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An Investigation of Direction of Arrival Estimation Scheme for Correlated Signals in Wireless Communication Systems

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Abstract

Over the past decades, a number of DOA estimation algorithms have been studied and the most outstanding subspace super resolution algorithms such as MUSIC and ESPRIT became favorable topics of study in estimating direction of arrival (DOA). MUSIC is the most commonly used super resolution algorithm due to its accuracy, high resolution and stability under certain conditions. However, MUSIC algorithm works on the premise that the signals are uncorrelated and can lead to degradation of the performance of MUSIC in multipath propagation of wireless communication systems that have highly correlated signals. This failure is due to loss of non-singularity property of the covariance matrix that is used in the signal model. This paper focuses on investigating a computationally efficient spatial smoothing technique using Uniform Linear Arrays (ULA) to solve the problem of correlation in wireless communication systems. The direction of arrival estimation is simulated on a MATLAB platform with a set of input parameters such as array elements, signal to noise ratio, number of snapshots and number of signal sources and the RMSE has been used to show the performance. Simulation has been conducted and the results show that forward-backward spatial smoothing (FBSS) method can achieve an accurate and efficient DOA estimation for correlated signals.

Keywords: MUSIC algorithm, DOA estimation, Uniform Linear Array (ULA), Forward/Backward Spatial Smoothing

INTRODUCTION

The function of the sensor arrays is to collect and process electromagnetic or acoustic signals. This has numerous applications in wireless communications, sonar, radar, earthquake, seismology, biomedicine, imaging and also for emerging technologies such as sensor networks [2].

As the name implies array signal processing deals with the processing of information bearing signals collected by an array of sensors operating in an environment of interest (This could be above the ground or even underwater) [3].

When array signal processing is compared with traditional orientation sensor, sensor array can control the beam flexibly with high signal gain and reduce interference and it is for this reason that array signal processing theory has become an active area of research [2].

For applications where a sensor array was to be used for locating a signal source for example finding the source direction of arrival (DOA), one of the key theoretical developments was the parametric vector space formulation introduced by Schmidt and others in 1980s [4]. Early contributions to array signal processing have been made mostly in the context of wireless communications and radar systems in the first half of the 20th century. In the second half of the 20th century, tremendous progress of digital processing hardware led to numerous new developments and applications [5].

The first approach to carrying out space-time processing of data sampled at an array of sensors was spatial filtering or beamforming. The conventional beamformer also known as the Bartlett beamformer dates back to the Second World War. The spatial filtering approach, however suffers from fundamental limitations for example the performance depends on the physical size of the array regardless of the available data collected and signal-to-noise ratio (SNR) [6].

Several investigations have been done in the field of array processing and have led to discovery of direction of arrival (DOA) estimation which is a technique of estimating the direction arrival of impinging signals on the antenna array. In general DOA estimation can be categorized into two groups; the conventional algorithm and the subspace algorithm. It has been proven that subspace algorithms are better in terms of performance compared to conventional algorithm since they do not rely on the physical size of the antenna array [7].

MUSIC is the most popular, simple and efficient technique of subspace algorithm used in direction of arrival (DOA) estimation. It is based on the assumption that the signals impinging on the array are uncorrelated. This makes the performance of MUSIC degrade when the signals are correlated in multipath propagation and this has led to numerous research works to make MUSIC work by modifying the covariance matrix through a suitable preprocessing scheme. With this point in view, this paper investigates a computationally efficient spatial smoothing method that resolves the uncorrelated condition in MUSIC by using uniform linear arrays (ULA) and determine the effect of one channel noise for three sources, the effect of each signal source having an independent channel noise. In this paper, a comparative study of DOA estimation using uniform linear array antennas is proposed and focuses on some of the factors that affect the accuracy and resolution of the system based on multiple signal classification (MUSIC) algorithm and the performance evaluation is based on ULA. This paper is organized as follows. In Section 2, the signal model of the uniform linear array is presented. In Section 3, the Eigen decomposition of array covariance is given. Section 4, shows DOA estimation for multipath signals and brief discussion on spatial smoothing, Section 5, shows the implementation steps for MUSIC algorithm, simulation and discussion of the results are presented in Section 6. Finally, conclusion and recommendation for future work is highlighted in Section 7.

UNIFORM LINEAR ARRAY (ULA) SIGNAL MODEL

Linear antenna array is an array composed of several array elements aligned in a straight line on a plane as shown in Figure 1. In general the linear array is easy to analyze compared to other array geometries because of its simple structure. However, there are two main disadvantages of linear antenna array that limits its usage for DOA estimation. The first disadvantage is that the capability of linear antenna array is limited to one dimension DOA estimation, which means it can only estimate the elevation angle (Θ) of the incoming signals. This is a limitation to practical implementation since the direction of incoming signals is defined by both elevation angles (Θ) and azimuth ($\boldsymbol{\varphi}$) [8].

