for stress and moment resultants are given in the following form

$$\begin{pmatrix}N_x\\N_\theta\\N_{x\theta}\\M_x\\M_\theta\\M_{x\theta}\end{pmatrix}=\begin{pmatrix}A_{11}&A_{12}&A_{16}&B_{11}&B_{12}&B_{16}\\A_{12}&A_{22}&A_{26}&B_{12}&B_{22}&B_{26}\\A_{16}&A_{26}&A_{66}&B_{16}&B_{26}&B_{66}\\B_{11}&B_{12}&B_{16}&D_{11}&D_{12}&D_{16}\\B_{12}&B_{22}&B_{26}&D_{12}&D_{22}&D_{26}\\B_{16}&B_{26}&B_{66}&D_{16}&D_{26}&D_{66}\end{pmatrix}\begin{pmatrix}\varepsilon_x\\\varepsilon_\theta\\\varepsilon_{x\theta}\\\kappa_x\\\kappa_\theta\\\kappa_{x\theta}\end{pmatrix}\tag{3}$$

where the strain-displacements for conical shell are:

$$\begin{aligned}
&\varepsilon_x=\frac{\partial u}{\partial x},\ \varepsilon_\theta=\frac{1}{x}u+\frac{1}{x\sin\alpha}\frac{\partial v}{\partial x}+\frac{1}{x\tan\alpha}w,\\
&\varepsilon_{x\theta}=\frac{1}{x\sin\alpha}\frac{\partial u}{\partial\theta}+\frac{\partial v}{\partial x}-\frac{1}{x}v,\kappa_x=-\frac{\partial^2 w}{\partial x^2},\\
&\kappa_\theta=\frac{1}{x^2\sin\alpha\tan\alpha}\frac{\partial v}{\partial\theta}-\frac{1}{x}\frac{\partial w}{\partial x}-\\
&\qquad\frac{1}{x^2\sin^2\alpha}\frac{\partial^2 w}{\partial\theta^2},\\
&\kappa_{x\theta}=\frac{1}{x\tan\alpha}\frac{\partial v}{\partial x}-\frac{2}{x^2\tan\alpha}v-\frac{2}{x\sin x}\frac{\partial^2 w}{\partial x\partial\theta}\\
&\qquad+\frac{2}{x^2\sin x}\frac{\partial w}{\partial\theta}.
\end{aligned}\tag{4}$$

The extensional stiffness, bending-extensional coupling stiffness, and bending stiffness are represented by A_{ij}, B_{ij} and D_{ij} respectively which are obtained by

$$A_{ij}=\sum_k Q_{ij}^{(k)}(z_k-z_{k-1});\ B_{ij}=\frac{1}{2}\sum_k Q_{ij}^{(k)}(z_k^2-z_{k-1}^2);$$
$$D_{ij}=\frac{1}{3}\sum_k Q_{ij}^{(k)}(z_k^3-z_{k-1}^3). \qquad (5)$$

The stiffness components A_{16}, A_{26}, B_{11}, B_{12}, B_{11}, B_{22}, B_{66}, D_{16} and D_{26} are identically zero since the antisymmetric angle-ply conical shells condition are considered. The displacement components are assumed to be in the following separable form:

$$u_0(x,\theta,t)=U(x)e^{(n\theta+i\omega t)}$$
$$v_0(x,\theta,t)=V(x)e^{(n\theta+i\omega t)} \qquad (6)$$
$$w(x,\theta,t)=W(x)e^{(n\theta+i\omega t)}$$

in which ω, t and n represent the angular frequency of vibration, time, and circumferential node number respectively. The non-dimensional parameters are introduced as follows:

$$X=\frac{x-a}{\ell},\ a\le x\le b \text{ and } X\in[0,1]$$
$$\lambda=\omega\ell\sqrt{\frac{I_1}{A_{11}}}, \text{ frequency parameter}$$
$$\beta=\frac{a}{b}, \text{the radii ratio}$$
$$\gamma=\frac{h}{r_a}, \text{ ratios of thickness to radius} \qquad (7)$$
$$\gamma'=\frac{h}{a}, \text{ ratio of thickness to length}$$
$$\delta_k=\frac{h_k}{h}, \text{ relative layer thickness of the } k\text{-th layer}$$

By substituting Eq. (4) into Eq. (3) and then applying the resulting equations into the governing Eq. (1), a set of second order partial differential equations in terms of displacement functions is obtained. Substitution of Eq. (6) and Eq. (7) into the obtained differential equations leads to the following matrix equation

$$\begin{bmatrix} L_{11} & L_{12} & L_{13}\\ L_{12} & L_{22} & L_{23}\\ L_{13} & L_{32} & L_{33}\end{bmatrix}\begin{bmatrix}U\\V\\W\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \qquad (8)$$

