International Journal of Applied Engineering Research ISSN 0973-4562 Volume 12, Number 17 (2017) pp. 6391-6398

Performance Analysis of Reed Solomon Code for various Modulation Schemes over AWGN Channel

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Abstract

The main objective of this paper is to analyse the performance of Reed-Solomon code for different digital modulation schemes and with different (n,k) combinations over a Additive White Guassian Noise (AWGN) channel. The simulation model provides error detection and correction using Reed-Solomon (RS) Code. The Original message is encoded and decoded using RS Code. MATLAB Simulink model is selected as the investigating tool. The performance of proposed GMSK and RS code combination for different code rates and different BT product is compared with un coded system with the constraint that the transmission bandwidth is constant .It has been observed that varying BT values in GMSK modulation for fixed (n,k) combination produces no significant effect over BER The results are presented by a plot between the bit error rate (BER) and signal to noise ratio .The results show that for a given bandwidth, it is beneficial to use GMSK modulation scheme with (63,k**)** code rate over other modulation schemes so as to obtain least BER for different signal to noise values. Same time it has been observed that performance of GMSK Modulation lies in between other modulation schemes for other (n,k) combinations. The BER performance also improves by decreasing code rate and by taking large block lengths or by increasing redundancy.

Keywords: AWGN, BER (Bit Error Rate),GMSK, QPSK , FSK, DPSK ,PSK, Matlab , RS Codes ,Galois field.

INTRODUCTION

To achieve consistent and reliable data from the information source to the destination is one of the main issue in communication system. The main objective of any communication system is transmission of data with the minimum error rate no matter whether it is digital or analog. The use of channel codes or Forward Error Correcting Codes in digital communication system is an integral part of ensuring reliable communication[1] even in the presence of noise. There are various ways to counter noise effect like making use of highly directional Antennas ,using Forward Error Correcting Codes & Spread Spectrum Communication. One Forward Error Correcting Code is required which is more suitable to work against Burst noise. Although there are various codes which works efficiently good for random errors but for Burst noise or Burst Error Reed Solomon Code is the best.

Reed Solomon are non binary code which is widely used in wireless communication, compact disc players and computers memories. Reed Solomon Codes are effective for deep fade channel and are considered as a structured sequence that is most widely used in Burst Error Control.

The main objective of this paper is to evaluate the performance of Reed-Solomon codes in error correction control system in term of bit error rate (BER). In proposed communication system the signal is transmitted using GMSK modulation technique in the presence of Additive White Gaussian Noise (AWGN). In GMSK, which is a subclass of continous phase modulation the digital data stream is first shaped with Gaussian filter before being applied to MSK modulator .By using Guassian filter, sideband power get reduced which in turn yields excellent performance in the presence of Inter Channel interference (ICI). Compressing the bandwidth although avoids ICI but it causes an expansion in time domain which results in Inter Symbol Interference(ISI). In order to remove ISI effect Equalizers are required at the receiver end.

GMSK modulation method, first proposed by K.Murota and K.Hirade [4], is a widely used modulaion scheme of cellular system due to its compact Power Spectral Density and excellent error performance. Although the performance of

GMSK has been analyzed by several researchers, coding for GMSK has received little attention[5].

This paper focuses on GMSK and Reed-Solomon (RS) coding. Error control codes insert redundancy into the transmitted data stream so that the receiver can correct errors that occur during transmission. Therefore, the bit interval of the coded bits is selected shorter in order to keep the information transmission rate constant. A shorter bit interval results in a larger transmission bandwidth. To remain the bandwidth of the coded system same as that of the uncoded system, the modulator used in the coded system must adopt a smaller value of B_b[3].

This paper is organized as follows. The next section i.e. section 2 gives an overview of the system including a description of GMSK modulation and RS codes. Section 3 gives our approach to the bandwidth allocation problem. Simulation results for GMSK modulation with different Reed-Solomon coding rates for maintaining same bandwidth are presented in Section 4 and performance of RS code based system with other modulation schemes is analysed , and the conclusions are given in Section 5.

SYSTEM OVERVIEW

RS/GMSK system model and Reed-Solomon coding system shown in Figure 1. The performance of various combinations of GMSK and RS codes is evaluated with the constraint that the total system bandwidth is constant. The bandwidth of GMSK canbe easily controlled by the parameter B_b. The uncoded system is also evaluated to serve as a benchmark.