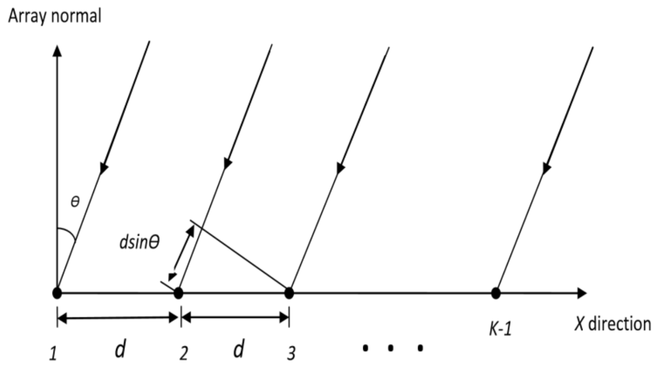

Figure 1: A uniformly spaced linear array model

Figure 1 illustrates a uniformly spaced linear array with *K* identical Omni-directional antenna elements located along the x-axis direction. The inter-element spacing is denoted by *d*. If a plane wave impinges upon the array at an angle Θ with respect to the array normal, as shown in figure 1, the wave front arrives at element *K+1* sooner than at element *K*, since the differential distance along the two ray paths is $d\sin\theta$. Therefore by setting the phase of the signal at the origin arbitrarily to zero, the phase leads of the signal at the first element 1 is:

$$\varphi = \frac{2\pi k}{\lambda} d\sin\theta \tag{1}$$

Where λ is the wavelength [34].

All elements outputs can be summed to provide the total array factor $F(\theta)$

$$F(\theta) = \sum_{K=0}^{K-1} w_k e^{\frac{j2\pi}{\lambda} k\, d\sin\theta} \tag{2}$$

The complex weight can be represented as

$$w_k = A(k)e^{jk\alpha} \tag{3}$$

Where the phase of k^{th} element leads that of the $(k-1)^{th}$ element byα, the array factor becomes:

$$F(\theta) = \sum_{K=0}^{K-1} A(k) e^{\frac{j2\pi}{\lambda} k\, d\sin\theta + jk\alpha} \tag{4}$$

If α is given by

$$\alpha = -j\frac{2\pi}{\lambda} d\sin\theta_0 \tag{5}$$

Where α is the phase term and θ_o is the direction angle the antenna is steered, a maximum response of $F(\theta)$ will result at θ_o. That is the antenna beam has been steered towards the wave source.

For illustration purposes as shown in Figure 1, by considering a three element that is aligned uniformly along a straight line as shown in Figure 1. The left most element is numbered as element *1* and taken as the element reference. The other two elements are numbered as element *2* and element *3* in a sequence from left to right. The distance between two neighboring elements is *d*. A source emits electromagnetic waves at such a far distance that the three propagation line paths from the source to the three elements can be considered parallel. The waves impinge on the array at an angle θ. As a result, the line paths from the source to elements *2* and *3* are longer than the path to element *1* by an extra distance of:

$$d_m = (m-1)d\sin\theta \qquad m = 0,1,2\ldots \tag{6}$$

Now assume that the wave signal received by the reference element is:

$$x_1(t) = S(t) \tag{7}$$

If noise is ignored, then the signals received by element *2* and *3* can be written as:

$$x_2(t) = S(t)e^{-j\beta d_1} = S(t)e^{-j\frac{2\pi d}{\lambda}\sin\theta} \tag{8}$$

$$x_3(t) = S(t)e^{-j\beta d_2} = S(t)e^{-j2\frac{2\pi d}{\lambda}\sin\theta} \tag{9}$$

Let the phase shift constant of the wave be:

$$\beta = \frac{2\pi}{\lambda} \tag{10}$$

The phase shift term $e^{-j\beta d_m}$ in *x* and *y* is the result of the signal propagating over an extra distance d_m in comparison with the path to the right most element 2. In a generalized way, the signals received by the three elements can be written as:

$$X = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} 1 \\ e^{-j\frac{2\pi d}{\lambda}\sin\theta} \\ e^{-j2\frac{2\pi d}{\lambda}\sin\theta} \end{bmatrix} S(t) = \begin{bmatrix} 1 \\ e^{-j\mu} \\ e^{-j2\mu} \end{bmatrix} S(t) = a(\mu)\, S(t) \tag{11}$$

Where $\mu = \frac{2\pi d}{\lambda}\sin\theta$ and $a(\mu) = [1 \ e^{-j\mu} \ e^{-j2\mu}]^T$, which is often called the array steering vector.

Since the discovery of array signal processing, uniform linear arrays (ULA) have received by far the most attention. This is due to the fact that their uniform spatial sampling which results in their array response vectors having a vandermonde form. This form is very essential to derivation of many important DOA estimation algorithms [8].

Well developed methods are existing for designing linear arrays with equal spacing of elements that produce a desired radiation pattern with reasonable accuracy. However, for conventionally designed arrays where all elements are equally spaced, there exists an upper limit to the element spacing if grating lobes are not to appear in the field pattern [10].

EIGEN DECOMPOSITION OF ARRAY COVARIANCE

For array output *X*, the covariance matrix R_x is given by:

$$R_x = E[XX^H] \tag{12}$$

Where $E\{.\}$ denotes expected value and *H* is the conjugate transpose matrix. Assuming signal and noise is uncorrelated, and the noise is zero mean white noise, substitute the value of *X* as given below:

$$X = AS + N \tag{13}$$

Where *A* is the steering matrix, *S* is the signal matrix and *N* is the noise matrix. Substitute (13) into (12).

$$R_x = E[(AS+N)(AS+N)^H] \tag{14}$$

$$R_x = AE[SS^H]A^H + E[NN^H] \tag{15}$$

$$R_x = AR_sA^H + R_N \tag{16}$$

Where

$R_s = E[SS^H]$ is called the signal correlation matrix and $R_N = \sigma^2 I$ is the noise correlation matrix, σ^2 is the noise power, *I* is the unit matrix of *M*×*M* dimension. In practical applications R_x usually cannot be directly obtained and only the sample covariance $\tilde{R}_x$ as shown in (17) can be used and $\tilde{R}_x$ is the time average estimate of R_x

$$R_x = \tilde{R}_x = \frac{1}{N}\sum_{n=1}^{N} X(t_n)\, X(t_n)^H = \frac{1}{N} X^H X \tag{17}$$

The above equation is a fundamental data matrix used in most descriptions of DOA estimation algorithms. It is this data matrix that contains important information about the signal source [10].

