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A Multiphase Queuing System with Assorted Servers by using Matrix Geometric Method

Roshli. Aniyeri and Dr. C. Ratnam Nadar

[1]Research Scholar, Department of Mathematics NoorulIslam University,
[2]Dean Students Welfare, NoorulIslam University,
[1,2]Kumaracovil, Thucklay, Kanyakumari district, Tamilnadu, India.

[1]Orcid: 0000-0001-6223-6199

Abstract

In this cram a matrix-geometric method is used for analyzing queuing model with multiphase queuing system. Passenger arrives at the terminal randomly following a Poisson process with state-dependent rates. Service times follows exponentially distributed. The main result of this paper is the matrix-geometric solution of the steady-state queue length from which many performance measurements of this queuing system like the stationary queue length distribution, waiting time distribution and the distribution of regular busy period, system utilization are obtained. Numerical examples are presented for both cases.

Key words: Matrix-Geometry methods, Queue size, steady state, Transition probability stationary probability.

INTRODUCTION

Queuing model with multiserver have a wide range of application in the last so many years. There are so many works related to multiserver queue. The study on multi-server queuing systems normally assumes the servers to be consistent in which the individual service rates are the same for all the servers in the system. This conjecture may be convincing only when the service process is mechanically or electronically controlled. In a queuing system with human servers, the above assumption can scarcely realized. It is frequent to observe that, server depiction service to identical jobs at different service rates. This reality leads to modeling such multi-server queuing systems with assorted servers that is the service time distributions may be different for different servers.

In multiserver queuing model have two servers .In this study of international Airport terminal in Kerala we stumble upon two servers with one server for bulk arrival and one server for individual passengers.

The matrix geometric method is a method for the analysis of quasi-birth–death processes, continuous-time Markov chain whose transition rate matrices with a repetitive block structure. It requires transition rate matrix with tridioganal block structure. This method was developed by Marcel F. Neuts and his students starting around 1975

Matrix-geometric methods approach is a useful tool for solving the more complex queuing problems of the rapid growth of the state-space introduced by the need to construct the generator matrix. Matrix-geometric method is applied by many researchers to solve various queuing problems in different frameworks. Neuts [19] explained various matrix geometric solutions of stochastic models. Matrix-geometric approach is utilized to develop the computable explicit formula for the probability distributions of the queue length and other system characteristic. Recently, Jain [17] analyzed a single server working vacation queuing model with multiple type of server breakdowns. They proposed a matrix-geometric method for computing the stationary queue length distribution. Recently, wang, Chen and Yang [32] studied the machine repair problem with a working vacation policy M/M/1, where the server may work with different repair rates rather than completely terminate during a vacation period. Ke [11] suggested the optimal control of an queuing system with server vacations, startup and breakdowns M/G/1.

Performance evaluation of queuing systems as a QBD (Quasi Birth and Death) process, having structured Markov chain can efficiently be done through matrix geometric method [1]. For infinite and finite capacity queue based on Markovian distribution can be analyzed for various performance measures using matrix geometric method [26]. Markovian queuing systems can efficiently be solved through matrix geometric method as compared to other numerical solution techniques and main block matrices can easily be obtained without constructing the Markov chain .The QBD process has a special tridiagonal structure in its transition matrix and it can be solved efficiently through the matrix geometric method, which is one of the matrix analytic methods [31]

Avi-Itzhak and Naor’ and Gave[4] have studied a single-server Poisson queue with server breakdowns. The service and repair times follow a general distribution. The steady-state waiting time distribution and average queue length have been obtained. However, the arrival rate is not dependent on operational state or repair state of the server. Yechiali and

