International Journal of Applied Engineering Research ISSN 0973-4562 Volume 12, Number 20 (2017) pp. 9697-9702

Reliability and Cost-Benefit Analysis of a Two-Unit Cold Standby System Used for Communication through Satellite with Assembling and Activation Time

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Abstract

This paper gives the reliability and cost benefit analysis of a two-unit cold standby communication system with assembling of both units at a time with activation time. In now-a-days, Communication devices are of great demand. Thus, it is essential to study the systems used for communication. Here, we deal with a two-unit cold standby communication system in which both the units are packed initially and are assembled immediately after the need arises. On need, the assembling process starts for both the units but one at a time and as soon as the units are assembled, one of them is made operative and other is kept as cold standby. On failure of the operative unit, some time (called activation time) is required to make the cold standby unit operative and the system works till its need exists or it gets failed completely. On failure, the system is got repaired and its working is resumed soon after the completion of repair. Various measures of system effectiveness have been obtained to get the optimum profit.

Keywords: Cold Standby, Communication through Satellites, Activation Time, Assembling, Cost-Benefit Analysis.

INTRODUCTION

Literature on reliability contains a large number of research papers wherein various measures of system effectiveness such as reliability, availability, busy period, etc have been obtained for single unit or standby systems. Cost-benefit analysis has alot of researchers. Parashar and Taneja [1] carried out the analysis for a PLC standby system with master and slave concept. Singh et al.[2] discussed the profit analysis of 2-out of-3 units system for an ash handling plant wherein situation of system failure did not arise. Bhatia et al. [3] dealt the reliability modeling of a 3-unit cold standby system working at full/ reduced capacity. Lakshminarayana and Kumar [4] optimized the reliability of an integrated reliability model using dynamic programming. Rizwan et al. [5] and Padmavathi et al. [6[analysed a seven unit system working in desalination plant. Malhotra and Taneja [7] gave the stochastic analysis of a two unit cold standby system with arbitrary distribution for life, repair and waiting times. Manocha and Taneja[8] analyzed the stochastic analysis of a two-unit cold standby system with arbitrary distribution for life, repair and waiting times. Sonia and Taneja [9] dealt with the multi backup path protection scheme for survivability in elastic optical network. Taj et al. [10] discussed the reliability analysis of a single machine subsystem of a cable plant with six maintenance categories.However,few researchers i.e. Adlakha and Taneja [11] and Adlakha et al. [12] have developed reliability models on systems used for communication through satellites . Adlakha et al. [12] studied two-unit standby system with assembling/disassembling but without taking into consideration the activation time. Whenever a cold standby unit is used, it takes some time to become operative as and its need arises and hence in the present paper, we carry out the reliability and cost-benefit analysis for a two-unit cold standby system used for communication through satellite with assembling and activation time for making the cold standby unit operative. Initially, both the units are packed and their assembling process is started as soon as the need arises. After assembling, one of the units is made operative and the other is kept as cold standby. On failure of the operative unit, the activation process of making the cold standby unit operative is started. As soon as the need is over, the units are disassembled and packed.

Model Description and Assumptions

1) Initially the system is packed and is assembled as soon as its need arises.

International Journal of Applied Engineering Research ISSN 0973-4562 Volume 12, Number 20 (2017) pp. 9697-9702

2) After assembling, one of the units is made operative and the other is kept as cold standby.

3) The cold standby unit takes some time (activation time) to become operative whenever it is needed.

4) Failure time is assumed to follow the exponential distribution whereas the times to repair. assemble, pack/unpack the system, assemble, disassemble, activate the cold standby to become operative whenever needed, to have the requirement of the system are arbitrarily distributed.

5) All random variables are independent.

6) After every repair, the unit becomes as good as new.

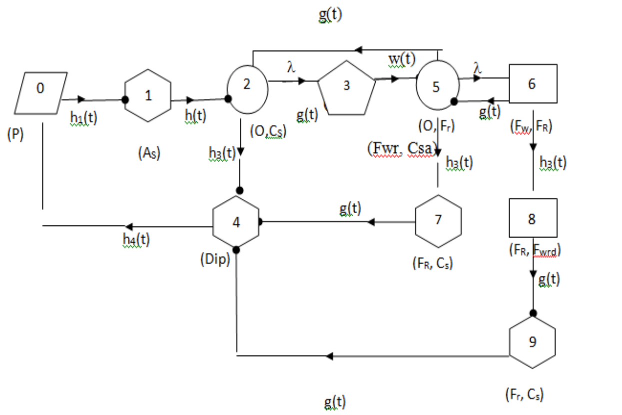
7) Repairman comes to the system as soon as a unit fails and repairs all the units which fail during his stay at the system.

