

Dynamic Modeling and Hybrid Control Design with Image Tracking for a Quadrotor UAV

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Abstract

The main purpose of the paper is to construct control laws for a quadcopter using different approaches. The first approach is the PID controller, that is very versatile and simple in comparison with other methods. On the other side, the LQR controller has robustness and good performance. Another important problem that considered in the paper is detection of moving objects using the camera installed on the quadcopter. The method of solution the problem is proposed. The results of its application are presented.

Keywords: control law, stability, tracking control

INTRODUCTION

In last years unmanned aerial vehicles (UAV) became an integral part of our lives. Due to the development of technologies, improvement of control systems and decreasing cost of modern microprocessors UAVs are easy available and procurable in the modern world. Quadcopters are the most frequent among unmanned aerial vehicles.

A quadcopter (or quadrotor helicopter, see Fig. 1) is a multirotor helicopter that has four rotors. Quadcopters are classified as rotorcraft, because their lift is generated by a set of rotors, namely vertically oriented propellers. Generally quadcopters has two pairs of identical fixed pitched propellers. Two of them are clockwise, another two are counterclockwise. To control the quadcopter we use independent variation of the speed of each rotor.



Figure 1: Quadcopter

The scope of their application is enormous: education, research, photography, journalism, sport, rescue operations and even military and law enforcement. This fact has a simple explanation: this model presents a very low moment of inertia and six degrees of freedom, which results in good stability of the quadcopter.

The most interesting problems related with quadcopters for scientists are investigation on kinematics and dynamics, questions on stability, guidance, navigation, trajectory

following and control.

In different literature there are presented various methods for quadcopter control. The main purpose of any controller is to stabilize the motion of the system and provide it effectively. Most commonly used approaches are PID (proportional–integral–derivative) controller [1], nonlinear controller [2], Kalman filter [3], feedback linearization [4], Lyapunov-based control [5], optimization-based control [6].

Work [3] describes the control based on the Kalman filter. This method, as mentioned in the work, gives good results in a quadcopter designed for indoor flight.

A comparison of Linear Quadratic Regulator and PID controller for a dynamic model of a quadcopter is given in [7–10]. In the study it is shown that both approaches give acceptable results for stabilization of the quadcopter motions. Other control strategies are considered in works [11–13].

Each of these approaches has some significant peculiarities. For example, Lyapunov function and corresponding stabilization controller [14] may be difficult to find in other specific cases. PID and LQR controllers are designed for linearized models, what limits the capability of such control scheme and requires for stay around the equilibrium point near which the linearization has been done.

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The first approach is the PID controller, that is very versatile and simple in comparison with other methods. On the other side, the LQR controller has robustness and good performance.

Another important problem that considered in the paper is detection of moving objects using the camera installed on the quadcopter. Detection of moving objects has become an important part of the solution to many applied problems in various fields, such as traffic monitoring, monitoring a large number of objects, autonomous navigation of robots and etc.

This paper discusses a method of determining moving objects and the direction of their movement on video with a moving camera. There are several approaches for detecting objects: methods, excluding the background, monitoring methods for singular points, methods of forming background, models with automatic movement. Algorithms that use a stationary camera to use the approach of separating the dynamic objects in the frame from the background frame. Based on the static points, that do not change their position on the image, you can determine the moving contours.

Adaptive background models are used because of their ability

to recognize changes in the frames, which are caused by illumination change in outdoor scenes or background changes due to camera motion. However, these methods are not effective with rapid scene changes and usually do not work correctly.

In the paper [15] a method that combines motion detection camera on the basis of specific points and comparison of optical flow to determine the points belonging to dynamic objects is proposed. It should be noted that the computation of the camera motion without any additional sensors. We will consider the algorithm, which is based on the algorithm [15] and allows to detect moving objects in the video image.

MATHEMATICAL MODEL OF THE QUADCOPTER

The quadcopter will operate in two coordinate systems. The first is the body reference frame, attached to the mass center of the quadcopter. The second is the inertial reference frame which is related to the ground. These frames are presented on the Fig. 2.

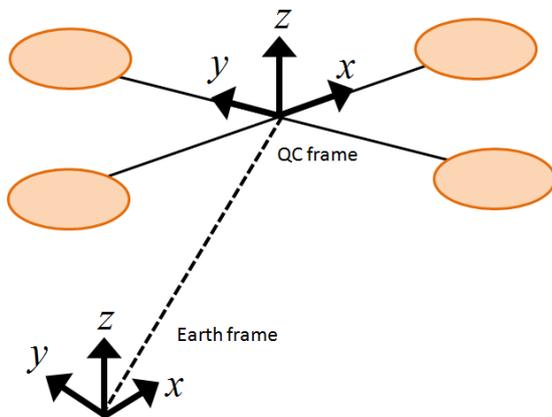


Figure 2: Earth-fixed frame and body-fixed frame

The quadcopter is basically consists of four propellers located orthogonally along the body frame.