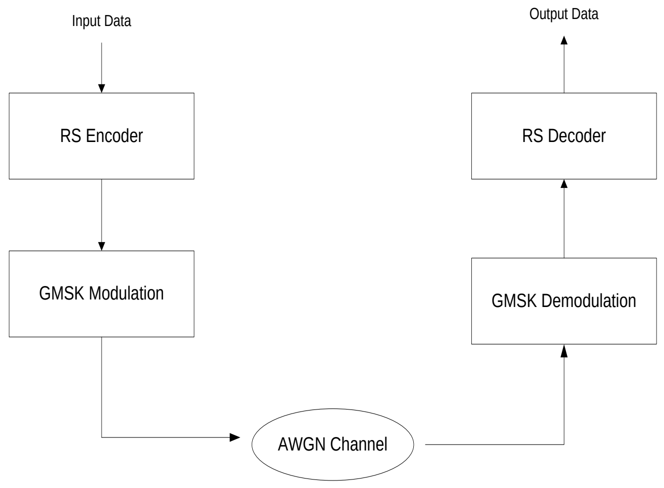

Figure 1: RS/GMSK Model

GMSK Modulation

GMSK, as its name suggests, is based on MSK and was developed to improve the spectral properties ofMSK by using a premodulation Gaussian filter. The filter impulse response is expressed as:

$$h(t) = \frac{1}{\sqrt{2\pi\sigma T}} exp\left(\frac{-t^2}{2\sigma^2 T^2}\right) \ldots\ldots(1)$$

$$where\ \sigma = \frac{\sqrt{ln2}}{2\pi B_b T}$$

The Gaussian filter is characterized by its B_bT product (B_b is the -3dB bandwidth of the Gaussian prefilter and T is the symbol period.) The lower the B_bT product, the narrower the modulation bandwidth. In this paper, we use $B_bT = 1.0$ and $B_bT = 0.5$ for the uncoded system along with other equivqlent B_bT values for coded system to maintain same required bandwidth . For transmission in an AWGN channel, the bit error rate of GMSK is given by

$$P = \frac{1}{2}\, erfc\left(\sqrt{\frac{d_{min}^2 E}{2N_0}}\right) \ldots\ldots\ldots(2)$$

Where d_{min} is the normalized minimum Euclidean distance between the signal representing “0” and the signal representing “1”, E is the energy per transmitted bit and $N_0/2$ is the power spectral density of the AWGN.

Reed-Solomon codes

Reed-Solomon codes are block-based error correcting codes with a wide range of applications in digital communications and storage. It is vulnerable to the random errors but strong to burst errors. Hence, it has good performance in fading channel which have more burst errors. In coding theory Reed–Solomon (RS) codes are cyclic error correcting codes invented by Irving S.Reed and Gustave Solomon [6]. They described a systematic way of building codes that could detect and correct multiple random symbol errors. By adding t check symbols to the data, an RS code can detect any combination of up to t erroneous symbols, and correct up to [t/2] symbols. As an erasure code, it can correct up to t known erasures, or it can detect and correct combinations of errors and erasures. Reed-Solomon codes are used to correct errors in many systems including:

- Storage devices (including tape, Compact Disk, DVD, barcodes, etc)
- Wireless or mobile communications (including cellular telephones, microwave links, etc)
- Satellite communications
- Digital television / DVB

- High-speed modems such as ADSL, xDSL, etc.

Block coding schemes involve dividing the input data into k-bit blocks and then mapping each k bit block into an n-bit block called a codeword, where n >k in the encoding process. (n-k) check bit blocks are added to each k-bit block. The ratio

r = k/n is called the code rate. The data is partitioned into symbols of m bits, and each symbol is processed as one unit both by the encoder and decoder. RS codes satisfy: $n \le 2^{m-1}$ and $n - k \ge 2t$, where t is the number of correctable symbol errors. Reed Solomon codes are polynomial codes over certain finite fields particularly useful in Burst error correction. Encoding & Decoding principles of nonbinary RS codes depends on Galois fields (GF).Symbols from extension galois field (2^n) are used in constructing RS codes.GF(2^n) is extension galois field with 2^n elements.Let β be a primitive element in GF(2^n)& G(Z) be the Generator polynomial with roots ($\beta,\beta^2,\beta^3,\ldots\ldots\beta^{N-M}$). Then $G(Z) = \sum_{i=1}^{N-M}(Z - \beta^i)$..........(3)

Let ($m_1,m_2,\ldots\ldots m_M$) be the message symbols where $m_i \epsilon GF(2^n)$ which is defined by a polynomial

$P(Z) = m_1+m_2Z+m_3Z^2+\ldots+m_MZ^{M-1}$.......(4)

&Hence Codeword Polynomial is

$C(Z) = P(Z)G(Z)$ ....(5)

If during transmission some additive errors are introduced due to noise which is described by error polynomial $e(Z) = \sum_{j=0}^{n-1} e_i Z^i$ ... .(6) then received polynomial becomes $R(Z) = C(Z) + e(Z)$.......(7)

Various algebric Decoding methods like Peterson-Gorenstein-Zierler(PGZ),Berlekamp- Massey Algorithm(BMA) and Euclidean method of Sugiyama are used for RS codes which are based on the idea of determining error location and error correction. Decoding algorithm for t error correcting RS codes is based on considering error polynomial(Z) which is

$e(Z) = e_{n-1}Z^{n-1}+e_{n-2}Z^{n-2}+\ldots e_1Z+e_0$..........(8)

here v is total errors that actually occurs & t is error correcting capability of RS codes.Let these errors occur at locations i_1, i_2, $i_3,\ldots i_v$. The error polynomial can then be written as