Repair is done on FCFS pattern.

O	:	Operative
P	:	System is packed
As	:	Assembling of the system in progress
Dip	:	Disassembling and packing in progress
Fwrd	:	Waiting for repair when need of the system is over
C_s	:	Cold standby unit
C_{sa}	:	Cold standby unit under activation
F_r, F_R	:	Failed unit under repair, repair is continuing from the previous state
λ	:	Constant failure Rate
$H_1(t)$, $h_1(t)$	:	cdf and pdf of time for unpacking the system
$H(t)$, $h(t)$	:	cdf and pdf of time for assembling the system
$H_3(t)$, $h_3(t)$	:	cdf and pdf of time during which requirement remains for using the system
$H_4(t)$, $h_4(t)$	:	cdf and pdf of time during which cold standby unit packed
$G(t)$,$g(t)$	:	cdf and pdf of repair time of failed unit
$W(t)$,$w(t)$	:	cdf and pdf of activation time of the standby unit

State Transition Probabilities and Mean Sojourn Times

State Possible transitions are represented in **Figure 1**. States 0, 1, 2, 3, 4, 5, 7 and 9 are regenerative states whereas 6 and 8 are non-regenerative as well as failed states. States 1, 3, 4, 7 and 9 are down states. State 0 represents that the system is packed.



Operative state ⬭ Downstate Regenerative point ⬡

FailedState □

Figure 1: State Transition Diagram

The state transition probabilities are:

$$q_{01}(t) = h_1(t); \quad q_{12}(t) = h(t);$$
$$q_{23}(t) = \lambda.e^{-\lambda t}.\overline{H_3}(t); q_{24}(t) = e^{-\lambda t}.h_3(t);$$
$$q_{35}(t) = w(t); \; q_{40}(t) = h_4(t) q_{52}(t) = e^{-\lambda t} g(t)\overline{H_3}(t) = e^{-\lambda t}E_1(t);$$
$$q_{55}^{(6)}(t) = \left(\lambda e^{-\lambda t} © 1\right)\overline{H_3}(t)g(t) = (1-e^{-\lambda t})E_1(t),$$
$$\text{where } E_1(t) = \overline{H_3}(t)g(t)$$
$$q_{56}(t) = \lambda.e^{-\lambda t}\overline{G}(t)\overline{H_3}(t) = \lambda.e^{-\lambda t}E_2(t),$$
$$\text{where } E_2(t) = \overline{G}(t)\overline{H_3}(t)$$
$$q_{54}^{(7)}(t) = \left(e^{-\lambda t}h_3(t) © 1\right)g(t) = E_3(t)(\text{say});$$
$$q_{5,9}^{(6,8)}(t) = \left(\lambda e^{-\lambda t}\overline{H_3}(t) © h_3(t) © 1\right)g(t) = E_4(t)(\text{say})$$
$$q_{74}(t) = g(t); q_{94}(t) = g(t)$$

The corresponding non-zero elements $p_{ij} = \lim_{s\to 0} q_{ij}^{*}(s)$ are given by

$$p_{01} = h_1^*(0) = 1, \; p_{12} = h_2^*(0) = 1, \; p_{23} = 1 - h_3^*(\lambda),$$
$$p_{24} = h_3^*(\lambda), \; p_{35} = w^*(0) = 1, \; p_{5,9}^{(6,8)} = E_3^*(0)$$
$$p_{56} = \lambda E_2^*(\lambda), \quad p_{55}^{(6)} = E_1^*(0) - E_1^*(\lambda);$$
$$p_{5,9}^{(6,8)} = E_4^*(0), \; p_{40} = 1, p_{74} = 1, p_{94} = 1$$