The differential operators, L_{ij} of the matrix are given as follow

$$L_{11}=\frac{d^2}{dX^2}+p\frac{d}{dX}-\left(S_3-S_{10}n^2\csc^2\alpha\right)p^2$$
$$L_{12}=S_{15}p\cot\alpha\frac{d^2}{dX^2}+(S_2n\csc\alpha+$$
$$S_{10}n\csc\alpha-2S_{15}p\cot\alpha-2S_{16}p\cot\alpha)p\frac{d}{dX}-$$
$$(S_3n\csc\alpha+S_{10}n\csc\alpha-2S_{15}p\cot\alpha-$$
$$2S_{16}p\cot\alpha-S_{16}n^2p\csc^2\alpha\cot\alpha)p^2$$
$$L_{13}=-3S_{15}np\csc\alpha\frac{d^2}{dX^2}+(S_2\cot\alpha+$$
$$2S_{15}np\csc\alpha+S_{16}np\csc\alpha)p\frac{d}{dX}-$$
$$(S_3\cot\alpha+2S_{15}np\csc\alpha+2S_{16}np\csc\alpha+$$
$$S_{16}n^3p\csc^3\alpha)p^2$$
$$L_{21}=S_{15}p\cot\alpha\frac{d^2}{dX^2}+(S_2n\csc\alpha+$$
$$S_{10}n\csc\alpha+2S_{15}p\cot\alpha+S_{16}p\cot\alpha)p\frac{d}{dX}+$$
$$(S_3n\csc\alpha+S_{10}n\csc\alpha+S_{16}p\cot\alpha+$$
$$S_{16}n^2p\csc^2\alpha\cot\alpha)p^2$$
$$L_{22}=-S_{15}\frac{d^3}{dX^3}-(2S_{15}+S_{16}+S_8np\csc\alpha\cot\alpha+$$
$$2S_{12}np\csc\alpha\cot\alpha)p\frac{d^2}{dX^2}-(S_{16}+$$
$$3S_{16}n^2\csc^2\alpha-S_{16}\cot^2\alpha+$$
$$S_9np\csc\alpha\cot\alpha)p\frac{d}{dX}+(S_3n\csc\alpha\cot\alpha+$$
$$2S_{16}n^2p\csc^2\alpha+S_{16}p\cot^2\alpha-$$
$$S_9n^3p^2\csc^3\alpha\cot\alpha)p^2 \qquad (9)$$

$$L_{23} = -S_{15}\frac{d^3}{dX^3} - (2S_{15} + S_{16} + S_8 np\csc\alpha\cot\alpha +$$
$$2S_{12}np\csc\alpha\cot\alpha)p\frac{d^2}{dX^2} - (S_{16} + 3S_{16}n^2\csc^2\alpha -$$
$$S_{16}\cot^2\alpha + S_9 np\csc\alpha\cot\alpha)p\frac{d}{dX} +$$
$$(S_3 n\csc\alpha\cot\alpha + 2S_{16}n^2 p\csc^2\alpha +$$
$$S_{16}\cot^2\alpha - S_9 np\csc\alpha\cot\alpha)$$
$$L_{31} = 3S_{15}np\csc\alpha\frac{d^2}{dX^2} - (S_2\cot\alpha -$$
$$2S_{15}np\csc\alpha - S_{16}np\csc\alpha)p\frac{d}{dX} -$$
$$(S_3\cot\alpha - S_{16}np\csc\alpha - S_{16}n^3 p\csc^3\alpha)p^2$$
$$L_{32} = S_{15}\frac{d^3}{dX^3} + (S_{15} - S_{16} + S_8 np\csc\alpha\cot\alpha +$$
$$2S_{12}np\csc\alpha\cot\alpha)p\frac{d^2}{dX^2} + (S_{16} + 3S_{16}n^2\csc^2\alpha -$$
$$S_{16}\cot^2\alpha - 2S_8 np\csc\alpha\cot\alpha - S_9 np\csc\alpha\cot\alpha -$$
$$4S_{12}np\csc\alpha\cot\alpha)p^2\frac{d}{dX} - (S_3 n\csc\alpha\cot\alpha -$$
$$S_{15}p + S_{16}p - 2S_{16}p\cot^2\alpha S_{16}n^2 p\csc^2\alpha -$$
$$2S_8 np^2\csc\alpha\cot\alpha - 2S_9 np^2\csc\alpha\cot\alpha -$$
$$S_9 n^3 p^2\csc^3\alpha\cot\alpha - 6S_{12}np^2\csc\alpha\cot\alpha)p^2$$
$$L_{33} = -S_7\frac{d^4}{dX^4} - 2S_7 p\frac{d^3}{dX^3} - (2S_8 n^2\csc^2\alpha -$$
$$S_9 + 4S_{12}n^2\csc^2\alpha)p^2\frac{d^2}{dX^2} + (4S_{16}n\csc\alpha\cot\alpha +$$
$$2S_8 n^2 p\csc^2\alpha - S_9 p + 4S_{12}n^2 p\csc^2\alpha)\frac{d}{dX} -$$
$$(S_3\cot^2\alpha + 2S_{16}np\csc\alpha\cot\alpha + 2S_8 n^2 p^2\csc^2\alpha +$$
$$2S_9 n^2 p^2\csc^2\alpha + S_9 n^4 p^2\csc^4\alpha +$$
$$4S_{12}n^2 p^2\csc^2\alpha)p^2 + \lambda^2$$