Naor[35] have considered a single-server exponential queuing model with arrival rate depending on operational state or breakdown state of the server. The steady-state mean queue length is obtained for a system with infinite capacity. Fond and Ross[7] analyzed the same model with the assumption that any arrival finding the server busy is lost, and they obtained the steady-state proportion of customers lost. Shogan[27] has dealt with a single-server queuing model with arrival rate dependent on server breakdowns. When the system is operational, it functions as a single-server Poisson queue with Erlang K-distributed service times and when it is not, no service takes place but customers continue to arrive according to a Poisson process with the arrival rate different from that when the system is operational. The operational period and failed period follow exponential and Erlang distributions, respectively. Steady-state waiting time distribution and average queue length have been obtained using the generating function technique. Neuts and Lucantoni [18] have studied a Markovian queuing model with N servers subject to breakdowns and repairs. Hur,S., Kim ,J., and Kang ,C[9] has analyzed a single-server Poisson queue with service time following a general distribution, time- and operation-dependent server failures, and arrival rate dependent on the state of the server. Matrix-geometric approach is utilized to develop the computable explicit formula for the probability distributions of the queue length and other system characteristic. Matrix-geometric methods approach is a useful tool for solving the more complex queuing problems of the rapid growth of the state-space introduced by the need to construct the generator matrix. Matrix-geometric method is applied by many researchers to solve various queuing problems in different frameworks. Neuts [19] explained various matrix geometric solutions of stochastic models. Matrix-geometric approach is utilized to develop the computable explicit formula for the probability distributions of the queue length and other system characteristic. Recently, Jain and Jain [17] analyzed a single server working vacation queueing model with multiple type of server breakdowns. They proposed a matrix-geometric method for computing the stationary queue length distribution.

The queueing systems with single or multiple vacations have been introduced by Levy and Yachiali [15]. The literature about vacation models is growing rapidly which includes survey papers by Teghem [29].We can find general models in Tian and Zhang [36]. In 2007, Li and Tian [first introduced vacation interruption policy and studied an M/M/1 queue. Recently, have driven a new elegant explicit solution for a two heterogeneous servers queue with impatient behavior. Author considers an M/M/R queue with vacations, in which the server works with different service rates rather than completely terminates service during his vacation period During the period, customers can be served at a lower rate. Using the matrix-analytic method, he analysis the necessary and sufficient condition for the system to be stable. Levy and Yechiali [15] have discussed the vacation policy in a multi-server Markovian queue. They have considered a model with 's' homogeneous servers and exponentially distributed vacation times. Using partial generating function technique, the system size has been obtained. Kao and Narayanan[14] have discussed the M/M/s queue with multiple vacations of the servers using a matrix geometric approach. Gray have discussed a single counter queueing model involving multiple servers with multiple vacations. A researcher have discussed an M/M/s queue with multiple vacation and 1-limited service. Neuts and Lucantoni [18] have analyzed the M/M/s queueing systems where the servers are subject to random breakdowns and repairs. Baba extended Servi and Finn's M/M/1/ queue to a GI/M/1/ queue. They not only assumed general independent arrival, they also assumed service times during service period, service times during vacation period as well as vacation times following exponential distribution. Furthermore, Baba derived the steady- state system length distributions at arrival and arbitrary epochs. Neuts and Takahashi[19] observed that for queueing systems with more than two heterogeneous servers analytical results are intractable and only algorithmic approach could be used to study the steady state behavior of the system. Krishna Kumar and Pavai Madheswari [12] analyzed M/M/2 queueing system with heterogeneous servers where the servers go on vacation in the absence of customers waiting for service. Based on this observation, Krishnamoorthy and Sreenivasan [13] analyzed M/M/2 queueing system with heterogeneous servers where one server remains idle but the other goes on vacation in the absence of waiting customers. In this paper we discuss an M/M/2 queueing system with multi servers without vacation using matrix geometric method.

This paper analysis matrix geometric method in multiphase queuing system. There are so many studies about matrix geometric method. But nobody studied matrix geometric method in the multiphase queuing system in International Airport terminals in Kerala.

In probability theory, the matrix geometric method is a method for the analysis of quasi-birth–death processes, continuous-time Markov chain whose transition rate matrices with a repetitive block structure. The method was developed by Marcel F. Neuts and his student around 1975.The method requires a transition rate matrix with tridiagonal block structure.

The objectives of this paper are

- Develop a queuing model
- Construct a structural Markov chain.
- Develop a generator matrix.
- To find steady state equation to determine the steady state probability distribution of the number of units in the system.

- To find the expected number of passengers in the system.
- To perform an analysis through numerical values.