Figure 3 shows the main parameters of the quadcopter: φ is roll angle, θ is pitch angle, ψ is yaw angle. Rotation of the UAV around the X axis is obtained when the speed of rotors 1 and 4 increases or decreases. Changing the roll angle we can obtain lateral acceleration. Rotation around the Y axis is obtained when the speed of rotors 2 and 3 increases or decreases. Changing the pitch angle we can obtain longitudinal acceleration. Rotation around the Z axis we can obtain by changing the speed of pairs if rotors 2, 3 and 1, 4 simultaneously.

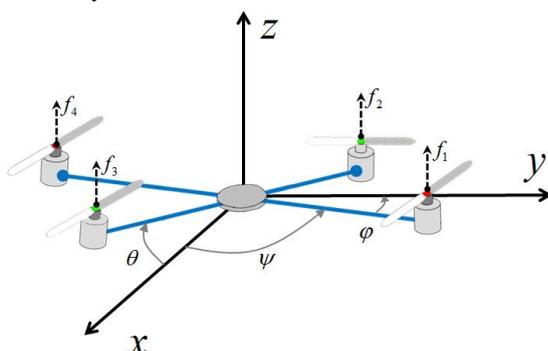


Figure 3: Quadcopter frames and vectors

Let us consider mathematical model of vertical motion of the quadcopter.

$$\begin{aligned} \dot{x} &= Ax + Bu, \\ y &= Cx, \end{aligned} \quad (1)$$

where x is a state vector, u is control, y is the output of the system, A , B and C are matrices with constant components. Quadcopter has 6 degrees of freedom, so matrix A is 6×6 matrix, B is the column 6×1 , C is the string 1×6 . Numerical values of constant matrices are shown below.

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0509 & 0.0509 & -0.0509 & 0.0509 \\ 0 & 0 & -5 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 & 0 & -5 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}.$$

Vector C is zero string, where value «1» is situated in the i -th place (i is the number of state, which is chosen as the output of the system).

The main purpose of the paper is to construct different control laws and to compare the results. Figure 4 presents the common scheme of integration the control law into the system.

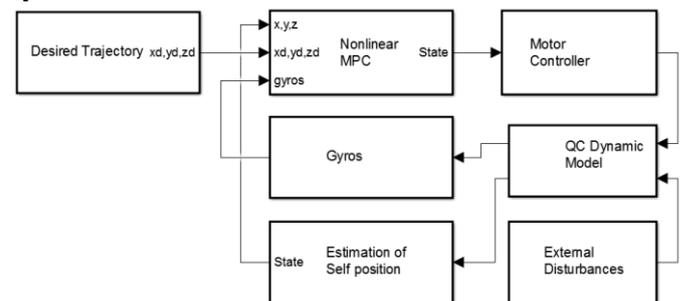


Figure 4: Control scheme

Linear Quadratic Regulator

For a continuous-time linear system (1) with a cost functional defined as

$$J = \int_0^{\infty} (x^T Q x + u^T R u) dt, \quad (2)$$

the feedback control law that minimizes the value of the cost is

$$u = -Kx, \quad (3)$$

where K is given by $K = R^{-1} B^T P$. Here P is found by solving the continuous-time algebraic Riccati equation

$$A^T P + PA - PBR^{-1}B^T P + Q = 0.$$

Let matrices Q and R take the following values:

$$Q = \begin{pmatrix} 10^9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad R = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}.$$

For numerical values Q and R the following vector of control coefficients was obtained

$$K = \begin{pmatrix} -5.8726 \cdot 10^4 \\ -2.4765 \cdot 10^4 \\ 2.1876 \\ -2.1876 \\ 2.1876 \\ -2.1876 \end{pmatrix}.$$

Using obtained control law the simulation of the vertical motion was performed. To examine the quality of the control law a step disturbance (e.g., wind) was used. As the result the following data were achieved: the transition time is four seconds, that is very good result. As we can see on the Fig. 5, there is no overshooting values, that make the transition process very smooth.