$e(Z) = e_{i_1}Z^{i_1} + e_{i_2}Z^{i_2} + \cdots e_{i_v}Z^{i^v}$ ....(9)

here e_{i_k} is the magnitude of k^{th} error.For error correction we must know two things error locations & magnitude of these errors. Thus, the unknowns are i_1, i_2, $i_3,\ldots i_v$&, $e_{i_1}\, e_{i_2},\, e_{i_v}$, which signify the locations & the magnitudes of the errors

respectively.The syndrome can be obtained by evaluating the

received polynomial at α

$S_1 = v(\alpha) = c(\alpha) + e(\alpha) = e(\alpha) = e_{i_1}Z^{i_1} + e_{i_2}Z^{i_2} + \cdots e_{i_v}Z^{i_v}$

..........(10)

If error magnitudes are defined as $Y_{k=}\, e_{i_k}$ for $k=1,2,\ldots v$ & error locations are $Z_{k=}\ \propto^{i_k}$ for $k=1,2,\ldots v$,where i_k is the location of k^{th} error & Z_k is the field element associated with this location then Syndrome can be written as $S_1=Y_1Z_1+Y_2Z_2+\ldots Y_vZ_v$............(11)

We can evaluate the received polynomial at each of the powers of α ,thus we have following set of 2t equations with v unknown error locations $Z_1,Z_2,\ldots,Z_v$ & the v unknown error magnitudes $Y_1,Y_2,\ldots,Y_v$,

$S_{1=}Y_1Z_1 + Y_2Z_2 + \cdots + Y_vZ_v$ ....(12)

$S_2 = Y_1Z_1^2 + Y_2Z_2^2 + \cdots + Y_vZ_v^2$ ....(13)

:

$S_{2t} = Y_1Z_1^{2t} + Y_2Z_2^{2t} + \cdots Y_vZ_v^{2t}$ ....(14)

If the error locator polynomial is defined as

$A(z) = A_vz^v + A_{v-1}z^{v-1} + \cdots A_1z + 1$ ....(15)

Then zeros of this polynomial are the inverse error locations $Z_k^{-1}$for $k=1,2,\ldots,v$ i.e.

$A(z) = (1 - zZ_1)(1 - zZ_2)\ldots(1 - zZ_v)$......(16)

So,if we know the coefficients of the error locator polynomial A(z),we can obtain the error locations $Z_1,Z_2,\ldots.Z_v$.Since error locations are now known these form a set of 2t linear equations these can be solved to obtain error magnitudes.

Performance Evaluation

In order to determine what combination of coding rate and B_b for the coded system results in the same bandwidth as the uncoded system, we must choose the measure of bandwidth.

International Journal of Applied Engineering Research ISSN 0973-4562 Volume 12, Number 17 (2017) pp. 6391-6398

In this paper, we have used the percent power containment bandwidth, denoted by B_x and defined as the bandwidth which contains x% of the signal power. B_{90}, B_{99} and $B_{99:9}$ are plotted in Figure 2.

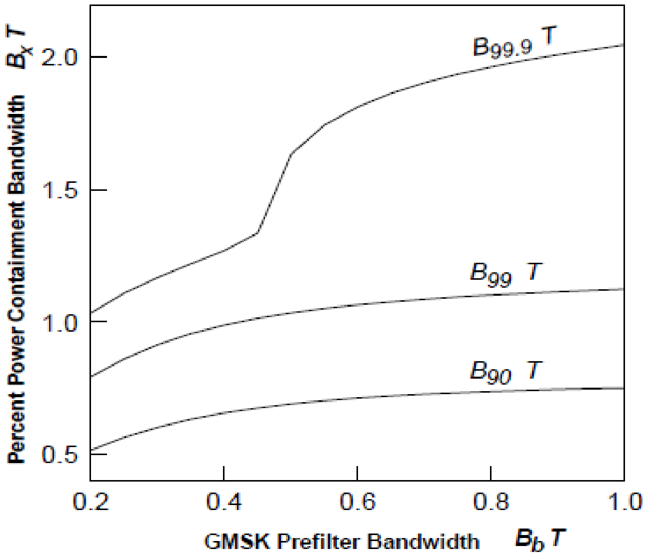

Figure 2: Percent power containment bandwidths for GMSK

$B_{99:9}$ for GMSK, which is the bandwidth that contains 99:9% of the signal power is used in the simulations. In Equation (2) for the uncoded system, the value of E is E_b, which is the energy per transmitted information bit. For the coded system, the value of E is set to be rE_b, since the energy for the coded bits is spread among the more numerous coded bits. This allows a fair comparison to be made between the uncoded and coded systems.

It is complicated to compute the bit error probability p_b by using Equation (2) and (3) because RS codes are non binary codes, so we use MATLAB. The simulation model is shown in Figure 3.

In the simulations the following parameters are used.