Model Description

Consider a M/M/2 queuing system where the service rate of the server are not identical. The server is modeled as oscillating b/w two queues a batch queue and individual queue. The passenger enter the terminal with single queue then after entering the terminal the batch queue passengers join the queue when the servers are doing batch service. Other passengers will join the individual queue. Arrivals of passengers follow Poisson distribution with parameter λ. The total number of passengers and the system capacity are assumed to be finite. There are two exponential servers with service rate μ_1 and μ_2 ($\mu_1 \neq \mu_2$) for server one and server 2 respectively. Passengers enter in to the terminal with a single queue then move to server 1 who is serving for bulk Passengers and server 2 for individuals. The arriving customers are served up on FCFS discipline. The queuing model with multiserver can be formulated by a continuous Markov chain .The possible states of the system at any epoch are represented by a double (i ,j) where i ≥0 denotes the number of passengers in the system j = 1,2,3 denotes the states of the server .The state (i,1) corresponds to i passengers are in the system and server one is busy. The state (i,2) corresponds to i passengers are in the system and server two is busy. The state (i,3) corresponds to i passengers are in the system and server one and server two is busy. The infinite state Markov chain for the model under study has a transition rate matrix Q which has a block tridiaogonal structure given by

$$Q = \begin{pmatrix} B_{00} & B_{01} & & & & \\ B_{10} & A_1 & A_2 & & & \\ & A_0 & A_1 & A_2 & & \\ & & A_0 & A_1 & A_2 & \\ & & & A_0 & A_1 & A_2 \\ & & & & \ddots & \ddots & \ddots \end{pmatrix}$$

- Matrix Q is infinitesimal generator of continuous time Markov chain and is the form of quasi birth death process. The sub matrix of order 3x3 is

- $A_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mu_1 & 0 \\ 0 & 0 & \mu_2 \end{pmatrix}$ $\quad A_2 = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}$

$$B_{00} = \begin{pmatrix} -\lambda & 0 \\ 0 & 0 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} -(\mu_1 + \mu_2 + \lambda) & \mu_1 & \mu_2 \\ 0 & -(\lambda + \mu_1) & 0 \\ 0 & 0 & -(\mu_2 + \lambda) \end{pmatrix}$$

$$B_{01} = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Clearly the continuous time Markov chain with generator A is reducible with absorbing state (3,3) and stationary probability vector π.The sub matrix form which satisfies the condition

$\pi A_2 e < \pi A_0 e$ is the necessary and sufficient condition for stability of QBD process where e denote the column vector with all its elements equal to one .Also the stability condition turns out to be

$$\lambda < \mu_1 + \mu_2 \quad \rightarrow \qquad (1)$$

under the stability condition ,the stationary probability vector X of the generator Q exist this stationary probability vector π partioned as $\pi = (\pi_0, \pi_1, \pi_2, \ldots..)$ is given by

$$\pi_0 B_{00} + \pi_1 B_{10} = 0 \quad \rightarrow \qquad (1.1)$$

$$\pi_0 B_{01} + \pi_1 A_1 + \pi_2 A_2 = 0 \rightarrow \qquad (1.2)$$

$$\pi_1 A_2 + \pi_2 A_1 + \pi_3 A_0 = 0 \rightarrow \qquad (1.3)$$

$$\pi_{i-1} A_2 + \pi_i A_1 + \pi_{i+1} A_0 = 0 \rightarrow (1.4)$$

Observe that the relationship

$$\pi_i = \pi_1 R^{i-1} \rightarrow \qquad (1.5)$$

and the normal equation is

$$1 = \pi_0 + \pi_1 e + (I - R)^{-1} \pi_2 e \rightarrow \qquad (1.6)$$

Where π_0 is the probability that no passengers in the system. The vector

$\pi_i = (\pi_{i0}, \pi_{i1}, \pi_{i2})$ and $\pi_1 = (\pi_{10}, \pi_{11}, \pi_{12})$ for $i \geq 2$ are probability for i passengers in the system in which π_{ij} refers to the joint probability that are i customers in the system and j = 0,1,2,3 corresponds to the status of the servers in the system. I is the identity matrix of order three and matrix R is minimal nonnegative solution with spectral radiusless than one of the matrix quadractic equation $A_0 + RA_1 + R^2 A_2 = 0 \rightarrow (2)$

Since the square matrix A_0, A_1, A_2 of order 3x3 are upper triangular R is also a 3x3 upper triangular matrix.now the relation $RA_2 e = A_0 e \rightarrow \qquad (3)$

Which implies the rate of transition from a state where there are i passengers to a state with i+1 matches the transition rate from i to i-1.To obtain R by equation (2)

$$R = A_0A_1^{-1} - R^2 A_2A_1^{-1} \rightarrow (4)$$

By taking the initial value of R = 0 the we can solve R and check the accuracy of this approximation by using equation (3) .The value of R will converge since

($-A_1^{-1}$) and $A_0 + R^2 A_2$ are positive .Hence each iteration the elements of R will increase monotonically. The boundary probabilities π_0 ,π_1, π_2 and πi i > 3can be obtained from equation (1.1) to (1.6) The stedy state probability are then used to find mean waiting time.

