Parameter Identification and Real-Time State Estimation in Hydraulic Actuators

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Abstract

This paper deals with the challenging aspect concerning the state estimation in hydraulic actuators characterized by not negligible nonlinearities. Particularly,. One of the most common hard nonlinearities in hydraulic actuator is the dead-zone. More specifically, dead-zone occurs in hydraulic control valves when its spool occludes the orifice with an overlap, so that for a range of spool positions there is no fluid flow. This paper focuses on an alternative nonlinear estimation method that is able to fully take into account dead-zone hard nonlinearity and measurement noise. The estimator is based on the State-Dependent-Riccati-Equation (SDRE).

Keywords: State estimation, Hydraulic actuators, Dead-zone

INTRODUCTION

The state estimation for hydraulic actuators is fundamental for their operation. Indeed, the hard working conditions and the complexity of the hydraulic systems [1 – 3] strongly exhort to take under consideration their reliability and, so, the state observers become functional for a fast and economical detection of faults. At the same time, they are a valid alternative to the installation of sensors to give feedback to the control laws in case of prohibitive costs or harsh environments.

The design of state observers should take into account the nonlinear dynamics of hydraulic actuators. One of the most common hard nonlinearities is the dead-zone. Dead-zone is a hard nonlinearity, frequently encountered in many actuators of industrial control systems, especially those containing some very common components, such as hydraulic or pneumatic valves and electric motors. The dead zone nonlinearity occurs in closed center valves when the land width is greater than the port width at neutral spool position. Control valves without such imperfections are costly to manufacture and their maintenance usually requires specialized personnel. The presence of dead zone in control valves degrades the achievable control performance or even destabilizes the closed-loop system if not properly dealt. Several approaches have been proposed in literature for the state estimation in hydraulic actuators. These methods include linear approaches, linearized model based techniques and robust observers based on nonlinear models. The high degree of parametric uncertainty that characterizes hydraulic actuators exhorts the researchers to follow dedicated approaches for the state estimation. Extended Kalman filter (EKF) has been adopted to detect the fault in hydraulic actuators by means of the state estimation. The approach is based on the local linearization of the nonlinear system and allows to closely track the state trajectories if compared with the linear approach of the Kalman filter. This paper investigates on an alternative nonlinear technique for the state estimates of the hydraulic actuators. The approach is based on the State-Dependent-Riccati-Equation (SDRE) nonlinear filtering formulation. The SDRE techniques are recently emerging for optimal nonlinear control and filtering techniques. The SDRE filter (SDREF) originates from a suboptimal nonlinear regulator technique that uses parameterization to bring the nonlinear system into a linear-like structure with state-dependent coefficients (SDC) and is characterized by the structure of the steady state Kalman filter and the Kalman gain is obtained by solving an algebraic Riccati equation. Simulations have been conducted in order to analyse the effect of the dead-zone characteristic on the novel estimator performance showing a comparison with the EKF technique.

MATHEMATICAL MODELLING

The hydraulic actuation system under consideration consists mostly of a double-rod hydraulic cylinder and a proportional valve. The hydraulic cylinder is linked to a mass that moves on a linear guide (Fig. 1).

For the derivation of the mathematical model, some assumptions are made: the tank pressure P_T is equal to zero, the fluid properties are not dependent on the temperature, the piston areas and the chamber volumes are equal, the internal and external fluid leakages are negligible.

The dynamics of the movable mass displacement (y) is governed by:

$$m\ddot{y} + \sigma\dot{y} + F_f(\dot{y}) = A_p P_L, \qquad (1)$$

where m is the mass of the load, σ is the viscous friction coefficient, F_f is the friction force, A_p is the piston area, $P_L =$

P_A-P_B is the load pressure, P_A and P_B are the pressures inside the two chambers of the cylinder.