- Input data 100000 symbols
- RS codeword length: 31,63, 127,255

SIMULATION PARAMETERS

Item	Value
Channel Model	AWGN
Modulation	GMSK
Channel Coding	Reed Solomon
Codeword Length	255, 127,63,31
Data Rate	1Mbps
Frequency	850 MHz

B_BT AND RS CODE PARAMETER COMBINATIONS THAT RESULT IN SYSTEMS WITH EQUAL BANDWIDTHS

n=31				
Uncoded	**Coded**			
	k=25	**k=23**	**k=21**	**k=19**
0.5	0.45	0.35	0.25	0.18
1.0	0.51	0.48	0.45	0.42

n=63				
Uncoded	**Coded**			
	k=57	**k=53**	**k=49**	**k=41**
0.5	0.47	0.46	0.40	0.22
1.0	0.64	0.54	0.49	0.45

n=127					
Uncoded	**Coded**				
	k=107	**k=103**	**k=99**	**k=87**	**K=79**
0.5	0.47	0.45	0.40	0.26	0.17
1.0	0.54	0.51	0.49	0.46	0.4

n=255				
Uncoded	**Coded**			
	k=239	**k=215**	**k=203**	**k=179**
0.5	0.48	0.4	0.3	0.22
1.0	0.77	0.54	0.52	0.5

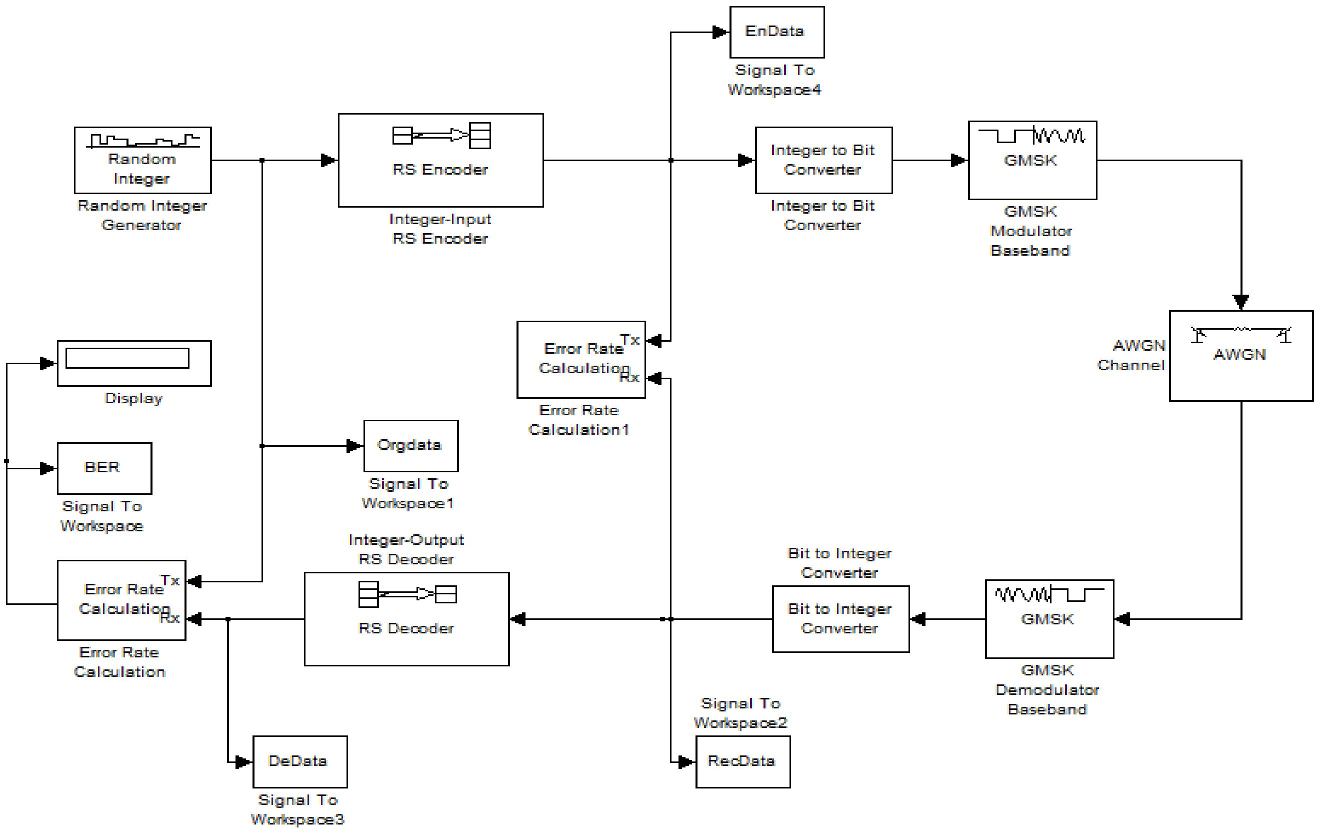

Figure 3: Proposed Simulation Model

RESULT & ANALYSIS

In this section, the parameters used in the simulation model are discussed. Simulation results are also presented. Proposed Simulation is used to evaluate the BER (Bit Error Rates) for different systems. The code parameters used in this simulation are RS (31,K) , RS (63, k) ,RS(127,K) , RS(255,K) for various modulation schemes . To ensure the bandwidths of the coded and un coded systems remain the same, first of all we have to calculate the value of B_bT when modulation scheme used is