Following are theorem related to above methodology.

Theorem

If the QBD process (X_k, J_k) k =0,1,2,….. is ergodic its limiting probabilities are given by $\pi_i = \pi_1 R^{i-1}$, $i = 2,3,.....$ where the rate matrix R is the minimal nonnegative solution to nonlinear equation $R = A_0 + RA_1 + R^2 A_2$ and the vectors $\pi_0$and $\pi_1$are the unique positive solution to the linear system.

Proof.

Let π be partitioned conformably with Q i.e. π = (π_0,π_1, π_2,π_3………) where π_i = ($\pi_{i1}\pi_{i2}$…………..π_{ik}) This gives the equation

$$\pi_0 B_{00} + \pi_1 B_{10} = 0$$

$$\pi_0 B_{01} + \pi_1 A_1 + \pi_2 A_2 = 0$$

$$\pi_1 A_2 + \pi_2 A_1 + \pi_3 A_0 = 0$$

..

…………………………………… $\rightarrow 1$

………………………………………

$$\pi_{I-1} A_2 + \pi_I A_1 + \pi_{I+1} A_0 = 0$$

In analogy with the situation there exist a constant matrix R such that $\pi_i = \pi_{i-1} R$, $i = 2,3,........ \rightarrow 2$ The sub vectors π_i are geometrically related to each other since $\pi_i = \pi_1 R^{i-1}$, $i = 2,3,..... \rightarrow 3$ Substitute equation (3)in to equation (1) we get

$$\pi_{i-1} A_2 + \pi_i A_1 + \pi_{i+1} A_0 = 0 \textit{ gives}$$

$$\pi_1 R^{i-2} A_2 + \pi_1 R^{i-1} A_1 + \pi_1 R^i A_0 = 0$$

$$\pi_i R^i (A_2 + A_1 R + A_0 R^2) = 0$$

so we can find R from

$$A_0 + A_1 R + A_2 R^2 = 0 \quad \rightarrow 4$$

$$\textit{i.e. } A_2 A^{-1} + R + R^2 A_0 A^{-1} = 0$$

$$\textit{i.e. } R = -(A_2 A^{-1} + A_0 A^{-1})$$

$$\textit{i.e.} R = -V - W R^2 \quad \rightarrow 5$$

$$R_0 = 0 \quad , \quad R_{k+1} = -V - W R_k^2 \quad k = 1,2,3,....$$

derivation of π_0 ,π_1 *first and second equation of* $\pi Q = 0$ *are*

$$\pi_0 B_{00} + \pi_1 B_{10} = 0$$

$$\pi_0 B_{01} + \pi_1 A_1 + \pi_2 A_0 = 0$$

replace π_2 *by* $\pi_1 R$ *we get*

$$(\pi_0 \ \pi_1) \begin{bmatrix} B_{00} & B_{01} \\ B_{10} & A_1 + RA_0 \end{bmatrix} = 0 \quad \rightarrow 6$$

can be solved for π_0 ,π_1 *with condition* $\pi e = 1$

$$1 = \pi e = \pi_0 e + \pi_1 e + \sum_{i=2}^{\infty} \pi_i e$$

$$= \pi_0 e + \pi_1 e + \sum_{2}^{\infty} \pi_1 R^{i-1} e$$

$$= \pi_0 e + + \sum_{1}^{\infty} \pi_1 R^{i-1} e$$

$$= \pi_0 e + \sum_{0}^{\infty} R^i e$$

$$= \pi_0 e + \pi_1 (\sum_{0}^{\infty} R^i) e$$

The eigen value of R lie inside the unit circle which means that I –R is non singular and hence that

$$\sum_{0}^{\infty} R^i = (I - R)^{-1} \quad \rightarrow 7$$

Normalize the vector π_0 and π_1 by computing $\alpha = \pi_0 e + (I-R)^{-1} \pi_1 e$ divide the computed vectors π_0 and π_1 by α

Proof

If the QBD is positive recurrent then the series $\sum_{n \geq 0} \pi_n$ must converge since $\pi_n = \pi_0 R^n$ for n >0 ,the series $\sum_{n \geq 0} R^n$ must converge ,and this implies that sp(R) <1.