The friction force is represented by the following equation:

$$F_f(\dot{y}) = \begin{cases} F_c \, \text{sgn}(\dot{y}) + \mu mg \, \text{sgn}(\dot{y}) & \dot{y} \neq 0 \\ Z \qquad |Z| \leq F_{c0} + \mu_0 mg & \dot{y} = 0 \end{cases} \qquad (2)$$

where F_c is the Coulomb friction force in the hydraulic actuator (assumed equal to its static value F_{c0}), μ is the Coulombian friction coefficient of the linear guide (assumed equal to its static value μ_0), g is the gravitational acceleration and Z is the net tangential force that acts on the actuator when it is not moving [4].
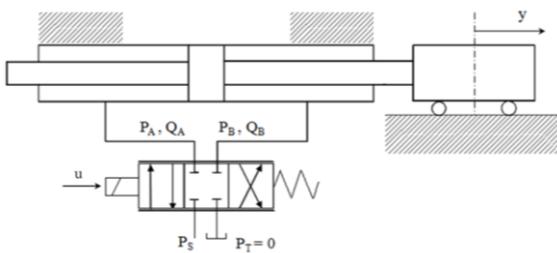

Figure 1: Schematic diagram of the hydraulic actuation system

The load pressure dynamics is given by:

$$\frac{V_0}{2\beta} \dot{P}_L = -A_p \dot{y} + Q_L \qquad (3)$$

where V_0 is the volume of each chamber for the centred position of the piston, $Q_L = (Q_A + Q_B)/2$ is the load flow and β is the effective Bulk modulus.

The load flow depends on the supply pressure, the load pressure and the valve spool position in accordance with the following:

$$Q_L = \Psi(v_e) v_e \sqrt{P_s - |P_L|} \qquad (4)$$

where v_e is the displacement signal of the spool valve and $\Psi(v_e)$ is a gain that depends on the geometry of the adopted proportional valve.

The analytical expression of $\Psi(v_e)$ can be assumed as:

$$\Psi(v_e) = \begin{cases} k_{qp}\left[1 + \left(\dfrac{k_{q0}}{k_{qp}} - 1\right)\dfrac{v_{ep}}{v_e}\right] & v_e > v_{ep} \\ k_{q0} & v_{en} \leq v_e \leq v_{ep} \\ k_{qn}\left[1 + \left(\dfrac{k_{q0}}{k_{qn}} - 1\right)\dfrac{v_{en}}{v_e}\right] & v_e < v_{en} \end{cases} \quad (5)$$

where v_{en}, v_{ep}, k_{q0}, k_{qn}, k_{qp} are parameters that must be identified experimentally. The equations in (5) represent an asymmetric bilinear function of the valve spool position. If $k_{q0} = k_{qp} = k_{qn}$, the function $\Psi(v_e)$ reduces to a constant gain. If $k_{q0} = 0$, the product $\Psi(v_e)v_e$ describes a dead zone function, where v_{en} and v_{ep} are the limits of the dead zone and k_{qn} and k_{qp} are the gains if v_e is negative or positive respectively. The adoption of $\Psi(v_e)$ is particularly efficient to describe a dead zone function if an overlapped valve is adopted.

The proportional valve dynamics can be well represented by a second order differential equation:

$$\frac{\ddot{v}_e}{\omega_{nv}^2} + \frac{2\xi_v}{\omega_{nv}} \dot{v}_e + v_e = k_e u + v_{e0} \qquad (6)$$

where parameters ω_{nv} and ξ_v are the natural frequency and the damping ratio of the valve respectively, v_{e0} is the spool position bias, k_e is the input gain and u is the valve command.

Finally, the equations governing the dynamics of the whole system (movable mass + hydraulic system) are:

$$\begin{cases} m\ddot{y} + \sigma\dot{y} + F_f(\dot{y}) = A_p P_L \\ \dfrac{V_0}{2\beta}\dot{P}_L = -A_p\dot{y} + Q_L \\ Q_L = \Psi(v_e)v_e\sqrt{P_s - |P_L|} \\ \ddot{v}_e = -\omega_{nv}^2 v_e - 2\xi_v\omega_{nv}\dot{v}_e + \omega_{nv}^2(k_e u + v_{e0}) \end{cases} \quad (7)$$

The developed fifth order model fully describes the nonlinear dynamical behaviour of the hydraulic actuation system and takes the nonlinear friction forces and the nonlinear flow rate distribution into account.