SOLUTION PROCEDURE

The displacement functions $U(X)$, $V(X)$ and $W(X)$ are approximated using cubic spline functions and can be expressed as

$$U(X) = \sum_{i=0}^{2} a_i X^i + \sum_{j=0}^{N-1} b_j \left(X - X_j\right)^3 H\left(X - X_j\right)$$
$$V(X) = \sum_{i=0}^{2} c_i X^i + \sum_{j=0}^{N-1} d_j \left(X - X_j\right)^3 H\left(X - X_j\right) \quad (10)$$
$$W(X) = \sum_{i=0}^{4} e_i X^i + \sum_{j=0}^{N-1} f_j \left(X - X_j\right)^5 H\left(X - X_j\right)$$

Here $H\left(X - X_j\right)$ is the Heaviside step function and a_i, c_i, e_i, b_j, d_j and f_j are unknown coefficients. The number of sub-intervals are represented by N of X divided within the range [0,1]. The knots of the splines or the collocation points are assumed at $X = X_s = s / N$, where $s = 0, 1, \ldots, N$. A set of $3N + 3$ homogenous equations with 11 unknown spline coefficients is obtained by satisfying Eq. (8) with the splines at the knots.

The boundary conditions are applied at the two ends of the conical shell. Considering Clamped-Clamped and Simply-Supported boundary conditions in this work, the condition are as follow:

(i) Clamped-Clamped (C-C) (both the ends are clamped)

$U = V = W = \Psi_X = \Psi_\Theta = 0$ at $X = 0$ and $X = 1$.

(ii) Simply-Supported (S-S) (both ends are simply supported)

$V = W = N_X = M_X = \Psi_\Theta = 0$ at $X = 0$ and $X = 1$

After applying boundary conditions, 8 equations on spline coefficients are obtained. Adding them with those obtained earlier, a set of $3N + 11$ homogenous equations, with the same number unknown are accumulated. Then, the system of equations are written in the form of eigenvalue problem as in

$$[P][q] = \frac{1}{\lambda^2}[Q][q] \quad (10)$$

In this relation, $[P]$ and $[Q]$ are square matrices, $[q]$ is a column matrix, λ is the frequency parameter and $[q]$ is the eigenvector.

RESULT AND DISCUSSION

The reliability of the present numerical formulation for conical shells is investigated by firstly, convergence tests, and secondly, comparisons with the existing results in the literature. Table 1 shows the convergence frequency parameter λ for a conical shell with $\alpha = 30°$, $\beta = 0.5$ and n=1. The computation is conducted by choosing N=2 as the initial value and the computation is carried on until it is found that *N*=16 is sufficient to achieve low value of percentage of change. The present results are compared with Irie et al. [1] for isotropic conical shell with parameters: $\alpha = 30°, 45°, 60°$, $\beta = 0.25, 0.5, 0.75$ and $\nu = 0.3$ which are depicted in Table 2 3 and 4. It can be seen from the table that the values of the fundamental frequency of present study are in good agreement with the results of Irie et al. [1].

Table 1: Convergence study on frequency parameter λ for antisymmetric angle ply laminated conical shells with C-C boundary conditions

N	λ_1	% change
4	0.535108	-
6	0.512124	-1.177
8	0.507776	-0.856
10	0.504841	-0.581
12	0.503078	-0.350
14	0.501962	-0.222
16	0.501214	-0.149
18	0.500707	-0.101

Table 2: Comparison of frequency parameter λ for isotropic conical shells under C-C boundary conditions: $\beta = 0.25$

β	0.25		
α	30^0	45^0	60^0
Present	0.9600	0.7595	0.5798
Irie [1]	0.9886	0.8712	0.6680

Table 3: Comparison of frequency parameter λ for isotropic conical shells under C-C boundary conditions: $\beta = 0.5$

β	0.5		
α	30^0	45^0	60^0
Present	0.9300	0.6576	0.6498
Irie [1]	0.9930	0.8731	0.6685

Table 4: Comparison of frequency parameter λ for isotropic conical shells under C-C boundary conditions: $\beta = 0.75$

β	0.75		
α	30^0	45^0	60^0
Present	0.9046	0.7131	0.7648
Irie [1]	1.0030	0.9484	0.9576

Table 5: The effect of circumferential node number *n* on the frequency parameter λ of C-C conical shells

n	C-C		
	λ_1	λ_2	λ_3
0	0.62358	0.75986	1.15747
1	0.63004	0.75904	1.15639
2	0.68800	0.78389	1.16954
3	0.68086	0.81293	1.19202
4	0.58426	0.83031	1.12444
5	0.41807	0.86007	0.96272

Table 6: The effect of circumferential node number *n* on the frequency parameter λ of S-S conical shells

n	S-S		
	λ_1	λ_2	λ_3
0	0.56404	0.67634	0.91740
1	0.57681	0.67297	0.91782
2	0.63313	0.73608	0.94580
3	0.61156	0.75240	0.90144
4	0.53301	0.61951	0.98427
5	0.44475	0.56249	0.86652

The effect of the variation of the circumferential node number, *n* on the values of frequency parameter for antisymmetric angle-ply composite laminated conical shells under C-C and S-S boundary conditions are demonstrated in Table 5 and 6. The parameters of the shells are fixed as: length ratio $\beta = 0.5$, ratio of thickness to a radius $\gamma = 0.05$ and cone angle $\alpha = 30°$. The material HSG and SGE are used which are arranged in the order of HSG-SGE. From the tables, it is found that the variation of circumferential node number leads to the up and down of the frequency parameter λ of conical shells.