When the number of samples *N*→∞, the process is ergodic hence the time average is a good approximation of the ensemble average, but in actual situations there are some errors because of the limitation of the samples number [1]. According to the theory that matrix can conduct eigenvalue decomposition to the array covariance matrix, first consider the ideal case where the noise does not exist

$$R_x = AR_sA^H \tag{18}$$

When the DOAs of the sources are different the columns of *A* are linearly independent to each other. Thus, *A* is a Vandermonde matrix [15].

For uniform linear array the matrix *A* is a Vandermonde matrix defined by

$$A = [a(\theta_1), (a(\theta_2), \ldots a(\theta_D))]^T \tag{19}$$

$$A = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-j\theta_1} & e^{-j\theta_2} & \cdots & e^{-j\theta_D} \\ \ldots & \ldots & \cdots & \ldots \\ e^{-j(M-1)\theta_1} & e^{-j(M-1)\theta_2} & \ldots & e^{-j(M-1)\theta_D} \end{bmatrix}$$

With

$$\theta_k = \frac{2\pi d}{\lambda}\sin\theta_k \tag{20}$$

$$\theta_i \neq \theta_k, i \neq j \tag{21}$$

Hence, if R_s is non-singular matrix Rank (R_s) = D, each signal source is independent, then;

$$\text{Rank}\left(AR_sA^H\right) = D \tag{22}$$

Since $R_x = E[XX^H]$ so

$$R_x^H = R_x \tag{23}$$

Rx is the Hermitian matrix, whose eigenvalue is real. Because R_s is positive definite, AR_sA^H is semi-positive definite, it has eigenvalues D.

In the presence of noise the covariance matrix is:

$$R_x = AR_sA^H + \sigma^2 I \tag{24}$$

Since $\sigma^2 > 0$, Rx is a full rank matrix, Rx has M positive real eigenvalues $\lambda_1, \lambda_2, \dots \lambda_M$ respectively corresponding to the M eigenvectors $v_1, v_2, \dots v_M$. for R_x is a Hermitian matrix where the noise and signal subspace are orthogonal. That is to say;

$$v_i^H v_j = 0 \quad i \neq j \tag{25}$$

In general, covariance matrix represents the cross covariance of noise corrupted signals that have impinged on the antenna array. The covariance matrix is an essential component in the determination of spatial power spectrum in order to estimate the direction of arrival [38].

DOA ESTIMATION FOR MULTIPATH SIGNALS

Much as DOA estimation is important there exists the problem of multipath which occurs when one or more signal reflections are received by the sensors. This means that a source in one direction is a scaled version of another with a different direction. $S_j(t) = re^{j\emptyset}S_i(t)$ for some i, j. In this case, the signal covariance matrix R_s is singular. Most of the DOA estimation algorithms completely fail in a multipath situation. However, some methods such as maximum likelihood methods can solve this problem but they are computationally expensive. Therefore forward-backward spatial smoothing can be used to de-correlate the signals and give more accurate estimates of the true DOAs [12].

Spatial smoothing

Spatial smoothing method for coherent signals was first proposed in [10]. In this paper an investigation of improved spatial smoothing method to solve the problem correlated signals in multipath situation is presented. The solution is based on a preprocessing scheme that partitions the total array of sensors into sub arrays and then generates the average of the sub array output covariance matrices. However other techniques such as Weighted Subspace Fitting (WSF) [6], spatial dithering based on mechanical dithering of the array exists[13].However, all these have a very high computational load, can be used with only uniform linear arrays (ULA) when compared to spatial smoothing method which can be used with other array configurations such as non-uniform linear arrays (NLA) to give more degrees of freedom and better performance when dealing with correlated signals.

The source covariance matrix R_S is non-diagonal and furthermore, if any two sources are fully coherent, the array manifold and R_z are also rank deficient and hence, they proposed a method to divide the ULA into overlapping sub arrays. Then, a smoothed covariance matrix is constructed by averaging over the covariance matrices of the sub arrays. More to the point, if the underlying ULA is divided into N_{tot} sub arrays each having N_{sub} physical sensors, then the smoothed covariance matrix is given by;

$$R_z = \frac{1}{N_{tot}}\sum_{i=1}^{N_{tot}} R_i \tag{26}$$

Where R_i is the $N_{sub} \times N_{sub}$ covariance matrix of the i^{th} sub-array.

The idea behind spatial smoothing is as follow;

Let a linear uniform array with M identical sensors be divided into overlapping forward sub arrays of size P, such that the sensor elements $\{1,\dots,P\}$ forms the first forward sub array and sensors $\{2,\dots,P+1\}$ forms the second forward sub array, let $x_k(t)$, denote the vector of received signals at K^{th} forward sub-array. We can now write;

$$x_k^f(t) = AF^{K-1}S(t) + n_k(t) \tag{27}$$

Where $F^{(K)}$ denotes the K^{th} power of the diagonal matrix.