The nonlinear system (7) can be written in the following form:

$$\begin{cases} \ddot{y} = -\dfrac{\sigma}{m}\dot{y} - \dfrac{F_f(\dot{y})}{mv_e}v_e + \dfrac{A_p P_L}{m} \\ \dot{P}_L = -\dfrac{2\beta A_p}{V_0}\dot{y} + \dfrac{2\beta\Psi(v_e)\sqrt{P_s - |P_L|}}{V_0}v_e \\ \ddot{v}_e = -2\xi_v\omega_{nv}\dot{v}_e - \left(1 - \dfrac{v_{e0}}{v_e}\right)\omega_{nv}^2 v_e + \omega_{nv}^2 k_e u \end{cases} \quad , \quad (8)$$

The system (8), whose state vector is given by $x = \begin{bmatrix} \dot{y} & y & P_L & \dot{v}_e & v_e \end{bmatrix}^{\text{T}}$, is nonlinear in the state and autonomous.

It is possible to note in the first and in the third equation of (8) a division for the variable v_e, as well as it happens in (5). In order to prevent divisions by zero, the variable $\tilde{v}_e = v_e + \varepsilon$ has been introduced, where ε is given by:

$$\varepsilon = \begin{cases} \Delta v_e & v_e = 0 \\ 0 & \text{otherwise} \end{cases} \qquad (9)$$

where the value of Δv_e can be assumed less than the discrimination threshold of the valve spool position sensor.

Replacing v_e with $\tilde{v}_e$, where v_e is at the denominator of fractions in (5) and (8), it follows:

$$\begin{cases} \ddot{y} = -\dfrac{\sigma}{m}\dot{y} - \dfrac{F_f(\dot{y})}{m\tilde{v}_e}v_e + \dfrac{A_p P_L}{m} \\[2mm] \dot{P}_L = -\dfrac{2\beta A_p}{V_0}\dot{y} + \dfrac{2\beta \Psi(\tilde{v}_e)\sqrt{P_s - |P_L|}}{V_0}v_e \\[2mm] \ddot{v}_e = -2\xi_v\omega_{nv}\dot{v}_e - \left(1 - \dfrac{v_{e0}}{\tilde{v}_e}\right)\omega_{nv}^2 v_e + \omega_{nv}^2 k_e u \end{cases} \quad . \quad (10)$$

The nonlinear equations of the system (10) have been adopted in order to derive the EKF and the SDREF [5 - 9].

Given the measurements of signals like the load displacement y, the load pressure P_L and the spool valve displacement v_e, the objective is the estimation of the states of the system.

EKF METHODOLOGY

The EKF is a mathematical tool for an optimal state estimation of nonlinear systems [12]. This algorithm can be used to estimate the unknown parameters by taking the parameters as additional states and augmenting state equations [13-16].

The system and the measurement equations are:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) + \boldsymbol{\psi}(t), \qquad (11)$$

$$\mathbf{z}(t) = \mathbf{h}(\mathbf{x}(t), \mathbf{u}(t)) + \mathbf{g}(t), \qquad (12)$$

being $\mathbf{x}$ the state vector, $\mathbf{u}$ the input vector, $\mathbf{f}$ a non-linear function, $\boldsymbol{\psi}$ the process noise with covariance $\mathbf{Q}_k$, $\mathbf{z}$ the measurement vector, $\mathbf{h}$ a non-linear function and $\mathbf{g}(t)$ the Gaussian white measurement noise with covariance $\mathbf{R}_k$.

The estimator has been implemented in a discrete time form with the state estimates and an estimation of the error covariance given by:

$$\hat{\mathbf{x}}_k^- = \mathbf{f}(\hat{\mathbf{x}}_{k-1}^+, \mathbf{u}_{k-1}), \qquad (13)$$

$$\mathbf{P}_k^- = \mathbf{A}_{k-1}\mathbf{P}_{k-1}^+\mathbf{A}_{k-1}^T + \mathbf{L}_{k-1}\mathbf{Q}_{k-1}\mathbf{L}_{k-1}^T, \qquad (14)$$

respectively.