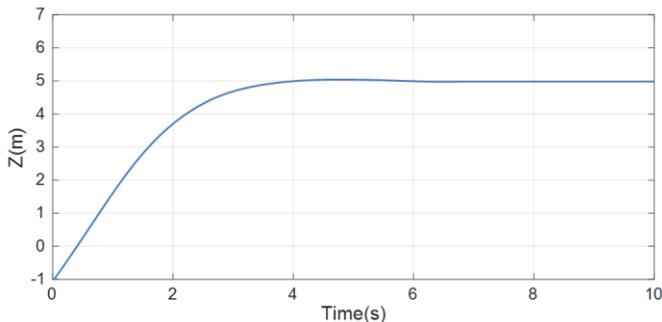


Figure 5: Step response for vertical motion

Figure 6 shows step response of the vertical speed using LQR controller.

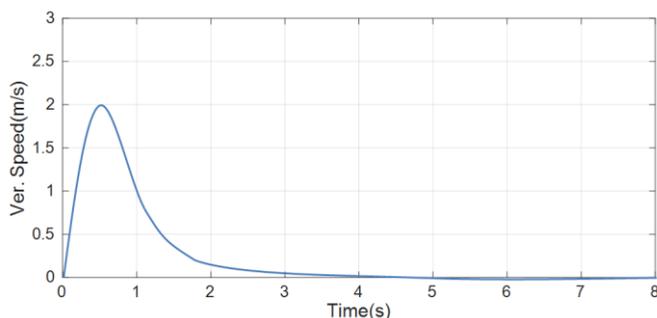


Figure 6: Step response for vertical speed

Similar results are obtained for other motions of the quadcopter.

PID controller

A PID controller continuously calculates an error value $e(t)$ as the difference between a desired setpoint and a measured

process variable and applies a correction based on proportional, integral, and derivative terms. Let us consider the vertical motion of the quadcopter. The transfer function of vertical motion equations is

$$H(s) = \frac{-0.0265}{s^2(s+5)}.$$

Transfer function of PID controller we will find in the form

$$H_u(s) = \frac{k_1 s^2 + k_2 s + k_3}{s}.$$

Transfer function of the closed-loop system is

$$H_{cl}(s) = \frac{k_1 s^2 + k_2 s + k_3}{s^3 + (5 + k_1) s^2 + k_2 s + k_3}.$$

Controller coefficients k_1, k_2 and k_3 are need to be found.

By equalling the coefficients in $H_{cl}(s)$ with optimal model coefficients we obtain k_1, k_2 and k_3 :

$$k_1 = 980, \quad k_2 = 10^4, \quad k_3 = 51.$$

Figure 7 shows the result of using the PID controller obtained above. As we can see, there is a little overshooting, but the transition time is less, then in system with LQR controller.

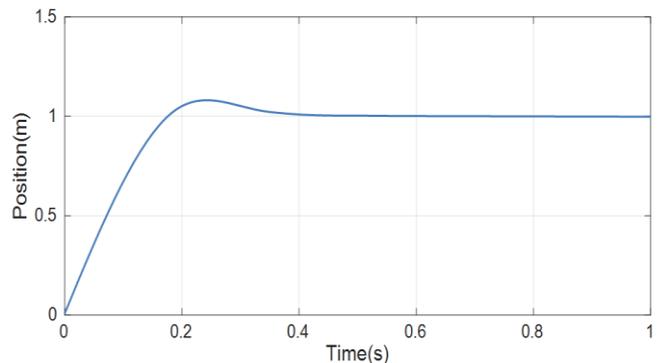


Figure 7: Step response for vertical speed

Figure 8 shows the result of using the PID controller obtained for vertical speed.

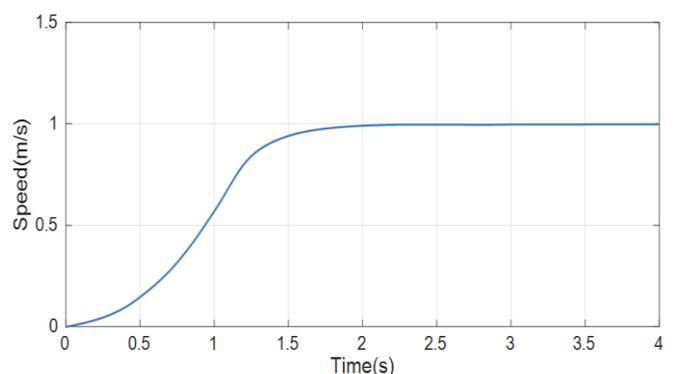


Figure 8: Step response for vertical speed

Similar results are obtained for other motions of the

quadcopter.

Detection of moving objects. Problem statement

One of the most important requirements for autonomous mobile systems, such as quadcopters, is visual scene understanding in outdoor environment. The detection and tracking of different moving obstacles like cars, bikes, pedestrians and etc. plays a significant role in safe navigation of quadcopters.