The main idea behind spatial smoothing is de-correlating the correlative signals to eliminate the singularity of the self-correlative matrix that can be processed by the MUSIC method [14].

Implementation steps of MUSIC algorithm

Considering the signal sources to be narrowband and each has the same center frequency ω_o. The number of testing signal source is D and antenna array is spaced linear array which consists of M (M>D) array elements; each element has the same characteristics and it is isotropic in each direction. The spacing is d and the array element interval is not larger than half the wavelength of the highest signal frequency. Each antenna element is the far field source, namely an antenna array receiving the signals coming from the signal source is a plane wave and both array and test signals are uncorrelated.

The implementation steps of MUSIC algorithm are shown below;

i) Obtain the following estimation of the covariance matrix based on the N received signal vector

$$R_x = \frac{1}{N}\sum_{i=1}^{N} X(i)X^H(i) \tag{28}$$

Or collect the data estimate $R_x = E[XX^H]$

ii)For the eigenvalue, decompose the above covariance matrix

$$R_x = AR_sA^H + \sigma^2 I \tag{29}$$

iii) According to the order of eigenvalues, take eigenvalue and eigenvector which are equal to the number of signal D as signal part of space; take the rest, M-D eigenvalues and eigenvectors, as noise part of space.

iv) Get the noise matrix

$$E_n: A^H V_i = 0, i = D+1, D+2, \dots, M.\ E_n = [V_{D+1}, V_{D+2}, \dots, V_M]$$

v) Vary θ; according to the formula

$$P_{MUSIC}(\theta) = \frac{1}{a^H(\theta)U_n a(\theta)U_n^H} \tag{30}$$

SIMULATION RESULTS AND DISCUSSION

This section shows computer simulations using MATLAB software provided to investigate the performance analysis of DOA estimation algorithm using uniform linear arrays (ULA) for the conventional MUSIC algorithm and also apply the forward/backward spatial smoothing algorithm for DOA estimation in multipath scenarios to decorrelate the correlated signals. In both cases, the impinging or incoming source signals are narrowband signals operating at the same center frequency of 2.4GHz commonly used in wireless communications and the azimuth arrival angle interval is set to be in the range of $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$. The additive background noise is assumed to be spatially and temporarily uncorrelated white complex Gaussian with the zero-mean. The space between two adjacent array elements for uniform linear arrays (ULA) is known to be half wavelength ($d = \frac{\lambda}{2}$).

The simulations are done by considering the variation of DOA with the number of elements, variation of spacing between array elements, effect on performance on using a single signal to noise ratio (SNR) channel, effect on performance on using a separate signal to noise ratio (SNR) for each signal source, MUSIC algorithm performance for correlated signals for three signal sources, MUSIC algorithm for correlated signals for three signal sources by applying forward/backward spatial smoothing (FBSS) and the effect of the degree of correlation on MUSIC performance for multipath signals. It is these factors that show the performance evaluation and resolution of any DOA estimation algorithm and therefore they will be applied for the case of uniform linear array (ULA). This is achieved by comparing the Root Mean Square Error (RMSE) values for the factors that affect direction of arrival estimation algorithms.

$$\text{RMSE}=\sqrt{\frac{1}{n}\sum_{k=1}^{n}(\theta - \widehat{\theta_k})^2} \tag{31}$$

With $k = 1, 2, \dots n$, where n is the number of trials, θ is the true angle of arrival and $\widehat{\theta_k}$ is a kth estimated angle of arrival.

Experiment 1: Validation of results

The MATLAB simulation results were done by comparing to the existing data in the literature. The blue, green and dashed red lines in Figure 2 are results presented by Pooja Gupta [16] and the black, yellow and magenta dashed lines are the present work with the increased number of snapshots. The Figure 2 gives the performance of MUSIC spectrum for six different variation of snapshot number. While keeping the other input parameters such as array size: M = 10 elements, signal to noise ratio: $SNR = 10dB$, true DOA: $20°$ and $70°$ and the signal frequency: $Freq = 2.4GHz$. In Pooja Gupta's work, the number of snapshots was varied from 10, 20 and 30 and for the current work; the number of snapshots is varied from 100, 500 and 1000. As observed, there is a good agreement between the simulated results and the published data by Pooja Gupta.

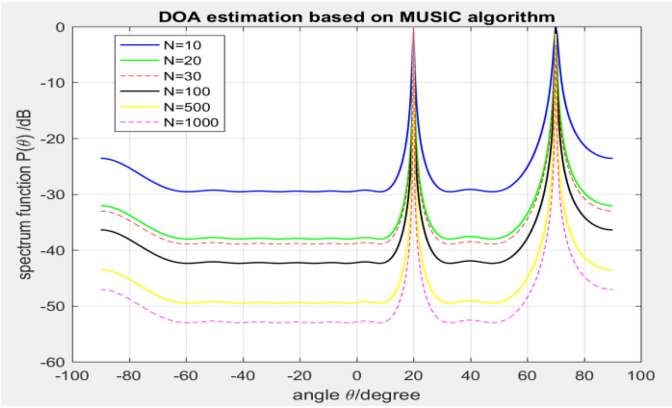

Figure 2: MUSIC spectrum for snapshot variation for ULA

The simulation results in Figure 2 show that the number of snapshots highly affects the performance of the MUSIC algorithm. It is clearly noted that the more the snapshots are increased the more the performance and accuracy of the direction of arrival estimation.