For the case of C-C conical shell, it is observed that the value λ_1 increases up to *n*=3 and then decreases afterwards. The value of λ_2 for *m*=2 are seen to decrease gradually from *n*=0 to *n*=1 and then increases afterwards. For frequency parameter λ_3, it can be seen that the value decreases and increases over the range $0 \le n \le 5$. The vibrational behaviour for S-S conical shell case gives similar pattern to the case before. As can be seen in the table, it is clear that the value of λ is higher for higher mode. Also, it is evident that the frequency parameter of C-C conical shells is greater than those of the S-S conical shells.

Figure 2 shows the effect of cone angle α $(10° \leq \alpha \leq 90°)$ on the frequency parameter λ of two-layered [30^0/-30^0] conical shells. The study is conducted for conical shells with C-C and S-S boundary conditions for frequency parameter λ with mode 1, 2 and 3 represented by Figs. 2(a), 2(b) and 2(c) respectively. The parameters such as n=1, $\beta = 0.5$, $\gamma' = 0.01$ and material HSG-SGE are fixed for both conical shells under C-C and S-S boundary conditions. From Fig. 2, it is observed that by increasing the cone angle of shells, the frequency parameter decreases. As is shown in Fig 2(a), 2(b) and 2(c), the frequency parameter is seen to drop significantly from $\alpha = 10°$ until $\alpha = 30°$ and then decreases steadily for $30° \leq \alpha \leq 90°$. For higher mode, the frequency parameter is higher. Also, the S-S conical shell has lower frequency parameter compared to C-C conical shell.

Figure 3 depicts the frequency parameter of C-C and S-S boundary conditions conical shells with parameters n=1, $\beta = 0.5$, $\gamma' = 0.01$ and material HSG-SGE. The influence of cone angle, α on the values of λ for two-layered [45^0/-45^0] conical shells is displayed. As one can see in Fig. 3, the frequency parameter shows a declining trend with the increase in cone angle which is similar as shown in Fig. 2. Observing frequency parameter for two-layered [30^0/-30^0] conical shells in Fig. 2 with frequency parameter for two-layered [45^0/-45^0] conical shells in Fig. 3, the difference in the values of λ are seen to become smaller with the increase of cone angle. The frequency parameter of antisymmetric angle-ply laminated conical shells under C-C and S-S boundary conditions for four-layered conical shells with ply-angle [30^0/-30^0/ 30^0/-30] and [45^0/-45^0/ 45^0/-45] are illustrated in Figs. 4 and 5 respectively. The materials are arranged in the form of HSG-SGE-SGE-HSG.

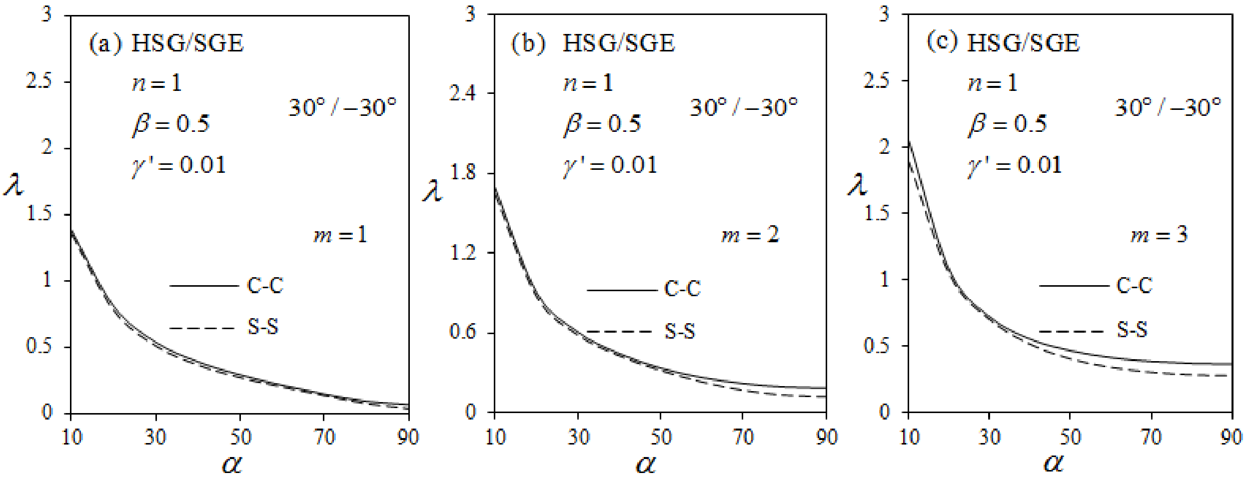

Figure 2. Variation of frequency parameter λ with respect to cone angle α for two-layered [30^0/-30^0] conical shells