GMSK.. Let us see how to calculate the value of B_bT, if we set the value of B_bT = X for the un coded system, then $B_{99:9}T$ = Y from Figure 2. When the RS coding rate is r, the coded $B_{99:9}T$ = Z is calculated from Z = Y × r. The corresponding value of B_bT is found from Figure 2. The parameters used in the simulations are shown in Table I and Table II.

The simulation results are shown in Figures 4–11. From Figure 4&5, it is clear that even if there are variations in the value of B_bT for a particular (n,k) combination BER values are still the same for given Eb/No and this is valid for all RS GMSK (n,k) combinations that shows increasing B_bT amount only reduces overshoots in Guassian filter but produces no significant improvement on BER. From fig 4&5 it is also clear that uncoded system gives lesser BER values for Eb/No upto 5dB but there is a marked reduction in BER values for Eb/No greater than 5dB for RS GMSK (63,k) system.From figures 4 to 8 it is very clear that (63,k) is best among all combinations of RS GMSK from BER point of view

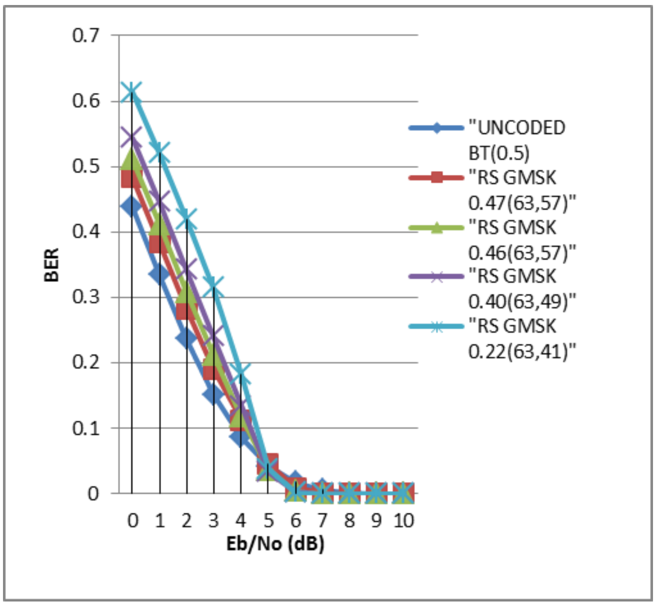

Figure 4: Performance for RS-GMSK(63:k) code combinations with the same bandwidth as an uncodedsystemwithB_bT = 0.5

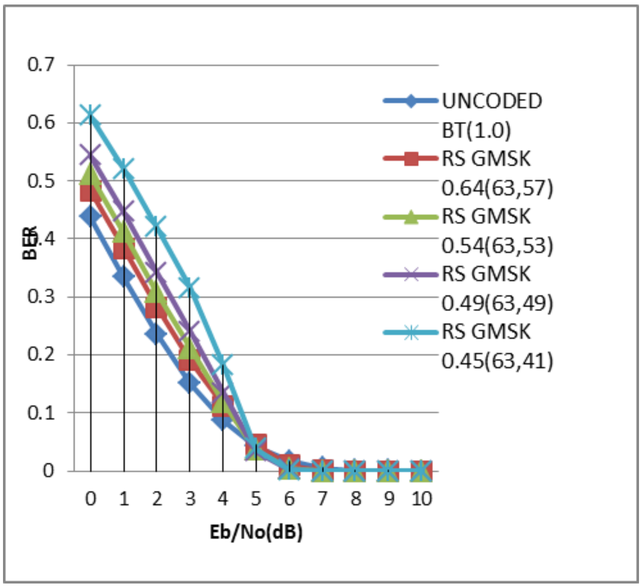

Figure 5: Performance for RS-GMSK(63;k) code combinations with the same bandwidth as an uncodedsystem with B_bT = 1.0

