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Application of Sumudu Decomposition Method to Solve Nonlinear System Fredholm and Volterra Integrodifferential Equations

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Abstract

In this paper, we use a method to obtain approximate solutions for nonlinear systems of Fredholm-Volterra integral differential equations with the help of Sumudu decomposition method (SDM) Where we convert Sumodu transform to nonlinear equations using Fredholm-Volterra integrodifferential Equations. The term nonlinear can be easily dealt with as It helps us in this system Adomian polynomial. We apply this system to three examples and the current results have close agreement with the approximate solutions that we were able to obtain with the help of Adomian decomposition method (ADM).

Keywords: Nonlinear system of Fredholem-Volterra Integro-Differential equations, Sumudu transform method, Adomian decomposition method.

Mathematics Subject Classification. 41A15, 65D07, 65M70, 65M12.

INTRODUCTION

The integral equations of linear and nonlinear Fredholm-Volterra solves many problems of mathematical physics, problems of population mechanics and so on in many scientific fields, The spread of epidemics, see [1]. It is not easy to find the exact solution because of the nonlinear part of the equation. Adomian decomposition method to solve Fredholm-Volterra integrodifferential equations provided by Adomian. In nonlinear cases of differential equations this technique has the advantage of dealing directly with the problem [2,3]. The basic ideas and previous works by Adomian and Wazwaz put their application on a nonlinear arrangement Fredholm-Volterra integrodifferential equations. These equations are resolved without been converted into an equivalent form. Linear avoidance, disturbance or any unrealistic suggestions we see [4,5]. He has also performed in [6] conditions of noise appearing in non homogeneous equations. He has done wazwaz [7] conditions necessary to ensure that noise conditions appear in non homogeneous equations. Sumudu transform has been used to solve many similar problems and apply it to many regular and partial differential equations in [8]. This conversion can also be applied to the neutron transfer equation in [9]. Some basic properties of Sumudu transform and conversion efficiency have been established in the solution Fredholm-Volterra integrodifferential equations, see [10, 12]. If

$$\phi(r)=\sum_{i=0}^{\infty} b_i r^i, \quad \text{then } F(z)=\sum_{i=0}^{\infty} i!\, b_i z^i , \tag{1}$$

BASIC DEFINITIONS

In the present paper, the intimate connection between Sumudu transform theory and decomposition method arising in the solution of nonlinear Fredholm-Volterra integrodifferential equation is demonstrated. During the study, we use Sumudu transform which is defined over the set of the following functions [14]:

$$A=\{\phi(r)\ :\exists M,\ \tau_1,\tau_2>0, |\phi(r)|<Me^{r/\tau_j}, \text{if } r\in(-1)^j\times[0,\infty)\} \tag{2}$$

By the following formula:

$$G(z)=S[\phi(r);z]=\int_0^{\infty}\phi(z\,r)e^{-r}dr, \quad z\in(-\tau_1,\tau_2). \tag{3}$$

Theorem 1.

Let $\phi(r)$ be in A, and let $G^i(z)$ denote Sumudu transform of ith derivatives, $\phi^i(r)$ of $\phi(r)$; then, for $i\geq 1$,

$$G^i(z)=\frac{G(z)}{z^i}-\sum_{\ell=0}^{i-1}\frac{\phi^{(\ell)}(0)}{z^{i-\ell}}. \tag{4}$$

For more details, see [13].

Recall that Sumudu transform of the convolution product $(\phi * g)(y)$ is given by

$$S[(\phi * g)(y;z)] = S\left[\int_0^y \phi(y-r)g(r)dr\right] = z\,F(z)G(z). \quad (5)$$

SUMUDU TRANSFORM METHOD

We consider general nonlinear Fredholm-Volterra integrodifferential equation:

$$\frac{d^i Z}{dy^i} = \phi(y) + \int_0^y K(y-r)F(Z(r))dr. \quad (6)$$

To solve the nonlinear Fredholem-Volterra integrodifferential equations by using Sumudu transform method, it is essential to use Sumudu transforms of the derivatives of $Z(y)$. We can easily

Show that

$$S\left[\frac{d^i Z}{dy^i}\right] = \frac{1}{z^i}S[Z(y)] - \frac{1}{z^i}Z(0) - \frac{1}{z^{i-1}}Z'(0) - \ldots - \frac{Z^{(i-1)}(0)}{z}. \quad (7)$$

Applying Sumudu transform to both sides of (7) gives

$$\begin{aligned}\tfrac{1}{z^i}S[Z(y)] - \tfrac{1}{z^i}Z(0) - \tfrac{1}{z^{i-1}}Z'(0) - \ldots - \tfrac{Z^{(i-1)}(0)}{z} &= S[\phi(y)] \\ &+ z\,S[K(y-r)]S[F(z(r))];\end{aligned} \quad (8)$$

By arrangement, we have

$$\begin{aligned}S[Z(y)] &= z^i S[\varphi(y)] + Z(0) + zZ'(0) + \ldots + z^{i-1}Z^{(i-1)} \\ &+ z^{i+1}S[K(y-r)]S[F(z(r))].\end{aligned} \quad (9)$$

The second step in Sumudu decomposition method is that we represent the solution as an infinite series given by

$$Z(y,\lambda) = \sum_{n=0}^{\infty} Z_n(y) \quad (10)$$

And the nonlinear term can be decomposed as

$$F(z(r)) = \sum_{n=0}^{\infty} A_n, \quad (11)$$

Where A_n are adomian polynomials [7] of $Z_0, Z_1, Z_2, \ldots, Z_i$ and they can be calculated by the following formula:

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\left(\sum_{n=0}^{\infty}\lambda^n z_n\right)\right]_{\lambda=0}, \quad n = 0,1,2,\ldots, \quad (12)$$

Where the so-called Adomian polynomials A_i can be evaluated for all forms of nonlinearity. General formula (12) can be easily used as follows.