The mean sojourn times (μ_i) are obtained using $\mu_i = E(T) = \Pr(T > t_i)$, where T is the sojourn time in state i, and are:

$$\mu_0 = \int_0^\infty t.h_1(t)\,dt \; ; \mu_1 = \int_0^\infty t.h_2(t)\,dt;$$
$$\mu_3 = \int_0^\infty t.w(t)\,dt; \; \mu_4 = \int_0^\infty t.h_4(t)\,dt$$
$$\mu_5 = \int_0^\infty e^{-\lambda t}\overline{H}_3(t)\,\overline{G}(t)\,dt = \int_0^\infty e^{-\lambda t}E_1(t)\,dt = E_1^*(\lambda)$$
$$\text{where } E_1(t) = \overline{H}_3(t)\,\overline{G}(t)$$
$$\mu_7 = \int_0^\infty t.g(t)\,dt \quad \text{and } \mu_9 = \int_0^\infty t.g(t)\,dt$$

The unconditional mean time taken by the system to transit for any regenerative state j when it is counted from the epoch of entrance into state i is mathematically states as

$m_{ij}=\int_0^{\infty} t.\,dQ_{ij}(t) = -q^{*}{}'_{ij}(0)$

$m_{01} = \mu_0;\ m_{12} = \mu_1$

$m_{23} + m_{24} = \mu_2$

$m_{52} = \int_0^{\infty} t\left\{e^{-\lambda t}\overline{H}_3(t).g(t)\right\}dt;\ m_{54}^{(7)} = \int_0^{\infty} t\left\{h_3(t).e^{-\lambda t}©1\right\}g(t)dt$

$m_{56} = \int_0^{\infty} t\left\{\lambda.e^{-\lambda t}\overline{H}_3(t)\,\overline{G}(t)\right\}dt$

$m_{5,9}^{(6,8)} = \int_0^{\infty} t\left\{\lambda e^{-\lambda t}\overline{H}_3(t)©h_3(t)©1\right\}g(t)\,dt$

$m_{54}^{(7)} + m_{56} + m_{52} = K_{51}(\text{say})$ and

$m_{54}^{(7)} + m_{5,9}^{(6,8)} + m_{55}^{(6)} + m_{52} = K_{52}(\text{say})$

Reliability and Mean Time to System Failure (MTSF)

The recursive relations for $\phi_i(t)$ are

$\phi_0(t) = Q_{01}(t)\ Ⓢ\phi_1(t)$

$\phi_1(t) = Q_{12}(t)\ Ⓢ\phi_2(t)$

$\phi_2(t) = Q_{23}(t)\ Ⓢ\phi_3(t) + Q_{24}(t)\ Ⓢ\phi_4(t)$

$\phi_3(t) = Q_{35}(t)\ Ⓢ\phi_5(t)$

$\phi_4(t) = Q_{40}(t)\ Ⓢ\phi_0(t)$

$\phi_5(t) = Q_{52}(t)\ Ⓢ\phi_2(t) + Q_{54}^{(7)}(t)\ Ⓢ\phi_4(t) + Q_{56}(t)$

Solving the above equations for $\phi_0^{**}(s)$ and by taking L.S.T. of the above equations, we get the Mean Time to system Failure (MTSF) as

$$MTSF = \lim_{s\to 0}\frac{1-\phi_0^{**}(s)}{s} = \frac{N}{D}, \text{where}$$

$$N = (\mu_0 + \mu_1)(1 - p_{23}.p_{52}) + \mu_2 + p_{23}(\mu_3 + K_{51}) + \mu_4\left(p_{54}^{(7)}p_{23} + p_{24}\right)$$

and $D = p_{23}.p_{56}$

Availability Analysis

By probabilistic arguments, we obtain the following recursive relations for the probability ($A_i(t)$) that the system up initially is up at time t given that it started at regenerative state i:

$A_0(t) = q_{01}(t)©A_1(t)$

$A_1(t) = q_{12}(t)©A_2(t)$

$A_2(t) = M_2(t) + q_{23}(t)©A_3(t) + q_{24}(t)©A_4(t)$

$A_3(t) = q_{35}(t)©A_5(t)$

$A_4(t) = q_{40}(t)©A_0(t)$

$A_5(t) = M_5(t) + q_{52}(t)©A_2(t) + q_{54}^{(7)}(t)©A_4(t) + q_{55}^{(6)}(t)©A_5(t) + q_{5,9}^{(6,8)}(t)©A_9(t)$

$A_9(t) = q_{94}(t)©A_4(t)$

where $M_2(t) = e^{-\lambda t}.\overline{H_3}(t), M_5(t) = e^{-\lambda t}.\overline{G}(t).\overline{H_3}(t)$