In the previous formula the following parameters are defined

$$\mathbf{A}_{k-1} = \left.\frac{\partial \mathbf{f}}{\partial \hat{\mathbf{x}}}\right|_{\hat{\mathbf{x}}_{k-1}^+}, \qquad (15)$$

$$\mathbf{L}_{k-1} = \left.\frac{\partial \mathbf{f}}{\partial \boldsymbol{\psi}}\right|_{\hat{\mathbf{x}}_{k-1}^+}. \qquad (16)$$

The filter gain can be computed as in (17) to evaluate the measurement residual. The updates of state estimates can be computed as in (18), while the estimation of the error covariance can be determined as in (19):

$$\mathbf{K}_k = \mathbf{P}_k^-\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_k^-\mathbf{H}_k^T + \mathbf{M}_k\mathbf{R}_k\mathbf{M}_k^T)^{-1}, \qquad (17)$$

$$\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + \mathbf{K}_k\left[\mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}_k^-, \mathbf{u}_k)\right], \qquad (18)$$

$$\mathbf{P}_k^+ = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_k^-, \qquad (19)$$

where

$$\mathbf{H}_k = \left.\frac{\partial \mathbf{h}}{\partial \hat{\mathbf{x}}}\right|_{\hat{\mathbf{x}}_k^-}, \qquad (20)$$

$$\mathbf{M}_k = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{g}}\right|_{\hat{\mathbf{x}}_k^-}. \qquad (21)$$

SDRE

SDRE techniques are used as control and filtering design methods and are based on state dependent coefficient (SDC) factorization. Infinite-horizon nonlinear regulator problem is a generalization of time invariant infinite horizon linear quadratic regulator problem where all system coefficient matrices are state-dependent. When the coefficient matrices are constant, the SDRE control method changes into the steady-state linear regulator. Filtering counterpart of the SDRE control algorithm is obtained by taking the dual system of the steady-state linear regulator and then allowing coefficient matrices of the dual system to be state-dependent. Starting from the nonlinear system (10), there are infinite solutions to transform this nonlinear system into an SDC form as:

$$\dot{\mathbf{x}}(t) = \mathbf{F}(\mathbf{x}(t), \mathbf{u}(t))\mathbf{x} + \boldsymbol{\psi} \qquad (22)$$

$$\mathbf{z}(t) = \mathbf{H}(\mathbf{x}(t), \mathbf{u}(t))\mathbf{x} + \mathbf{g} \qquad (23)$$

where

$$\mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) = \mathbf{F}(\mathbf{x}(t), \mathbf{u}(t))\mathbf{x}$$

$$\text{and } \mathbf{h}(\mathbf{x}(t), \mathbf{u}(t)) = \mathbf{H}(\mathbf{x}(t), \mathbf{u}(t))\mathbf{x}, \qquad (24)$$

$\boldsymbol{\psi}$ is the process noise with covariance $\mathbf{Q}$, and $\mathbf{g}$ is the Gaussian white measurement noise with covariance $\mathbf{R}$.

Starting from the SDC form, the derivative of the state estimate is given by:

$$\dot{\hat{\mathbf{x}}} = \mathbf{F}(\hat{\mathbf{x}}, \mathbf{u})\hat{\mathbf{x}} + \mathbf{K}_{\mathbf{f}}(\hat{\mathbf{x}})[\mathbf{z}(\mathbf{x}) - \mathbf{H}(\hat{\mathbf{x}}, \mathbf{u})\hat{\mathbf{x}}] \qquad (25)$$

where

$$\mathbf{K}_{\mathbf{f}}(\hat{\mathbf{x}}) = \mathbf{P}(\mathbf{x})\mathbf{H}^{\mathbf{T}}(\hat{\mathbf{x}}, \mathbf{u})\mathbf{R}^{-1} \qquad (26)$$

and **P** is the positive definite solution of the algebraic Riccati equation (27).

$$F(\hat{x},u)P(\hat{x})+P(\hat{x})F^T(\hat{x},u)+$$
$$-P(\hat{x})H^T(\hat{x},u)R^{-1}H(\hat{x},u)P(\hat{x})+Q=0 \qquad (27)$$

