

A Review on Media Clogging in Attached Growth System

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Abstract

Attached growth system is widely used nowadays as a biological wastewater treatment which used a support medium as a physical bed for biomass growth such as wood, gravel, sand, rock and wide range of synthetic plastic material. However, limitation in attached growth system is rarely been discussed in details. Based from the review, it shows that the major drawback of attached growth system is media clogging. Bio film thickness, slow water flow, selection of media (media type, media size and surface area, and media void), lack of aeration, lack of backwashing and mechanical failure has been reported caused media clogging. This paper presents a review of one of the problems encountered with attached growth system which is media clogging in approaches to enhance the performance of the biological process in wastewater treatment.

Keywords: Attached growth; clogging; media; reactor; wastewater treatment

INTRODUCTION

Advanced attached growth process to treat wastewater has been in practice in recent years. It has been considered as an attractive option for wastewater treatment. Biofilm reactors are alternatives for efficient and good performance treatment plant designs than conventional wastewater treatment systems [1]. The main advantage of attached growth treatment systems is that they sustain a high concentration of microorganisms which is resulting in high removal rates at minimize hydraulic retention times [2]. Attached growth system has been tested with different type of wastewater for treatment such as municipal [3], industrial [4], domestic [2], and agricultural [5]. In the attached growth treatment process, microorganisms are attached on a bio media surface which is forming a biofilms. Substrates in the wastewater are seeped into the biofilm and gradually degraded by the microorganisms [6]. Bio media is a physical bed for biomass growth. There are few types of bio media that can be used such as rocks, glass, plastic, and wood.

Various investigations have been determines the efficiency of the attached growth system to remove organic in wastewater treatment. However, limitation in attached growth system is rarely been discussed in details due to media clogging. Thus, it is important to overcome the limitations in order to improve the efficiency of attached growth system. This paper presents a review of one of the problems encountered with attached growth system which is media clogging in approaches to enhance the performance of the biological process in wastewater treatment.

MEDIA CLOGGING

Numerous investigations have been done and common problems of attached growth system has been reported which led to a conclusion that the key factor of the treatment system limitation is media clogging. Bio film thickness, slow water flow, media selection (media type, media size and surface area and media void), lack of aeration, lack of backwashing and mechanical failure are most of the factors that contribute to media clogging. The consequences of media clogging are excess head loss, short circuiting and frequently of backwashing is increasing [7]. Media clogging also caused malodor problem due to anaerobic condition [1]. Even though there are some theories proposed to clarify the limitation factors of attached growth system, but verification of required experimental to prove the theories proposed is still not sufficient [8].

BIOFILM THICKNESS

Attached growth system require a great number of microbes on the bio media bed which create a biofilm is crucial aspect of wastewater treatment system's performance [9]. Flow rate and nutrient concentrations are the key factors that can affect the bio film thickness [10]. Surface characteristics of surface area, porosity, and surface roughness are other factors that influence the biofilm thickness [11]. Ødegaard et al., 1999 stated that since the full depth of the biofilm which the substrates have penetrated is normally less than 100 µm, the perfect biofilm is thin and uniformly distributed on the surface of bio media [12]. It is important to maintain a minimized

thickness of biofilm to prevent clogging of the bio media [13]. According to Shahot et al., 2014, common problem with biofilm is high organic loading due to the proliferation of slime bacteria which caused clogged film [1].

A formed of slime layer of the microorganisms occurred on the surface of media inside the bed. The microorganisms will grow and increase the thickness of slime layer. Before the adsorbed organic matter can reach the microorganisms near the media surface, it will degrade. Therefore, the microorganisms will enter into endogenous phase of growth due to starving. The ability to cling on the media surface will loosen because the organic matter could not enter the inner layer of the media. The slime layer will be washed out by wastewater and new layer of microorganisms will grow. The process of losing slime layer is called sloughing. It is a mainly a function of the organic and hydraulic loading in the filter [14]. Kevin 1992, stated the sloughing process is to make sure the microorganisms keep fresh and decrease the layer of anaerobic thickness which is usually developed [15]. While, Pipeline, 2004, also added that if the sloughing action did not occurred, the media will clog and develop anaerobic conditions [16].

At highest organic loading rate of 3.2 kg COD/(m³d), it was found that media plastic got clogged due to excessive biomass growth which caused the process diffusion of oxygen and substrate through the biofilm become challenging [17]. Based on the study conducted by Cabije et al., 2009, shows that high organic loading reduces the COD removal of 65.68 %, while low organic loading increases the COD removal of 67.89 % [18]. From observation, it indicates that the average of biofilm thickness at high organic loading was measured to be 7.71 µm and 2.81 µm at lower organic loading [18]. It shows that high organic loading will formed thick bio film and eventually led to media clogging which failed the treatment process.