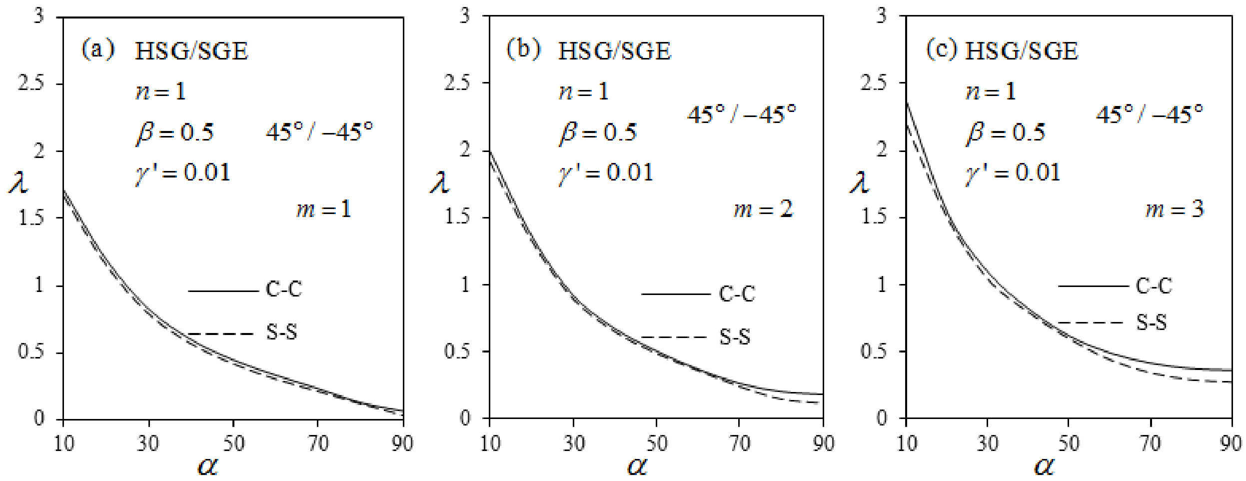

Figure 3. Variation of frequency parameter λ with respect to cone angle α for two-layered [45^0/45^0] conical shells

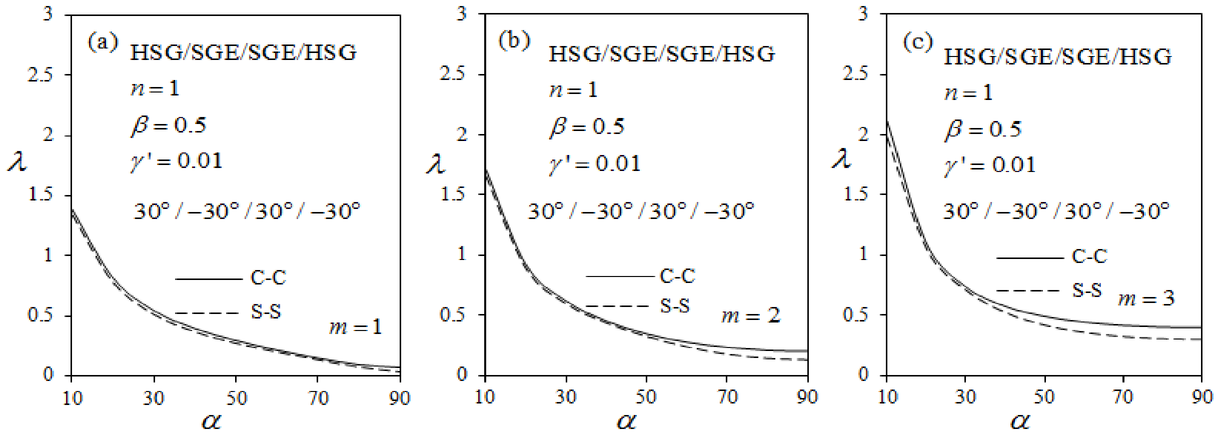

Figure 4. Variation of frequency parameter λ with respect to cone angle α for four-layered [30^0/-30^0/ 30^0/-30] conical shells