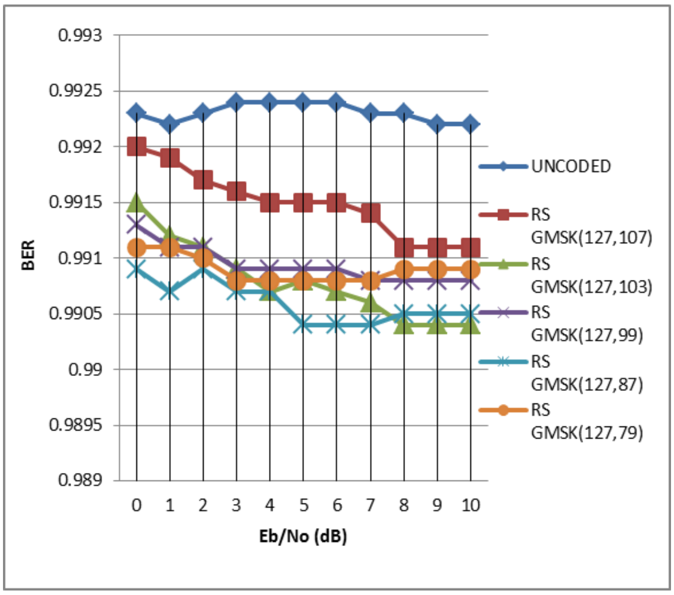

Figure 7: Performance for RS-GMSK(127:k) code combinations with the same bandwidth as an uncodedsystem with B_bT = K

GMSK-RS(31,K) ,GMSK-RS(127,K) ,GMSK-RS(255,K) code combinations has higher amounts of BER as compared to GMSK-RS(63:k) code combination which shows that it is the best because BER is least for it.

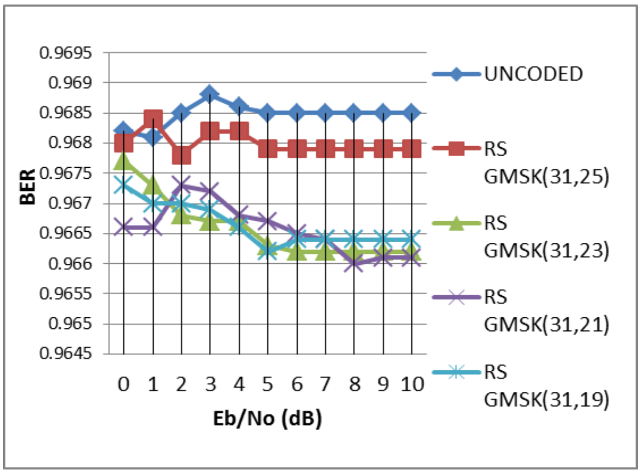

Figure 6: Performance for RS-GMSK(31;k) code combinations with the same bandwidth as an uncodedsystem with B_bT = K

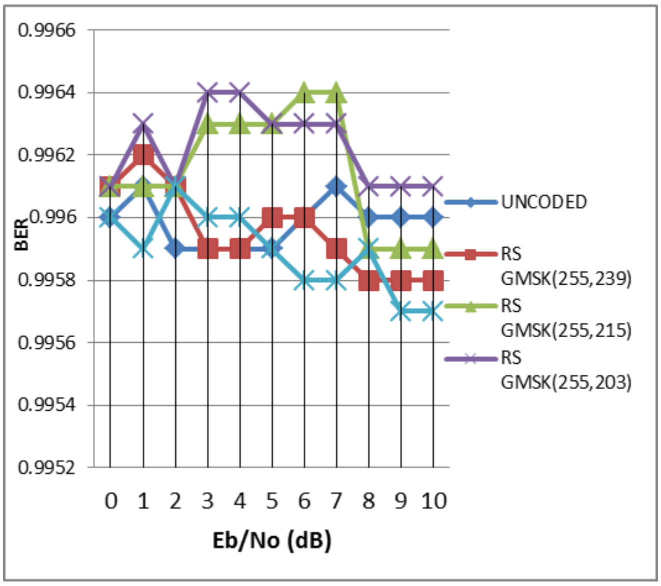

Figure 8: Performance for RS-GMSK(255:k) code combinations with the same bandwidth as an uncodedsystem with B_bT = K

Along with the use of RS Code with GMSK modulation scheme it is also combined with other modulation schemes like PSK,DPSK,QPSK and FSK and it has been observed thatRS GMSK combination is best for (63,K) and (127,k) code combinations but RS GMSK combination is worse than others for (31,k) and (255,k) code combinations which makes it clear that it is better to prefer GMSK modulation over PSK,DPSK,QPSK &FSK modulation methods for moderate values of " n" nor for too high nor too low as visible in fig.9 to fig.12.

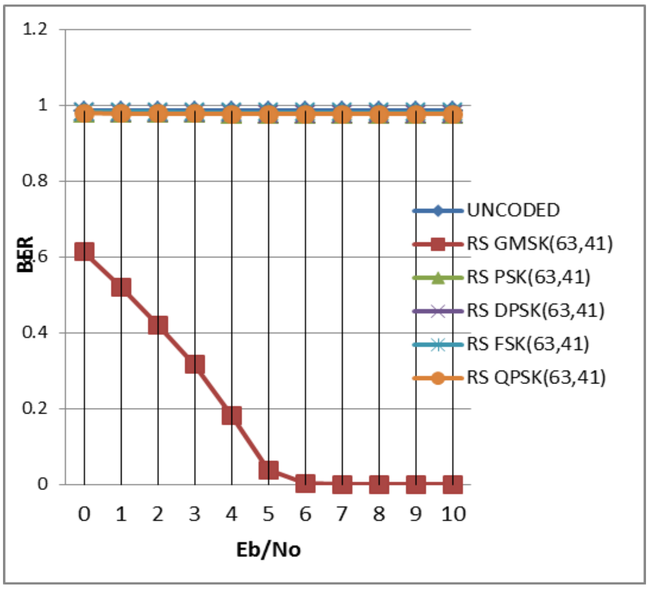

Figure 9: Performance of RS(63:k) code combinations with various Modulation schemes