The results from fig 2 are summarized in Table 1 with 5 trials to be able to analyze the error using $RMSE$. The true angles of arrival are designed as θ_1 and θ_2 and then the estimated angles of arrivals are designed as $\widehat{\theta_1}$ and $\widehat{\theta_2}$. It can be noted that with N = 10 snapshots, N= 20 snapshots and N=30 snapshots, the Root Mean Square Error: $RMSE$ = 1.5865, $RMSE$ = 0.6626 and $RMSE$=0.5398 respectively for the arrivals at $\theta_1 = 60°$ and $\theta_2 = 70°$. These results show a good performance since the $RMSE$ used as a measure of accuracy reduces as the number of snapshots are increased as shown in table 1. This gives a further reduction in $RMSE$ value if the number of snapshots are increased to N=100 snapshots, N=500 snapshots and N=1000 snapshots.

Table 1: Estimated DOA with snapshots and error analysis

No of snapshots	θ_1	$\widehat{\theta_1}$	$Error_1$ $(\theta_1 - \widehat{\theta_1})$	θ_2	$\widehat{\theta_2}$	$Error_2$ $(\theta_2 - \widehat{\theta_2})$
10	20	20.0000	+0.0000	70	69.0000	+1.0000
	20	20.5000	+0.5000	70	71.5000	+1.5000
	20	20.0000	+0.0000	70	69.9000	+0.5000
	20	19.5000	+0.5000	70	71.2000	+1.2000
	20	20.5000	-0.5000	70	68.5000	+1.5000
			RMSE=1.5865			
20	20	20.0000	+0.0000	70	70.5000	-0.5000
	20	20.0000	+0.0000	70	70.5000	-0.5000
	20	20.5000	+0.5000	70	70.0000	+0.0000
	20	20.0000	+0.0000	70	69.9000	+0.1000
	20	19.9000	+0.5000	70	69.7000	+0.3000
			RMSE=0.6626			
30	20	20.0000	+0.0000	70	69.0000	+0.5000
	20	20.0000	+0.0000	70	70.0000	+0.0000
	20	20.0000	+0.0000	70	70.5000	-0.5000
	20	19.5000	+0.5000	70	70.0000	+0.0000
	20	20.0000	+0.0000	70	70.0000	+0.0000
			RMSE=0.5398			

Experiment 2: Variation of DOA with the number of array elements

The variation of the DOA estimation with the number of array size is simulated in experiment 2. In Pooja Gupta's work the number of array element was varied from M = 10, M = 20 and M = 30 in steps of 10 elements and the results of the DOA for M = 10, 20 and 30 elements in Pooja Gupta's are compared to M = 35, M =40 and M = 45 in the current work as shown in Figure 3. This simulation is based on MUSIC algorithm for uniform linear array (ULA). During the simulation of this experiment, the following signal and array parameters are kept constant: True DOA: θ_1 = 20° and θ_2 = 70°, signal frequency: $Freq$ = 2.4GHz, snapshots: N = 300, SNR = 10dB.

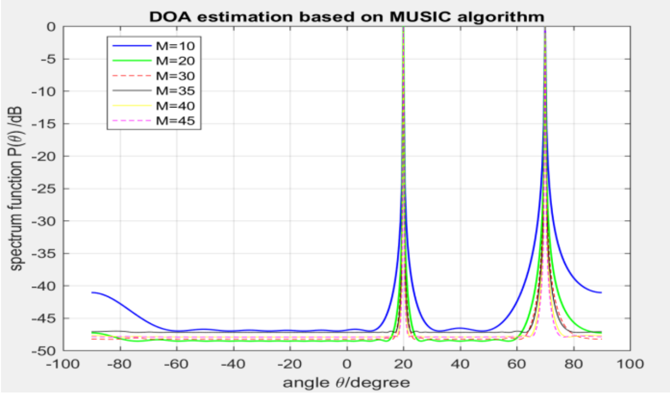

Figure 3: DOA estimation with variation of array elements for ULA

The simulation results in Figure 3 show that the number of array elements highly affects the performance of the MUSIC algorithm. It is clearly noted that the more the number of elements are increased the more the performance and accuracy of the direction of arrival estimation. The above simulation further illustrates that Pooja Gupta data is in agreement with the current work.

The results from Figure 3 are summarized in Table 2 and Table 3 with 5 trials to be able to analyze the error using $RMSE$. The true angles of arrival are designed as θ_1 and θ_2 and then the estimated angles of arrival are designed as $\widehat{\theta_1}$ and $\widehat{\theta_2}$. It can be noted that with M = 10 elements, M= 20 elements and M=30 elements, the Root Mean Square Error: $RMSE$ = 3.1794, $RMSE$ = 1.1804 and $RMSE$=0.4960 respectively for the arrivals at $\theta_1 = 60°$ and $\theta_2 = 70°$. These results show a good performance since the $RMSE$ used as a measure of accuracy reduces as the numbers of elements are increased as shown in table 2.