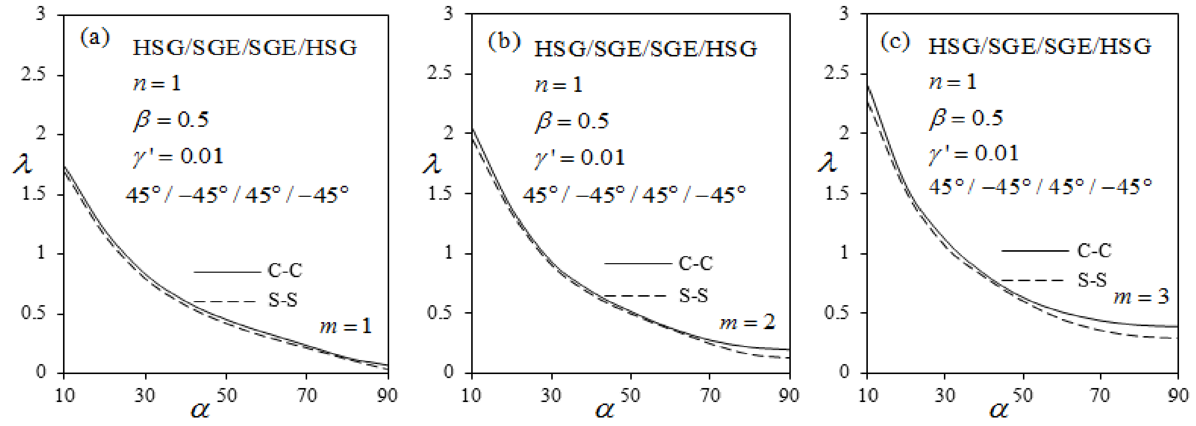

Figure 5. Variation of frequency parameter λ with respect to cone angle α for four-layered [45^0/-45^0/ 45^0/-45] conical shells

It is seen that the values of λ for conical shells with ply-angle [45^0/-45^0/ 45^0/-45] are higher for smaller cone angle when compared with ply-angle [30^0/-30^0/ 30^0/-30] but the values do not have much difference as cone angle increases.

For both figures, the frequency parameter for C-C conical shells is greater than S-S conical shells case. Furthermore, the results from Figs. 2, 3, 4 and 5 shows that the number of layers and different ply-angles have considerable effect on the frequency parameter of conical shells. In Fig. 6, the effects of length ratio β on the angular frequency ω of C-C conical shells are investigated. The length ratio, β is varied from 0.1 to 0.9. The conical shells parameters are taken to be *n*=1, $\alpha = 30°$ and $\gamma = 0.05$ with material orientation HSG-SGE. The study is conducted for three different ply-angles: [30^0/-30^0], [45^0/-45^0] and [60^0/-60^0] depicted by Figs. 6(a), 6(b) and 6(c) respectively. From the figures, it is found that as β increases, the values of angular frequency increase. It can be seen the angular frequency increases slowly up to certain value of β and then climbs rapidly afterwards. Also, the values of ω are slightly difference for different ply angles for $0.1 \le \beta \le 0.5$ but the change in the values of ω are more pronounced for $\beta = 0.5$ onwards. The maximum value of ω of conical shells occurs for ply-angle [30^0/-30^0] at $\beta = 0.9$. Figure 7 presents the variation of angular frequency for S-S conical shells against length ratio β with parameters *n*=1, $\alpha = 30°$ and $\gamma = 0.05$. The results are presented for three different ply-angles. It can be observed that the vibrational behaviour of conical shells in Fig. 7 has the same pattern as shown in Fig. 6. Observing the results in Figs. 6 and 7, it is noted that the values of ω are lower for S-S conical shells than the values of ω for C-C conical shells. Also, the frequencies are lower for higher ply-angle as the values of β increases. Figures 8 shows the angular frequency for C-C and S-S conical shells with respect to length ratio for two different cone angle, $\alpha = 45°$ and $\alpha = 60°$. It can be seen that the effect of length ratio on the frequencies of conical shells are more pronounced for higher cone angle ($\alpha = 60°$). Figures 9 depicts the angular frequency values of C-C and S-S four- layered conical shells with two different ply orientation ; [30^0/-30^0/ 30^0/-30] and [45^0/-45^0/ 45^0/-45]. It is concluded that the ply-angles gives influence on the frequency parameter of four-layered conical shells. With the increase in the length ratio, the difference in the value of ω for [30^0/-30^0/ 30^0/-30] conical shells are higher than conical shells with ply-angle [45^0/-45^0/ 45^0/-45].

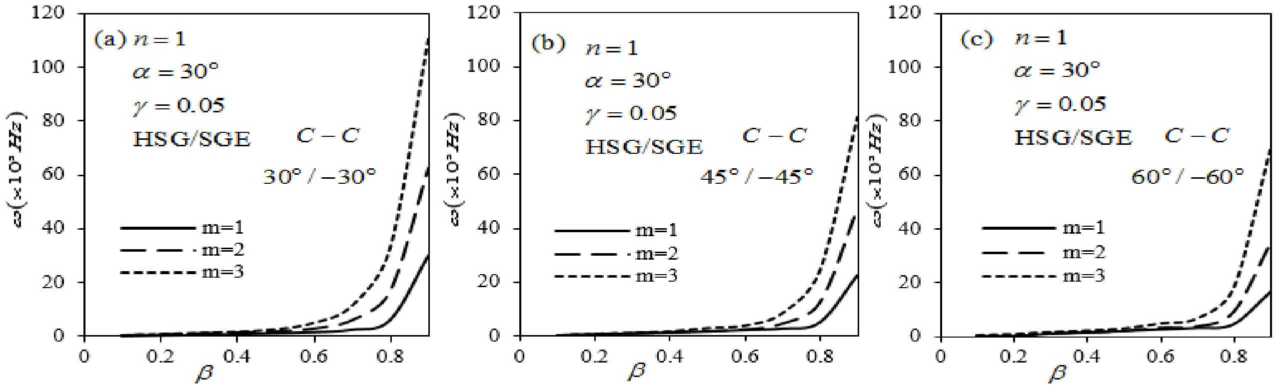

Figure 6. Variation of angular frequency ω with respect to length of cone β for different lamination angles under C-C boundary conditions