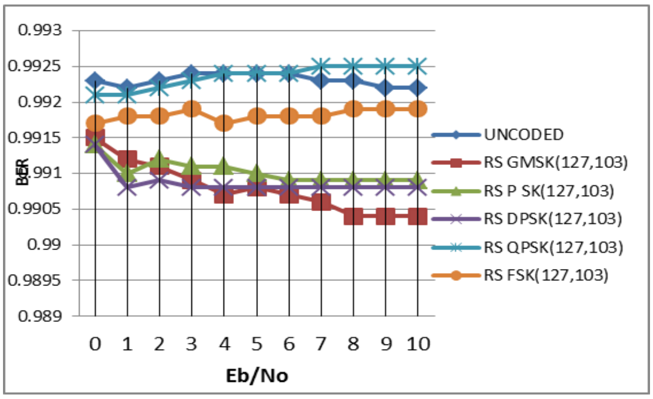

Figure 10: Performance of RS(127:k) code combinations with various Modulation schemes

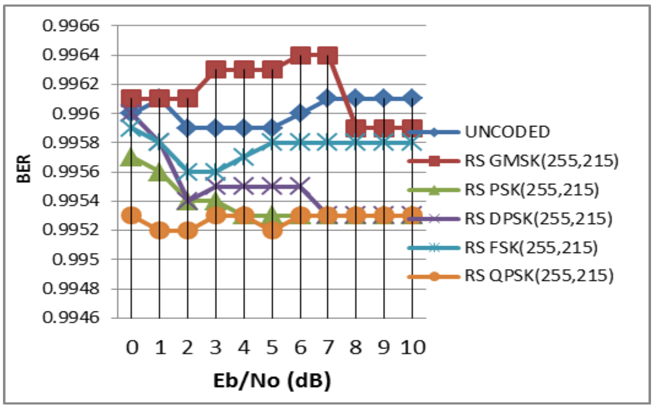

Figure 11: Performance of RS(255:k) code combinations with various Modulation schemes

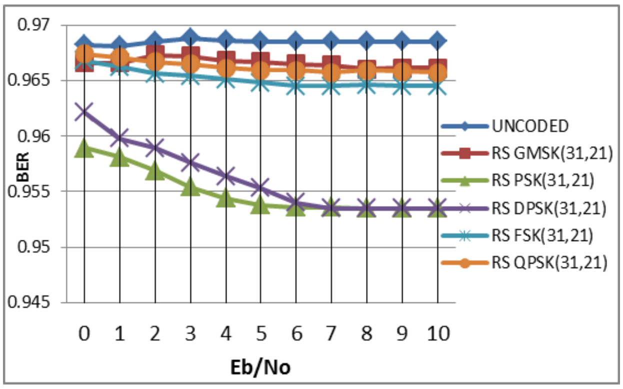

Figure 12: Performance of RS(255:k) code combinations with various Modulation schemes

RS Code with one particular modulation scheme with constant code rate is analysed for various (n,k) combinations and it has been observed that for a constant code rate in all modulation schemes (31,21) is the best because it gives least BER. For a constant code rate(k/n) in various (n,k) combinations RS code is analysed with various modulation schemes and it is observed that (31,21) combination is best no matter what the modulation scheme is.

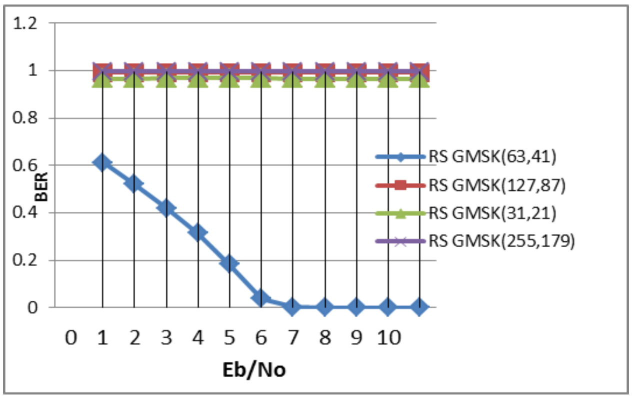

Figure 13: Performance of RS GMSK (n:k) code combinations with constant code rate .