SLOW WATER FLOW

Slow water flows will inhibit the removal of excess microorganisms [19]. According to Ødegaard, H. et al., 1999, if the turbulence is too low, it may cause insufficient shearing of the biofilm and build up more biomass than it supposed to be and blocked the voids of bio media as it will restrict free passage of wastewater and substrates into the inner layer of bio film [12]. System down of operation reactor also caused a disruption of water flow which is eventually obstructing the biological treatment process [20]. If the water flow is slow down or blocked, oxygen transfer into the bio media will decreased and eventually clogged the media. Therefore, it is important for a media to have a uniform water flow in order to avoid dead zones and channels which eventually will reduce the nitrification rate [21].

SELECTION MEDIA

The behavior and performance of biological treatment system are associated with different sizes, types, surfaces and voids of bio media. Therefore, the choice of selection and appropriate bio media used such as media type, media size and surface area, and media void is crucial. Selection of media is essential as it will affect the capability of biofilm and removal efficiency as the media replacement is supposedly not required during operation of treatment plant [22]. The ideal bio media is the material that has criteria such as high specific surface area, high void fraction or high porosity, lightweight and high durability.

A. Media Type

Type of media consists of floating media or submerged media. The floating media offer less specific gravity while submerged media has greater specific gravity than water [23]. Structured plastic media has been modernized in a form of design and operation in biofilm reactors. Synthetic plastics are widely use nowadays as it were arranged to have high porosity, maximum specific surface area and light in weight. These types of media are easy to carry and install as well as offering clog resistant media and high void fraction. Instead of using plastic as a media, there are other media has been used as a medium for bacterial growth such as peat [24], fibrous rope modified PVC [15], polyurethane sponge [25], and recycled glass [26] have been applied in attached growth systems.

For example, a usage of loofa sponge as a bio media by Nabizadeh et al., 2008, meets the characteristics as a packing media [27]. However, the loofa sponge at the higher loading rates of 2.4 kg/m³d was clogging and deformation. The fibre structure also destroyed. Due to anaerobic condition in the inner layer of bio film, the bacteria will use carbon of which it will have a tendency to the deterioration of media. Therefore, loofa cannot be classified as a good choice of medium as the sponges will decay after a long time [27]. Lekang and Kleppe 2000, found that Leca clay (submerged media) showed a tendency to clog compared to plastic media (floating media) [21]. Hence, plastic is one of a good choice bed media. This is due to plastic (floating media) has larger surface area which can allow more bacteria growth on the surface media and it is more economical [23]. Besides that, polyethylene plastic media were used in MBBR for river purification treatment and successfully removed 58 % of COD, 80 % of TSS and 75 % of ammoniac hydrogen [28].

B. Media Size and Surface Area

In attached growth system, the smallest size of medium offers a large surface area per unit of volume size for bio film process and the required reactor volume will be minimized [29].

Based on conducted experiment by Nguyen et al., 2010, shows the result of small (1 cm³ of volume size and 2580 cm² of surface area) and middle (8 cm³ of volume size and 1296 cm² of surface area) sponge had significantly higher COD removal efficiencies as compared to large (27 cm³ of volume size and 864 cm² of surface area) sponge size [25]. The maximum COD removal efficiencies of small and middle sizes at day 10 were 54.9 and 69.9 %, respectively. Meanwhile, the removal efficiency of the large sponge size was only 28.8 % after 3 days, followed by a steady state. It shows that small size of media will increase the organic removal in efficiently. However, the middle sponge size of 8 cm³ shows the best performance of COD removal [25]. Meanwhile, Lekang and Kleppe 2000, found that the smallest particle size of 2 - 4 mm clay with higher specific surface area of 500 - 1000 m²/m³ showed a tendency to clog compared to plastic media with the specific surface area of 220 - 500 m²/m³ [21]. It indicates that it is important to select the size of media with the optimum surface area. However, it must be determined before it can be used as a bed media in a reactor.

According to Hodkinson 1999, high specific surface area offer more bacterial growth and allows high removal of organic rate [30]. It has been found that the rate of nitrification per volume of media will be increasing with the higher specific surface area of media [31]. A researched by Kevin 1992, using a media of fibrous rope modified PVC has lower a specific surface area compared to other submerged type of media studies which were reported typically 100 – 150 m²/m³ and lower rate of nitrification were found in his study [15]. From a laboratory work by Balakrishnan and Eckenfelder 1969, also shows the increasing of specific surface area of ceramic media from 190 m²/m³ to 249.5 m²/m³ resulted with higher nitrification efficiency [32].

In overall, it shows a relationship of media size, surface area and specific surface area in order to achieve higher organic removal rate. It is important that the small size of media must be chosen with the optimum surface area as it has been proven if the surface area is too high, it will have a tendency of media clogging or low organic removal rate.

C. Media Void

Joo, S.H., et al., 2000, explained that pores provide high surface area for the nitrifying bacteria to growth and increase the biomass concentration [33]. According to Wang, R.C., et al., 2005, pores of media in the reactor are easily clogging due to biofilm growth [7]. Therefore, increasing of void ratios will allow air move freely in the media and eventually reduce the clogging of media [34]. Loupasaki, E. and Diamadopoulos, E., 2012, mentioned that the pore size or the voids of bio media carrier are essential in determining which microorganisms will be established and dominate the system [35].