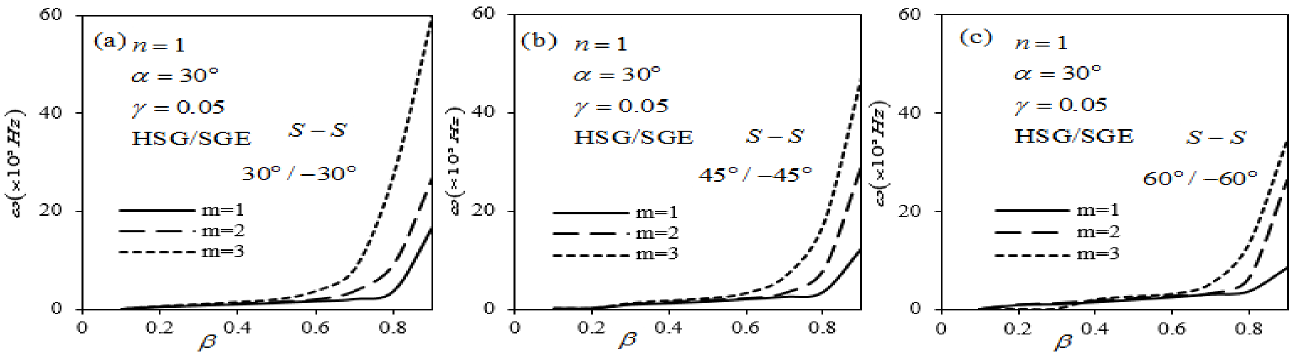

Figure 7. Variation of angular frequency ω with respect to length of cone β for different lamination angles under S-S boundary conditions

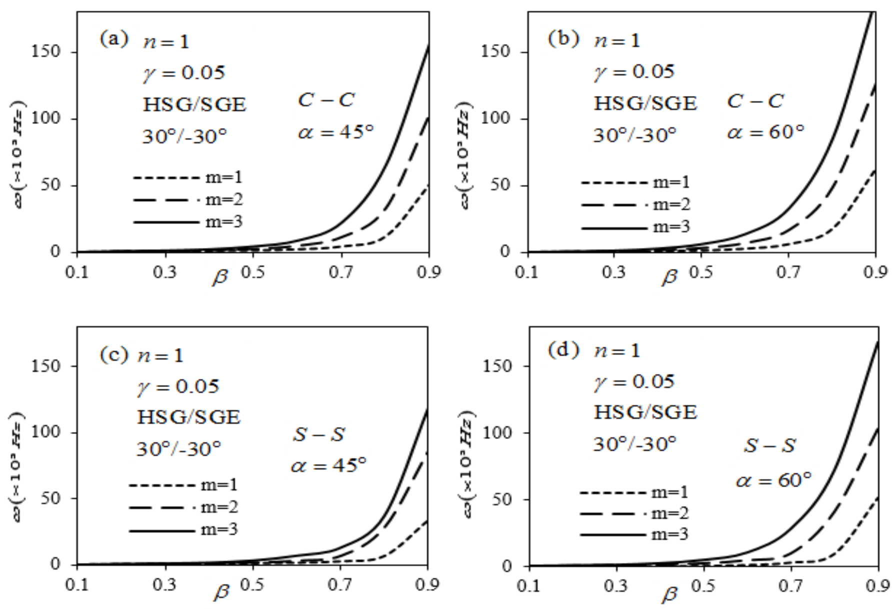

Figure 8. Variation of angular frequency ω with respect to length of cone β for $\alpha=45$ and $\alpha=60$ under C-C and S-S boundary conditions