Table 2: Estimated DOA with elements and error analysis

No of Elements	θ_1	$\widehat{\theta_1}$	$Error_1$ $(\theta_1 - \widehat{\theta_1})$	θ_2	$\widehat{\theta_2}$	$Error_2$ $(\theta_2 - \widehat{\theta_2})$
10	20	19.2500	+0.7500	70	71.0000	-1.0000
	20	21.0000	-1.0000	70	65.5000	+4.5000
	20	21.0000	-1.0000	70	67.0000	+3.0000
	20	20.0000	+0.0000	70	69.9000	+0.1000
	20	19.5000	+0.5000	70	69.5000	+0.5000
			RMSE=3.1794			
20	20	19.2000	+0.8000	70	70.9000	-0.9000
	20	20.0000	+0.0000	70	71.0000	-1.0000
	20	20.0000	+0.0000	70	70.0000	+0.0000
	20	20.9000	+0.9000	70	70.0000	+0.0000
	20	20.0000	+0.0000	70	70.5000	-0.5000
			RMSE=1.1804			
30	20	20.0000	+0.0000	70	70.5000	-0.5000
	20	20.0000	+0.0000	70	70.0000	+0.0000
	20	20.0000	+0.0000	70	70.9000	-0.9000
	20	20.0000	+0.0000	70	70.1000	-0.1000
	20	20.0000	+0.0000	70	70.4000	-0.4000
			RMSE=0.4960			

As the number of elements are further increased to M = 35 elements, M= 40 elements and M=45 elements. The results are summarized in Table 3. This gives a further reduction in the Root Mean Square Error: $RMSE$ = 0.4733, $RMSE$ = 0.1483 and $RMSE$=0.1342 respectively for the arrivals at $\theta_1 = 60°$ and $\theta_2 = 70°$ as summarized in table 3.

Table 3: Estimated DOA with elements and error analysis

No of Elements	θ_1	$\widehat{\theta_1}$	$Error_1$ $(\theta_1 - \widehat{\theta_1})$	θ_2	$\widehat{\theta_2}$	$Error_2$ $(\theta_2 - \widehat{\theta_2})$
35	20	20.0000	+0.0000	70	69.5000	+0.5000
	20	20.0000	+0.0000	70	70.5000	-0.5000
	20	20.0000	+0.0000	70	69.5000	+3.0000
	20	20.0000	+0.0000	70	69.4000	+0.6000
	20	20.0000	+0.0000	70	69.9000	+0.1000
			RMSE=0.4733			
40	20	20.0000	+0.0000	70	70.3000	-0.3000
	20	20.0000	+0.0000	70	69.9000	+0.1000
	20	20.0000	+0.0000	70	69.9000	+0.1000
	20	20.0000	+0.0000	70	70.0000	+0.0000
	20	20.0000	+0.0000	70	70.0000	+0.0000
			RMSE=0.1483			
45	20	20.0000	+0.0000	70	69.8000	+0.2000
	20	20.0000	+0.0000	70	70.2000	-0.2000
	20	20.0000	+0.0000	70	70.0000	+0.0000
	20	20.0000	+0.0000	70	69.9000	+0.1000
	20	20.0000	+0.0000	70	70.0000	+0.0000
			RMSE=0.1342			

Experiment 3: Variation of spacing between array elements

The variation of spacing between array elements for DOA estimation is presented in experiment 3. In Pooja Gupta's work the variation is from d = *lambda*/6, d = *lambda*/2 and d = *lambda* and this same information is used to determine the performance when the spacing is varied below d = *lambda*/2 and also above d = *lambda*/2. The results in the current work are shown in Figure 4. This simulation is based on MUSIC algorithm for uniform linear array (ULA). During the simulation of this experiment, the following signal and array parameters are kept constant: True DOA: θ_1 = 20° and θ_2 = 70°, signal frequency: *Freq* = 2.4*GHz*, snapshots: N = 300, *SNR* = 10*dB*.

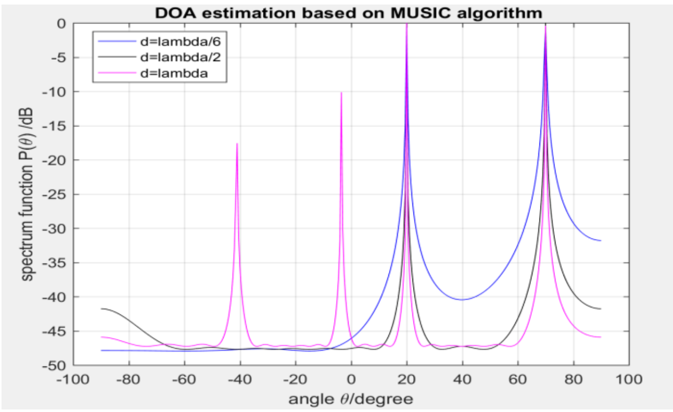

Figure 4: DOA estimation with variation of array element spacing for ULA

A spectral function for element spacing of *lambda*/6, *lambda*/2 and lambda is shown using solid blue line, black line and magenta line. It can be understood that we obtain better and narrower peaks when the distance increases from *lambda*/6 to *lambda*/2. But the spectrum losses efficiency when the element spacing is increased beyond lambda/2 as can be seen in Figure 4. Some false peaks are observed which are not the true DOAs defined by θ_1 = 20° and θ_2 = 70°. So the maximum allowable spacing in order to have the best efficiency is *lambda*/2 for uniform linear array (ULA).

Experiment 4: MUSIC algorithm for incoherent signals for three signal sources having the same channel SNR

This simulation shows the performance when three impinging signals are to be detected and one channel signal to noise ratio (SNR) is also considered to check the performance.

During the simulation of this experiment, the following signal and array parameters are kept constant: Signal frequency: *Freq* = 2.4*GHz*, Elements: M=8, Snapshots: N = 300, *SNR* = 0*dB*, DOAs of θ_1 = -20°, θ_2 = 60° and θ_3 = 70°.