The highly nonlinear equations (10) have been parameterized in SDC form with the following choice:

$$F(x,u)=\begin{bmatrix} -\dfrac{\sigma}{m} & 0 & \dfrac{A_p}{m} & 0 & -\dfrac{F_f(\dot{y})}{m\tilde{v}_e} \\ 1 & 0 & 0 & 0 & 0 \\ -\dfrac{2\beta A_p}{V_0} & 0 & 0 & 0 & \dfrac{2\beta\Psi(\tilde{v}_e)\sqrt{P_s-|P_L|}}{V_0} \\ 0 & 0 & 0 & -2\xi_v\omega_{nv} & -\left(1-\dfrac{v_{e0}}{\tilde{v}_e}\right)\omega_{nv}^2+\dfrac{\omega_{nv}^2 k_e u}{\tilde{v}_e} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad (28)$$

$$H(x,u)=H=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \qquad (29).$$

SIMULATION RESULTS

In order to evaluate the benefits of the proposed estimator, simulations have been carried out on the mathematical model of a hydraulic actuator. The hydraulic actuator has been modelled by means of (10), in which the parameters have been determined by means of an identification procedure and are:

$$m=440 \text{ kg}, \ \sigma=23555 \ \frac{\text{N}\cdot\text{s}}{\text{m}}, \ F_C=F_{C0}=950 \text{ N},$$

$$\mu=\mu_0=0.01, \ A_p=0.01 \text{ m}^2, \ V_0=0.004 \text{ m}^3, \ \beta=1\text{e}9 \text{ Pa},$$

$$k_{q0}=1\text{e}-12 \ \frac{\text{m}^3}{\text{s}\cdot\text{V}\cdot\text{Pa}^{\frac{1}{2}}}, \ k_{qp}=6.15\text{e}-7 \ \frac{\text{m}^3}{\text{s}\cdot\text{V}\cdot\text{Pa}^{\frac{1}{2}}},$$

$$k_{qn}=5.86\text{e}-7 \ \frac{\text{m}^3}{\text{s}\cdot\text{V}\cdot\text{Pa}^{\frac{1}{2}}}, \ v_{e0}=0.01\text{V}, \ k_e=0.49,$$

$$\omega_{nv}=152 \text{ rad/s}, \ \xi_v=0.92, \ P_S=6\text{e}6 \text{ Pa}.$$

The performance of the SDREF will be evaluated through a comparison with the EKF estimator and the real state. The observers have been designed taking into account as input the signal u and the measurements given by displacement y, the load pressure PL and the spool valve displacement v_e.

The simulated results have been obtained by means of open loop tests employing a sine wave as input (Fig. 2). The parameters adopted in the observers are the same of the hydraulic actuator model.
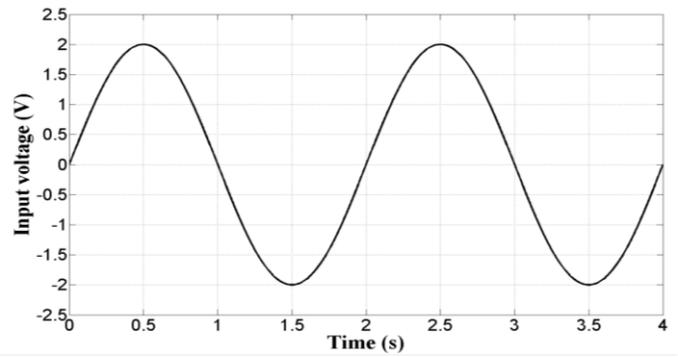

Figure 2: Input voltage

This test refers to the realistic condition of asymmetric dead-zone ($v_{ep}=0.43 \text{ V}, v_{en}=-0.21 \text{ V}$) and it is very functional to highlight the limits of linearization based techniques. Indeed, it appears more clear the capability of the SDREF to estimate the load velocity (Fig. 3).
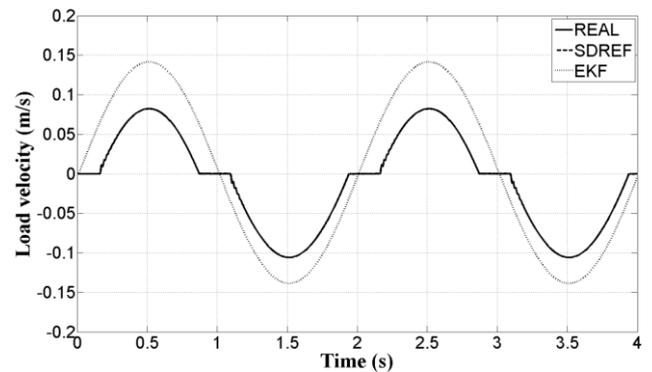