Suppose that we have a video containing imagery of the earth's surface, recorded with a camera rigidly fixed on a moving object, for example, the quadcopter. Analyzing frames of the video image, we need to determine a direction vector and speed of object motion with the camera, and if the frames are moving objects, determine the position and speed of moving objects. It is assumed that the plane of flight of a moving object parallel to the plane of the earth, and the camera lens is situated so that the vector, coming out of it, is perpendicular to the plane of flight and directed to the ground plane.

Detection of moving objects. Problem solution

The main idea of the method proposed consists in the comparison of artificial optical flow based on the movement of the camera, with real optical flow, and tracking variations in them.

Let us consider the main stages of the algorithm:

- 1) The calculation of the actual optical flow.
- 2) The calculation of the artificial optical flow.
- 3) Identification of dynamic points and filter them.

Let us study these stages more detailed.

First stage. The calculation of the actual optical flow is implemented by the algorithm Lucas – Canada [16]. The result of this algorithm is the optical flow, we present, in turn, a set of vectors. This set shows how to change the position of the point in the current frame relative to the previous one.

Second stage. Artificial flow is represented by the position of all points received for a valid optical flow is mathematically projected onto the next frame, given the motion of the camera. In other words, calculating an artificial optical flow, we can determine the vector direction of the camera. The camera motion is represented by a rotation and offset, which are the result of the algorithm PTAM [17]. To produce artificial flow uses a homographic projection. Usually homography projects the position of the point from the plane of one camera into the plane of the other. In our case, several assumptions to simplify and accelerate the process of projection were made. The main assumption is that the Earth's surface has a significant slope, i.e., the average slope can be considered close to zero. Small changes in topography of the order of 40 – 50 cm will not affect the efficiency of the algorithm.

Third stage. In this problem, as a representation of the optical flow are used sets of vectors. It is worth noting that the vast majority of vectors artificial optical flow are mutually parallel. Summing them we can get the resultant vector which will show the direction of the camera movement in the scene.

The dynamic nature of points becomes clear in the comparison of the above optical flows. Dynamic objects are identified by a set of valid vectors of the optical flow having the different from the motion vector of the camera direction and/or magnitude. To continue calculations you need to group

the vectors in the image. It may happen that some of the vectors will be isolated relative to the other. Often they can be caused by uncontrolled variations or other errors. In general case, these vectors are not taken into account. The group of vectors that are in a certain neighbourhood of each other is taken for a dynamic object.

It is proposed to abandon the use of the PTAM algorithm because of its complexity and resource-intensive. The task which was undertaken by the PTAM algorithm, will decide the method of [18]. The MATLAB environment was selected as the instrument for the implementation of the algorithm. The reason for choosing this environment was the presence of libraries written for most popular processors. Library use instructions and other instruments for the processor to work efficiently with matrix calculations.

In the primary implementation of the decision of tasks of detection of moving objects was used in pyramidal algorithm of Lucas – Canada. The speed of the algorithm depends on the size of the processed frames: the larger the frame the longer it is processed. On the other hand, the better the frame and the more time it measures, the more accurate will be determined by optical flow. Therefore, depending on the specific tasks and the available computing power you need to choose quality and speed calculations will be made.

Cause inaccurate operation of the algorithm Lucas – Canada can serve various noises on the image. The appearance of noise depends on the camera making shooting. The algorithm Lucas – Canada before processing the image, for optimality it is often discolor.

Because of the peculiarities of the algorithm, before working with image, for best performance, to it apply anti-aliasing. Approved the application of the smoothing Gaussian and median filter. However, these tools are not appreciably affected the quality of results.

Instead of the pyramidal algorithm of Lucas – Canada is more efficient use of an iterative algorithm Lucas –Canada. The main difference is that when working with the current frame, it uses the information accumulated during the period from the previous frames. This type of algorithm works several times faster and better.

The results of implementation of the method is shown in the Fig. 9.



Figure 9: Example of output of the method

CONCLUSION

Considering the results obtained in the paper it is obviously that the plant was controlled with two different control methods presenting satisfactory results. Both of these

controllers gave good response of the system. It is widely known that the LQR controllers are robust and produce a low steady state error. The main disadvantage of LQR controllers is big transition time. From the other side, a PID controller gives a very fast response but unlike the LQR controller a PID controller do not have robustness. These controllers are also can be used in the solution of other problems [19 – 26]. Additionally in the paper a method of detection of moving objects using onboard quadcopter camera is proposed. The results of its application are presented.

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