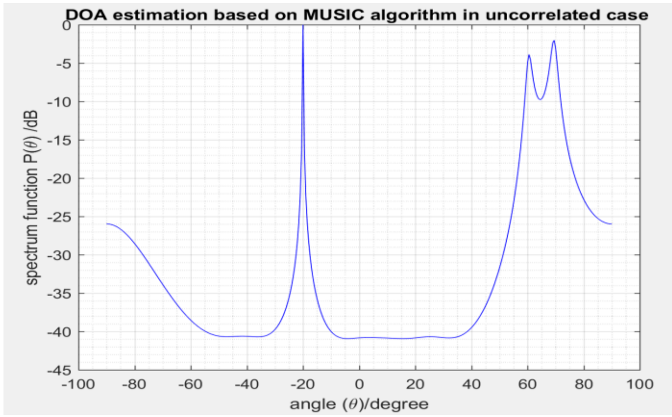

Figure 5: MUSIC spectrum for three signals with same channel noise

In Figure 5 it is clear that the three signals are also detected and are well defined at the given DOAs. It also shows that the more the angle differences the more the performance and resolution of DOA estimation algorithm.

Experiment 5: MUSIC algorithm for incoherent signals for three signal sources with different SNR respectively

This simulation shows the performance when three impinging signals are to be detected and each signal is assumed to have its own independent SNR when impinging the array and the performance is compared with that of Figure 5. To check the performance of the two different scenarios. The signal to noise ratio (SNR) is varied as: SNR_1=-5 *dB*, SNR_2=0 *dB* and SNR_3=5 *dB* respectively.

During the simulation of this experiment, the following signal and array parameters are kept constant: Signal frequency: *Freq* = 2.4*GHz*, Elements: M=8, Snapshots: N = 300, DOAs of θ_1 = -20°, θ_2 = 60° and θ_3 = 70°.

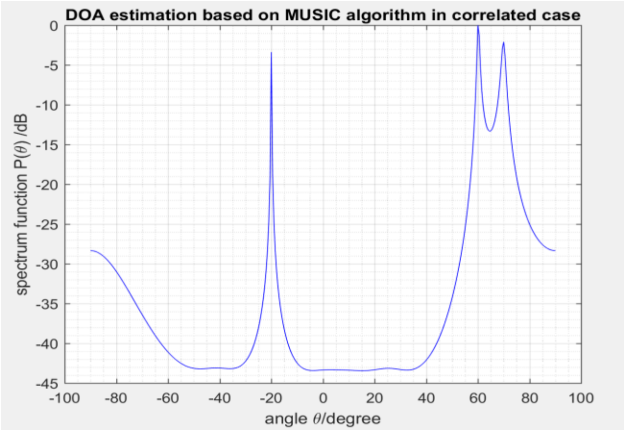

Figure 6: MUSIC spectrum for three signals with different SNR for each signal

This simulation in Figure 6 shows the performance when three impinging signals are to be detected and each signal is assumed to have its own independent SNR channel when impinging the array and the performance is compared with that of Figure 5. The MUSIC spectrum for Figure 6 shows an improvement in resolution as the three spectra become narrower and that shows that if we configure each signal at a specific SNR it is possible to obtain better results for any DOA estimation algorithm.

Experiment 6: MUSIC algorithm for coherent signals for three signal sources

This simulation shows the performances when three impinging signals are to be detected. The simulation is for the case of multipath scenario and shows what happens when the signals are correlated.

During the simulation of this experiment, the following signal and array parameters are kept constant: Signal frequency: *Freq* = 2.4*GHz*, Elements: M=8, SNR=0 *dB,* Snapshots: N = 300, DOAs of θ_1 = -20°, θ_2 = 60° and θ_3 = 70°.

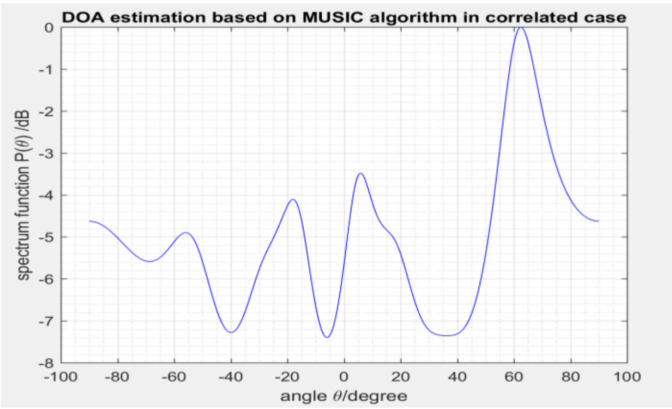

Figure 7: MUSIC spectrum for three signals for the case of correlation

The simulation in Figure 7 is when correlated signals are impinging on a uniform linear array (ULA) and shows that the MUSIC algorithm completely fails to estimate the direction of arrival θ_1 = -20°, θ_2 = 60° and θ_3 = 70°. The spectra are clearly undefined and this is due to the loss in the rank of the covariance matrix and also the covariance matrix is no longer non-singular.

Experiment 7: MUSIC algorithm for coherent signals for three signal sources by applying Forward/Back Spatial Smoothing (FBSS)

This simulation shows the performances when three impinging signals are to be detected. The simulation is for the case of multipath scenario and a technique called Forward/Backward Spatial Smoothing (FBSS) is applied to de-correlate the correlated signals in multipath environments. This technique restores the rank of the covariance matrix and also makes the matrix non-singular again and therefore corrects the problem experienced in multipath situation as shown in Figure 7.