Figure 3: Load velocity

Fig. 4 exhibits the comparison in terms of load displacement. The differences are lightly visible because of the absence of strong nonlinear effects. Differently, the nonlinear approach of the SDREF can be appreciated with reference to the load pressure (Fig. 5).
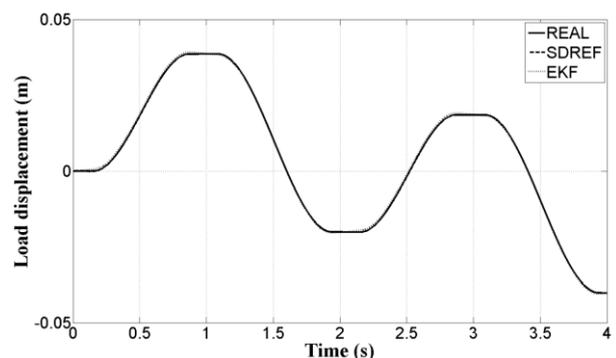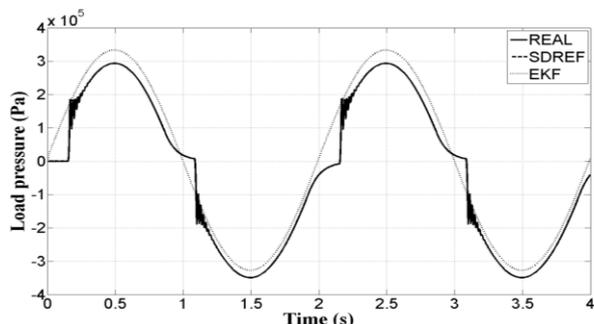

Figure 4: Load displacement

Figure 5: Load pressure

Figg. 6 and 7 refer to the spool velocity and to the spool displacement respectively.
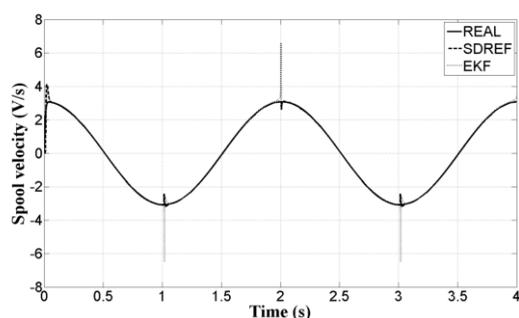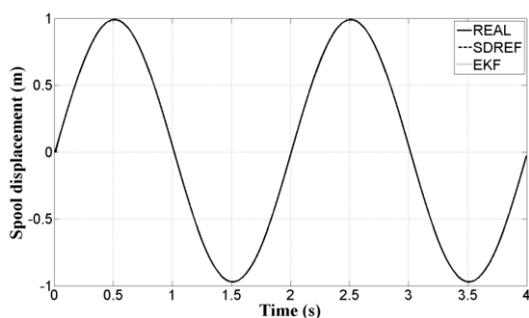

Figure 6: Spool velocity.

Figure 7: Spool displacement.

The illustrated simulation results are functional to validate the SDREF application for nonlinear system such as the hydraulic actuator and allow to consider the technique a valid tool for the fault detection algorithm and a functional alternative to the employment of sensors. The advantages of the SDREF are clear for the states dependent on hard nonlinearities, where the EKF approach fails because of the linearization procedure.

CONCLUSION

A state-dependent-Riccati-equation based Kalman filter has been proposed for the state estimation of a hydraulic actuator.

A fifth order nonlinear model has been derived and adopted to design the estimator. The effectiveness of the filter has been evaluated by means of simulations that show a comparison with the EKF technique. The results show the advantages in terms of state estimates for all the states depending on nonlinearities. Consequently, the effectiveness of the SDREF is clearly visible for the load velocity and for the load pressure estimates. Taking into account the performances of the technique, its main contributions can be found in fault detection algorithms or as alternative solution to the installation of velocity sensors.

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