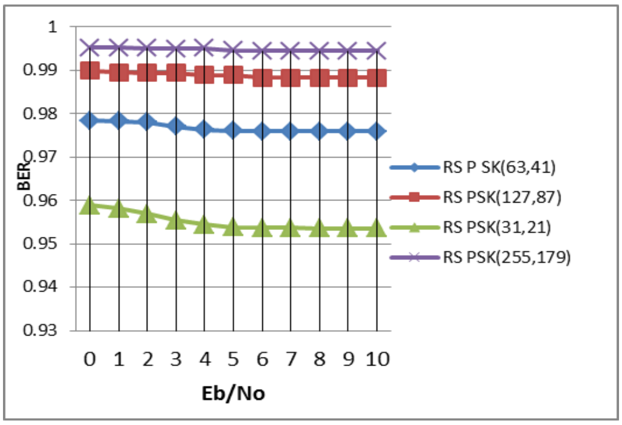

Figure 14: Performance of RS PSK (n:k) code combinations with constant code rate .

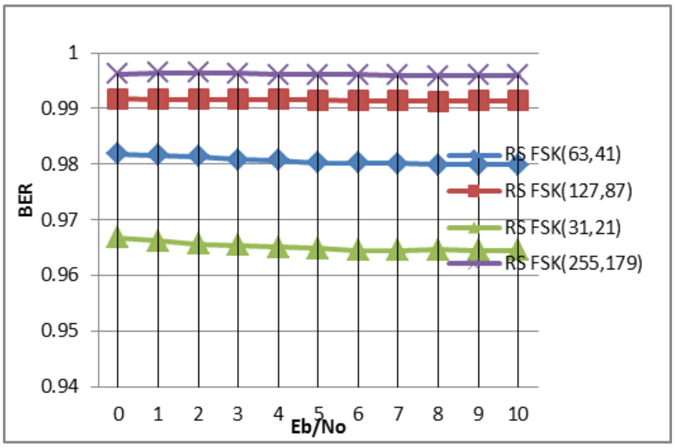

Figure 15: Performance of RS FSK(n:k) code combinations with constant code rate .

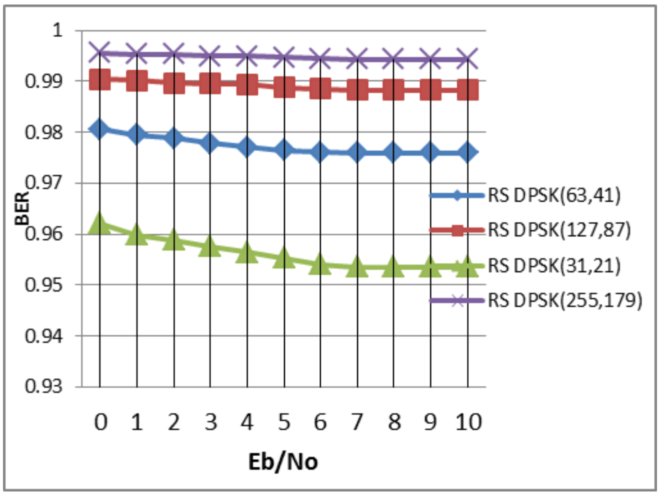

Figure 16: Performance of RS DPSK (n:k) code combinations with constant code rate .

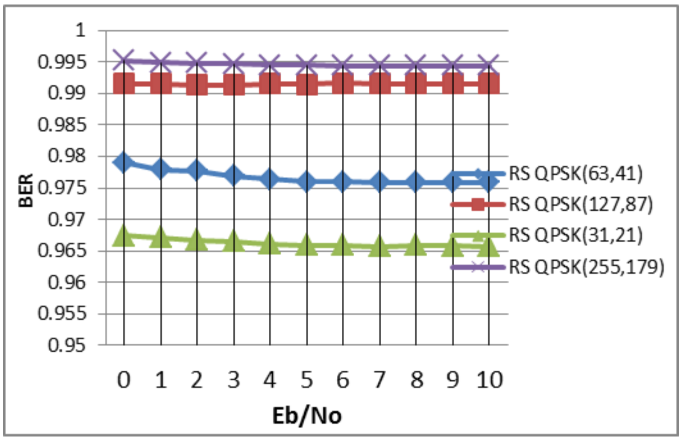

Figure 17: Performance of RS QPSK (n:k) code combinations with constant code rate .

CONCLUSION

In this paper, we examined the performance of communication system over GMSK modulation which employ RS channel coding. Under a constant bandwidth constraint, we optimized the combination of coding a RS (n,k) combinations and modulation.

The Proposed system results show that for same bandwidth as un coded system, a coded system bit error rate probability performance can be improved for a given bandwidth by taking smaller code rate. For all RS GMSK (n,k) combinations varying B_bT product produces no significant improvement on BER. (63,k) is best among all combinations of RS GMSK from BER point of view. It is better to prefer GMSK modulation over PSK,DPSK,QPSK &FSK modulation methods for moderate values of " n" nor for too high nor too low.At a constant code rate, over other (n,k) combinatios (31,21) combination is best no matter what the modulation scheme is. We also performed the simulations for different code rates and different block length with fixed no of error correction capabilities and result shows that the BER performance can also be improved by decreasing code rate but for large block lengths. In the proposed technique MATLAB simulation is selected as the investigating tool.

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