During the simulation of this experiment, the following signal and array parameters are kept constant: Signal frequency: *Freq* = 2.4*GHz*, Elements: M=8, SNR=0 *dB,* Snapshots: N = 300, DOAs of θ_1 = -20°, θ_2 = 60° and θ_3 = 70°.

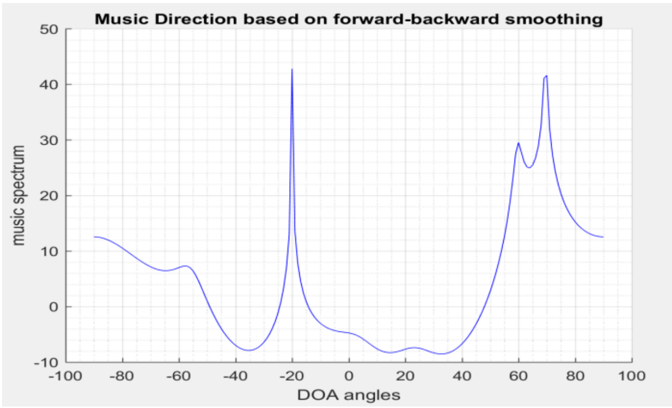

Figure 8: MUSIC spectrum for correlated signals using FBSS

In Figure 8 it is clear that the three DOAs (θ_1 = -20°, θ_2 = 60° and θ_3 = 70°) have been detected by using the FBSS technique to de-correlate the correlated signals from multipath signal environments. This FBSS technique uses the existing array structure and divides it into sub-arrays whose covariance matrix size is less than that of the original covariance matrix. After dividing the ULA into sub-arrays the corresponding covariance matrices are summed up to obtain a high rank matrix.

Experiment 8: Effect of the degree of correlation on DOA estimation for coherent signals of three signal sources by applying Forward/Back Spatial Smoothing (FBSS)

This simulation shows the performances when three impinging signals are to be detected. The simulation is for the case of multipath situation and shows what happens when the signals are highly, moderately and weakly correlated respectively as shown in Figure 9 and a technique called Forward/Backward Spatial Smoothing (FBSS) is applied to de-correlate the correlated signals in multipath environments. This technique restores the rank of the covariance matrix and also makes the matrix non-singular again.

During the simulation of this experiment, the following signal and array parameters are kept constant: Signal frequency: $Freq = 2.4GHz$, Elements: M=8, SNR=0 dB, Snapshots: N = 300, DOAs of θ_1 = -20°, θ_2 = 60° and θ_3 = 70°.

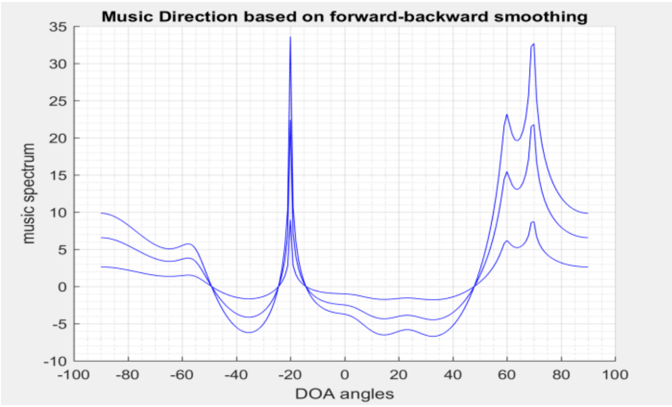

Figure 9: MUSIC spectrum for highly, moderately and weakly correlated signals using FBSS

It is seen from Figure 9 that the highly correlated signals are easily detectable compared to moderately and weakly correlated signals respectively. This is due to the fact that the higher the correlation coefficient (say $r \geq 0.5$) the more the signals are correlated and when the correlation coefficient decreases (say $r \leq 0.5$) then the signals are weakly correlated and it won't be easily detectable as shown in Figure 9. The correlation coefficient (r) should satisfy the following criteria: $0 \leq r \leq 1$ which means that the correlation coefficient should be defined within that range

CONCLUSION

The MUSIC algorithm for DOA estimation uses the Eigen values and Eigen vectors of the signal and noises to estimate the direction of arrival of the incoming signals by considering the fact that they are orthogonal to one another and therefore becomes easier to separate the signals from noise as the Eigen vectors for signal and noise subspace are orthogonal. MUSIC works efficiently when the signals that are being incident on the array of sensors are uncorrelated. The performance and resolution of MUSIC algorithm can be improved by increasing the inter element spacing, increasing the number of antenna elements, signal to noise ratio (SNR), number of snapshots and configuring each impinging signal at its own SNR channel further improves the resolution and performance. For coherent signals the conventional MUSIC algorithm fails to obtain narrow and sharp peaks. However, a spatial smoothing technique called Forward/Backward Spatial Smoothing (FBSS) technique of the MUSIC algorithm discussed in this paper can be implemented for coherent signals to be able to detect correlated signals in multipath scenarios. This FBSS technique achieves sharp peaks and makes the estimation process much accurate. However, more research is still needed in improving the performance and resolution in multipath signals by applying the FBSS technique while using non-uniform linear arrays (NLA) which would improve the performance and resolution for the same aperture while reducing the number of elements at the same time increasing the degrees of freedom.

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