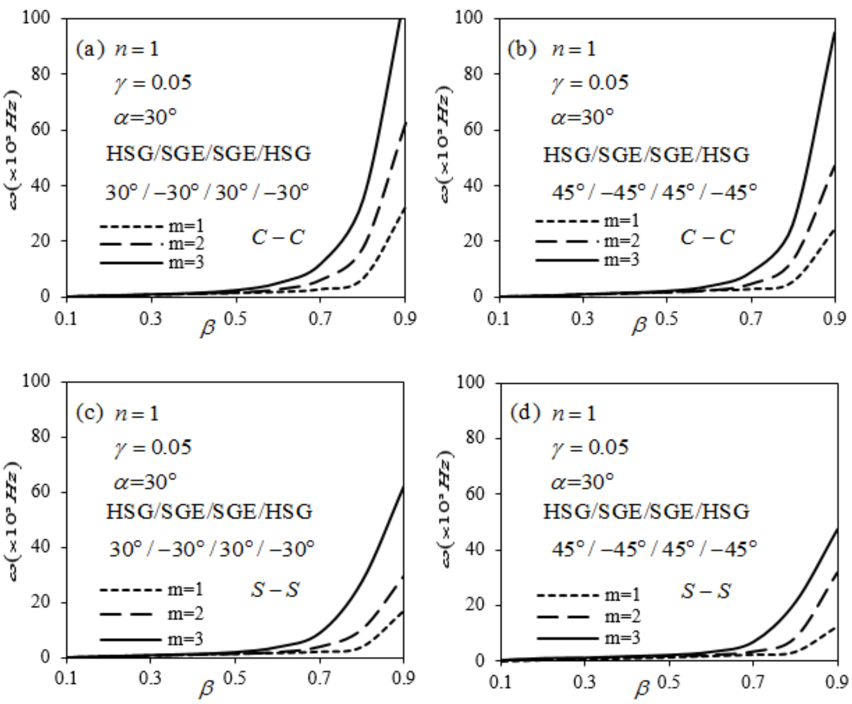

Figure 9. Variation of angular frequency ω with respect to length of cone β for four-layered conical shells under C-C and S-S boundary conditions

CONCLUSION

In this work, free vibration of antisymmetric angle-ply composite laminated conical shells under C-C and S-S boundary conditions are investigated. Spline method is used to approximate the solution into a set of algebraic equations which is then solved using eigensolution technique. From the results of the study, it is concluded that among two considered boundary conditions for conical shell, natural frequencies in C-C boundary conditions have higher value compared to S-S boundary conditions. In addition, geometric properties such as circumferential node number, cone angle, length ratio, ply orientation, number of layers and materials give appreciable effect to the frequency values of the conical shells. The frequencies are increased with the decrease of cone angle. Also, by increasing the length ratio, the frequencies are seen to decrease. The obtained results could serve as reference solutions for investigators and engineers.

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REFERENCES

[1] Irie, T., Yamada, G., and Kaneko, Y., 1982, " Free Vibration of Conical Shells with Variable Thickness", J. Sound Vib., 82(1), pp. 83-94.

[2] Irie, T., Yamada, G., and Tanaka, K., 1984, " Natural Frequencies of Truncated Conical Shells," ", J. Sound Vib., 92(3), pp. 457-453.

[3] Sankaranarayanan, N., Chandrasekaran, K., and Ramaiyan, G., 1988, " Free Vibrations of Laminated Conical Shells of Variable Thickness," J. Sound Vib., 123(2), pp. 357-371.

[4] Tong, L., 1993, " Free Vibration of Orthotropic Conical Shells," Int. J. Engng Sci, 31(5), pp. 719-733.

[5] Ng, T. Y., Li, H., and Lam, K. Y., 2003, " Generalized Differential Quadrature for Free Vibration of Rotating Composite Laminated Conical Shells with Various Boundary Conditions," Int. J. Mech. Sci, 43, pp. 567-587.

[6] Jankowski, J., and Kubiak, T., 2010, " Dynamic Response of the Truncated Conical Shell Subjected to Pressure Pulse Loading," Mech. Mech. Engng., 14(2), pp. 215-222.

[7] Ghasemi, F. A., Ansari, R., and Paskiaby, R. B., 2012, " Free Vibration Analysis of Truncated Conical Composite Shells using The Galerkin Method," J. Appl. Sci., 12(7), pp. 698-701.

[8] Xie, K., Chen, M., and Liu, Z., 2017, " An Analytic Method For Free And Forced Vibration Analysis of Stepped Conical Shells with Arbitrary Boundary Conditions," Thin-Walled Structures., 111, pp. 126-137.

[9] Souza, V. D., and Saraiva, J., 1995, " Analysis of Free Vibrations Of Conical Shells using Donnell's Approximations," Appl. Mech. Rev., 48(11), pp. 84-89.

[10] Jin, G., Ma, X., Shi, S., and Liu, Z., 2014, " A Modified Fourier Series Solution for Vibration Analysis of Truncated Conical Shells with General Boundary Conditions," Appl. Acoust., 85, pp. 82-96.

[11] Heydarpour, Y., Aghdam, M. M., and Malekzaech, P., 2014, " Free Vibration Analysis of Rotating Functionally Graded Carbon Nanotube-Reinforced Composite Truncated Conical Shells," Composite Structures., 117, pp. 187-200.

[12] Viswanathan, K. K., Navaneethakrishnan, P. V., 2014, " Free Vibration of Layered Truncated Conical Shell Frusta of Differently Varying Thickness by the Method of Collocation with Cubic and Quintic Splines," Int. J. Solids Struct., 42, pp. 1129-1150, 2005.

[13] Soedel, W., 2004, Vibration of Shells and Plates, West Lafayette, Indiana: Purdue University.