In steady state, the availability of the system is given by,

$$A_0 = \lim_{s\to 0} s.A_0^{*}(s) = \frac{N_1}{D_1}, \text{where}$$

$$N_1 = \mu_2(1-p_{55}^{(6)}) + \mu_5 p_{23}$$

$$D_1 = (\mu_0+\mu_1+\mu_4)\left(1-p_{55}^{(6)}-p_{23}p_{52}\right) + (1-p_{55}^{(6)})\mu_2 + p_{23}p_{52}\mu_3 + p_{23}K_{52} + p_{23}p_{5,9}^{(6,8)}\mu_9$$

Expected Time for Assembling/Disassembling

By probabilistic arguments, the recursive relations for $AD_i(t)$, probability that assembling/disassembling going on at time t given that the system started from state i at time t = 0, are obtained as

$AD_0(t) = q_{01}(t)©AD_1(t)$

$AD_1(t) = W_1(t) + q_{12}(t)©AD_2(t)$

$AD_2(t) = q_{23}(t)©AD_3(t) + q_{24}(t)©AD_4(t)$

$AD_3(t) = q_{35}(t)©AD_5(t)$

$AD_4(t) = q_{40}(t)©AD_0(t) + W_4(t)$

$AD_5(t) = q_{52}(t)©\ AD_2(t) + q_{54}^{(7)}(t)©AD_4(t) + q_{55}^{(6)}(t)©AD_5(t) + q_{5,9}^{(6,8)}(t)©AD_9(t)$

$AD_9(t) = q_{94}(t)©AD_4(t)$, where

$W_1(t) = \overline{H_2}(t)$ and

$W_4(t) = \overline{H_4}(t)$

The expected fraction of time during which the assembling/disassembling of the system goes on, is given by

$$AD_0 = \lim_{s\to 0} s.AD_0^{*}(s) = \frac{N_2}{D_1}$$

where

$$N_2 = \mu_1(1-p_{55}^{(6)}-p_{23}p_{52}) + \mu_4(1-p_{55}^{(6)}-p_{23}p_{52})$$

Proceeding in the similar fashion as above, the other measures of the system effectiveness have been obtained and the results are obtained and are given as follows:

Other Measures of System Effectiveness

Expected Time during which the System Remains Packed(SP_0 **)=** $\frac{N_3}{D_1}$, where $N_3 = \mu_0(1 - p_{55}^{(6)} - p_{23}.p_{52})$

Busy Period Analysis for Repair($B_0) = \frac{N_4}{D_1}$, where $N_4 = p_{23}(\mu_5 + \mu_9.p_{5,9}^{(6,8)})$

Expected Number of Visits by the Repairman $(V_0) = \frac{N_5}{D_1}$, where $N_5 = p_{23}(1 - p_{55}^{(6)})$

Expected Activation Time $(AT_0) = \lim_{s \to 0} s.AT_0^*(s) = \frac{N_6}{D_1}$, where $N_6 = \mu_3.p_{23}(1 - p_{55}^{(6)})$

Costs-Profit Analysis

In steady state, the profit per unit time incurred to the system is given by

Profit (P) = $C_0\ A_0 - C_1\ AD_0 - C_2\ SP_0 - C_3\ B_0 - C_4\ V_0 - C_5\ AT_0$, where

C_0 is revenue per unit up time,

C_1 is rent/expenses paid for assembling/disassembling per unit time,

C_2 is the cost incurred during the period when the system remains packed,

C_3 is the cost per unit time during which the system remains under repair,

C_4 is the cost per visit of the repairman,

C_5 is the cost per unit time during which the unit is activated.

Numerical Results and Graphical Analysis

The following particular case is considered for numerical results:

Let

$$h_1(t)=\beta_1.e^{-\beta_1 t};\ h(t)=\beta_2.e^{-\beta_2 t};$$
$$h_3(t)=\beta_3.e^{-\beta_3 t};h_4(t)=\beta_4.e^{-\beta_4 t};$$
$$g(t)=\alpha.e^{-\alpha t};\quad w(t)=\beta.e^{-\beta\ t}$$

Taking β_1=2, β_2=1, β_3=0.5, β_4=6, β=0.01, λ=0.01, α=1, C_0=1000, C_1=500, C_2=100, C_3=100, C_4=100 and C_5=100; the values of various measures of system effectiveness are obtained as:

Mean time to system failure (MTSF) = 43018.17

Availability (when the component is operative) = 0.399799

Expected Assembling/disassembling time of the unit=0.23320061

Expected time during which system remains packed= 0.09994288

Busy period of the repairman= 0.00267702

Expected Number of Visits by the Repairman=0.003971423

Expected profit incurred to the system= 232.8258

Behaviour of MTSF and other measures of system effectiveness have been observed with respect to different values of different rates by plotting graphs which revealed that

- Mean Time to System Failure (MTSF) decreases as failure rate increases (λ) of the operative unit but gets increased for higher value of repair rate (α).
- Availability (A_0) decreases with increases in the value of failure rate (λ), but its value gets increased with the increased in the value of repair rate (α).
- Expected busy period of the repairman and expected number of the visits of the repairman increases with the increase of failure rate.
- Behaviour of the other measures with respect to various rates is as usual.

Figure 2 depicts the behaviour of profit with respect to failure rate (λ) for different values of repair rate (α).We observe the following from the graph:

i) The profit decreases with the increase in the values of failure rate (λ) and has higher values for higher values of repair rate (α) when λ< 0.006 but after that trend is reversed with respect to repair rate

ii) For α=0.002, the profit is either positive or zero or negative according as λ is < or = or > 0.006714 and hence failure rate should not be less than .006714.

iii) For α=0.004, the profit is positive or zero or negative according as λ is < or = or > 0.006623 and hence failure rate should not be less than .006623.

iv) For α=0.006, the profit is positive or zero or negative according as λ is < or = or > 0.006401 and hence failure rate should not be less than 0.006401.

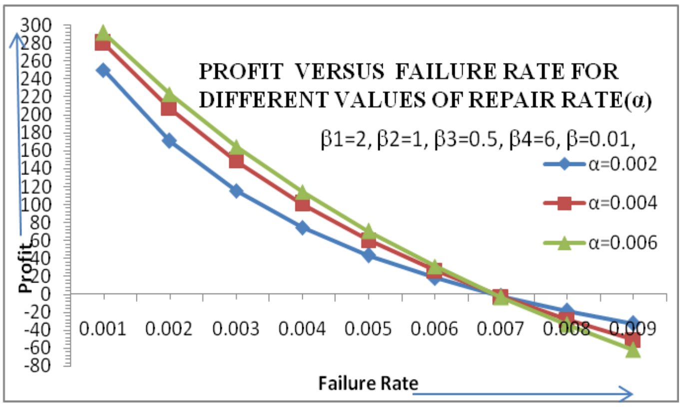


Figure 2: Profit versus Failure Rate

Figure 3 represents the behaviour of profit (P) with respect to revenue per unit time (C_0) for different values of assembling/disassembling rate (C_1)

We observe the following from the graph:

i) The profit increases with the increase in the values of revenue rate (C_0) and has lower values for higher values of repair rate (C_1)

ii) For C_1=100, the profit is negative or zero or positive according as C_0< or = or > 69.24 and hence revenue rate should not be less than 69.24

iii) For C_1=1100, the profit is negative or zero or positive according as C_0< or = or > 124.72 and hence revenue rate should not be less than 124.72

iv) For C_1=2100, the profit is negative or zero or positive according as C_0< or = or > 188.46 and hence revenue rate should be fixed not less than 188.46

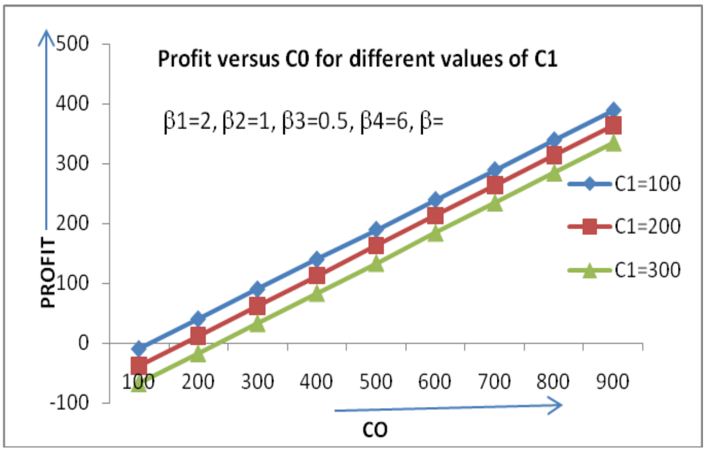


Figure 3: Profit versus C_0

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