Theorem

The Markov process Q is ergodic if and only if $\pi A_2 e < \pi A_0 e$ *where e is the column vector of unity and* π *is the solution of* $\pi (A_0 + A_1 + A_2) = 0$ *and* $\pi e = 1$

Theorem

The Markov process *with* the generator Q is positive *recurrent if and only if*

$\lambda < c\mu$.

Performance indices

The performance prediction in terms of system measures is an important aspect of queuing modeling. The performance characteristic is used to bring out the qualitative behavior of the queuing model under study .We use matrix–geometric method, to find the following performance measures:

Expected number of passengers in the queue $E(n) = \pi_1 R(I-R)^{-2} e$

Numerical illustration

In this fragment numerical results have obtained by the method described in this paper to gain and understanding of the performance of this queue, the effect of parameter λ, μ_1, μ_2 0n the specific probabilistic description ,mean number of passengers in the system. The applications of multiphase queuing models in International Airport illustrate using matrix geometric method analytically. The analytical results established can be obtained numerically by taking a suitable illustration. We use the following values of the parameters:

$\lambda = 1$ $\mu_1 = 4$ $\mu_2 = 5$

-1 0 0 0	1 0 0 0 0 0			
0 0 0 0 4 0 0 0 5	-10 4 5 0 -5 0 0 0 -6	1 0 1 0 1 0 0 0 1		
	0 0 0 0 4 0 0 0 5	-10 4 5 0 -5 0 0 0 -6	1 0 0 0 1 0 0 0 1	......
		0 0 0 0 4 0 0 0 5	-10 4 5 0 -5 0 0 0 -6	1 0 0 0 0 1 0 0 1

The matrix obviously has the correct QBD structure. Since in the above matrix each transition rate matrix row sum equal to zero. This implies above block matrix chosen is correct.

Now to check the Markovian is ergodic.i.e we have to check

$\pi A_2 e < \pi A_0 e$

Let us take $A = A_0 + A_1 + A_2$ and $\pi A = 0$ *and* $\pi e = 1$

$$A_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mu_1 & 0 \\ 0 & 0 & \mu \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} -(\lambda+\mu_1+\mu_2) & \mu_1 & \mu_2 \\ 0 & -(\mu_1+\lambda) & 0 \\ 0 & 0 & -(\lambda+\mu_2) \end{bmatrix}$$

$$= \begin{bmatrix} -10 & 4 & 5 \\ 0 & -5 & 0 \\ 0 & 0 & -6 \end{bmatrix}$$

Therefore $A = A_0 + A_1 + A_2 = \begin{bmatrix} -9 & 4 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Solving $\pi A = 0$ $\pi e = 1$we get stationary probability vector

$\pi_i = (0.3077 \quad 0.6923 \quad 0)$

and = $\pi A_2 e = 1$,

$\pi A_0 e = 2.7692$

Therefore $\pi A_2 e < \pi A_0 e$ Which implies markovian is ergodic.

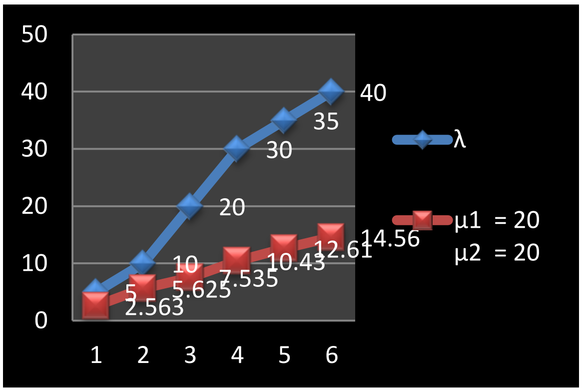
CONCLUSION

In this Paper an M/M/C/N Queue is studied. This study extended single server concept to the multi-server queue. The steady-state probability and the closed-form expression of the rate matrix were obtained using matrix-analytical method . Using QBD process and matrix-geometric solution method to derive the distribution for the stationary queue length and waiting time of a customer of a system. Finally, numerical results were provided. For real life situations, where the arrival of customer or job depends on the server status, such performance indices established. may be very helpful in the designing and development of many system in the field of computer system, manufacturing system and telecommunication networks, etc.