For example, Tapiz et al, 2007, used Polyethylene Terephthalate (PET) plastic bottles as the media [36]. Their results showed that media with pores has higher COD removal of 79.3822 % compared to bio reactor media without pores is 76.1494 %. Therefore, it proves that high pores or voids of media will prevent media clogging.

D. Lack of Aeration

Based on the experiment by Kevin 1992 proved that by using a synthetic fibrous rope modified PVC bio media experienced limited amount of sufficient oxygen into the attached bacteria within the biofilm due to diffusional resistance to oxygen transfer into the biofilm [15]. Qureshi et al., 2005, stated that penetration of oxygen which sufficiently supplied to the inner layer of biofilm thickness is crucial. If oxygen is not transferred to the microorganism in the inner layer, it may kill them and it can lower the performance of the treatment process [37]. Referring to Ødegaard et al., 1999, diffusion of compound in and out of biofilm is very important as to maintain a low thickness of the biofilm by shearing force and make sure the substrate evenly distributed over the surface of the media. In order to obtain this, the turbulence caused by the high air flow is necessary to maintain oxygen of 3 mg O₂/L in aerobic reactors at high organic loads [12].

According to Shohreh Azizi, et al., 2013 arrangement of packed bed bio film reactor will help to overcome the possible limitation of oxygen supplied efficiency and offers a relatively greater effluent quality at a higher organic loading rate [38]. In the reactor, arrangement of vertical pipe between the layered strata on the packed bed of biofilm was made to ease the effluent flow. Thus, it will avoid choking of sludge. As for air distribution, different header pipes to various levels were provided for uniform distribution of air in the reactor as it will increase the oxygen transfer efficiency for the entire reactor.

Aeration is an important part to supply oxygen sufficiently and may prevent media clogging. Zhu et al., 2013, applied the supplementary aeration device in their reactor where the water is drained intermittently with a rhythmical water and air movement [39]. Thus, allowed oxygen is supplied efficiently into the inner layer of media bed [39]. Xie et al., 2004, found that the optimum aeration volume for an aerobic biological filtration process with floating filter media of polystyrene foam pellets was 2.5 m³/m³ per h [40]. However, aerated systems are expensive as it requires extra operational cost due to increasing of energy consumption [35]. Therefore, an optimum design of aeration is essential as it will affect the performance of treatment system in overall.

E. Lack of Backwashing

Backwash system is required to prevent the reactor from clogging and to maintain the nitrification rate and it is needed to remove solids of clogging and excessive growth of biomass on the media [23]. For example, a recycled glass used as

tertiary filter medium in trickling filters has a tendency of plugging up or clogging problems due to backwashing needed [26].

The frequency and volume of backwash cycle are the factors that affect the efficiency of backwash system [23]. Based on observation by Xie et al., 2004, low backwash frequency causing media clogging [40]. Under 24 backwashes, the BOD removal rate and T-N nitrification rate was 67.1 % and 40.9 % respectively while the BOD removal and T-N nitrification is dramatically increasing after backwashing exceeded 30 times. Therefore, to prevent the media clogging, a sufficient frequency of backwash system is essential as it will improve attachment and biofilm growth process [23].

F. Mechanical Failure

It has been reported some of bio reactor had a disadvantage. Based from Ødegaard et al., 1999, they stated that rotating biological contactors (RBC) always experience with mechanical failure [12]. Fluidized bed bioreactor, FBR, have a problem of system shut down where a monitoring during start up is needed [20].

In the attached growth system, efficiency and performance of the bio reactor depending on the choice of bio media [25]. Lekang and Kleppe 2000, showed that the trickling filter with Leca clay media have a tendency to clog when it was operating at the maximum hydraulic load [21]. The reactor also exposed to fouling and a regular calibration twice a week is needed. However, plastic media do not show any problem of media clogging even at higher loading rate. Other than that, Nabizadeh et al, 2008 also found that by using aerated submerged fixed-film reactor, the loofa sponge media clogging at higher loading rate [27]. It proves that selection of media type is important as it gives impact to the effectiveness of reactor.

CONCLUSION

Based from the review, it shows that the major drawback of attached growth process is media clogging. It can be caused by designing of bio reactor and bio media. A selection of bio media will give impact to overall performance of bio reactor.

Bio film thickness, slow water flow, selection of media (media type, media size and surface area, and media void), lack of aeration, lack of backwashing and mechanical failure has been reported causing clogging of bio media. It was found that the consequences of media clogging are excess head loss, short circuiting, high frequent of backwashing and malodor problem.

The result obtained from the review shows that one of the problem of attached growth system which is media clogging caused poor performance of treatment system. Thus, it could lead to a low quality of clean water produce.

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