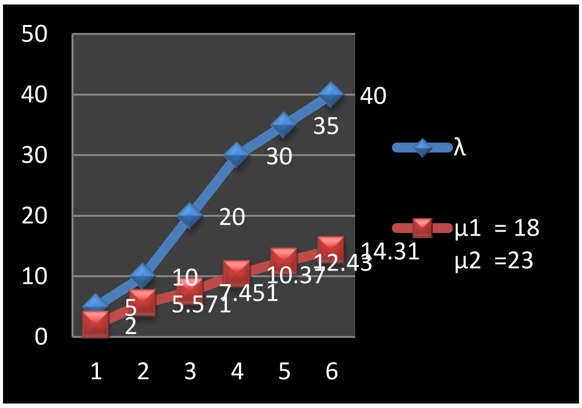
Mean system size E(N) versus arrival rate λ

Mean number of passengers in the system					
λ	μ_1 = 15 μ_2 = 18	μ_1 =16 μ_2 = 20	μ_1 = 18 μ_2 =23	μ_1 = 20 μ_2 =20	μ_1 = 22 μ_2 =19
5	2.04	2.352	2	2.563	2.517
10	5.53	5.115	5.571	5.625	5.89
20	7.13	7.245	7.451	7.535	7.125
30	10.2	10.26	10.37	10.43	10.68
35	12.1	12.12	12.43	12.61	12.53
40	14.0	14.22	14.31	14.56	14.72

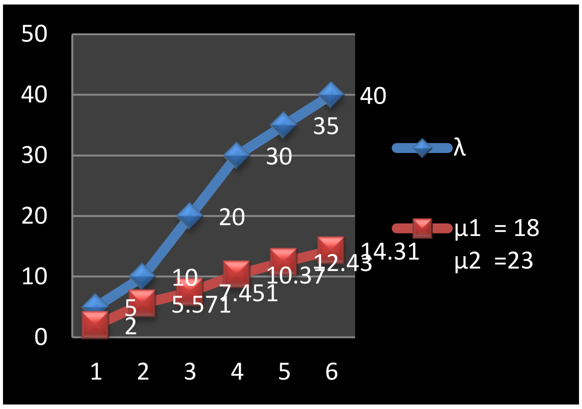
From table we can observed that the expected queue length increases when the system capacity increases



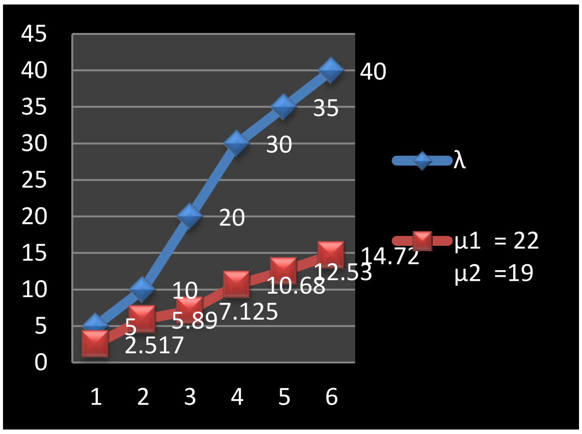
Average queue length over different values of λ if μ_1 = 15 and μ_2 =18



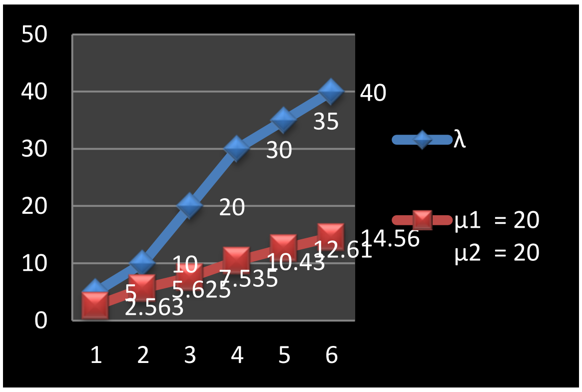
Average queue length over different values of λ if μ_1 = 16and μ_2 =20



Average queue length over different values of λ if μ_1 = 18and μ_2 =23



Average queue length over different values of λ if μ_1 = 22 and μ_2 =19



Average queue length over different values of λ if μ_1 = 20 and μ_2 =20

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