Assuming that the nonlinear function is $F(z(y))$, therefore, by using (12), Adomian polynomials are given by

$$\begin{aligned}&A_0 = F(Z_0), \quad A_1 = Z_1F'(Z_0), \quad A_2 = Z_2F'(Z_0) + \tfrac{1}{2!}Z_1^2F''(Z_0)\\ &A_3 = Z_3F'(Z_0) + Z_1Z_2F''(Z_0) + \tfrac{1}{3!}Z_1^3F'''(Z_0)\\ &A_4 = Z_4F'(Z_0) + (\tfrac{1}{2!}Z_2^2 + Z_1Z_3)F''(Z_0)\\ &\quad + \tfrac{1}{2!}Z_1^2Z_2F'''(Z_0) + \tfrac{1}{4!}Z_1^4Z_2F^{iv}(Z_0).\end{aligned} \quad (13)$$

Substitution of (10) and (11) into (9) yields

$$\begin{aligned}S\left[\textstyle\sum_{n=0}^{\infty}Z_n(y)\right] &= z^iS[\phi(y)] + Z(0) + zZ'(0) + \ldots + z^{i-1}Z^{(i-1)}(0)\\ &+ z^{i+1}S[K(y-r)]S\left[\left(\sum_{n=0}^{\infty}A_n\right)\right].\end{aligned} \quad (14)$$

On comparing both sides of (14) and by using standard ADM we have

$$S[Z_0(y)] = z^iS[\varphi(y)] + Z(0) + zZ'(0) + \ldots + z^{i-1}Z^{(i-1)}(0). \quad (15)$$

Then it follows that

$$\begin{aligned}S[Z_1(y)] &= z^{i+1}S[K(y-r)]S[A_0(y)],\\ S[Z_2(y,r)] &= z^{i+1}S[K(y-r)]S[A_1(y)],\end{aligned} \quad (16)$$

More generally way, we have

$$S[Z_{n+1}(y)] = z^{i+1}S[K(y-r)]S[A_n(y)], \quad n \geq 0 \quad (17)$$

Combined Sumudu transform-Adomian decomposition method for solving nonlinear Fredholm-Volterra integrodifferential equations of the second kind will be illustrated by studying the following example.

Example 1.

Consider solving the nonlinear Fredholm-Volterra integrodifferential equation by using combined Sumudu transform-Adomian decomposition method ([14], [15]).

$$Z'(y)-\int_0^y \cos(y-r)Z^2(r)dr=-2\sin(y)-\tfrac{1}{3}\cos(y)-\tfrac{2}{3}\cos(2y),$$
$$Z(0)=1 \quad (18)$$

By applying Sumudu transform of both sides of (18) we obtain

$$S[Z(y)]=1-\frac{2z^2}{1+z^2}-\frac{z}{3(1+z^2)}-\frac{2z}{3(1+4z^2)}+\frac{z^2}{1+z^2}S[Z^2(y)] \quad (19)$$

Substituting the series assumption for $Z(z)$ and Adomian polynomials for Z^2 as given above in (10) and (12), respectively, by using the recursive relation equation (14), we obtain

$$S[Z_0(y)]=1-\frac{2z^2}{1+z^2}-\frac{z}{3(1+z^2)}-\frac{2z}{3(1+4z^2)} \quad (20)$$

Recall that Adomian polynomials for $F(z(y))=z^2(y)$ are given by

$$A_0=Z_0^2,\qquad A_1=2Z_0Z_1,$$
$$A_2=2Z_0Z_2+Z_1^2,\qquad A_3=2Z_0Z_3+2Z_1Z_2. \quad (21)$$

Taking the inverse Sumudu transform of both sides of the first part of (20) and using the recursive relation equation (20) give

$$Z_0(y)=1-0.96666y-y^2+0.5y^3+0.08333y^4-0.09166y^5$$
$$-2.777*10^{-3}y^6+8.531*10^{-3}y^7+4.960*10^{-5}y^8-\ldots,$$
$$Z_1(y)=0.5y^2-0.32222y^3-0.17129y^4+0.12777y^5$$
$$+0.02237y^6-0.03386y^7+3.295*10^{-3}y^8+\ldots. \quad (22)$$

By using (10), we obtain the series solution as follows:

$$Z(y)=1-0.96666y-0.5y^2+0.17778y^3-0.08796y^4+0.03611y^5$$
$$+0.01959y^6-0.025329y^7+3.3446*10^{-3}y^8+\ldots. \quad (23)$$

The exact solution of this example is $Z(y)=\cos(y)-\sin(y)$ ([14], [15]).

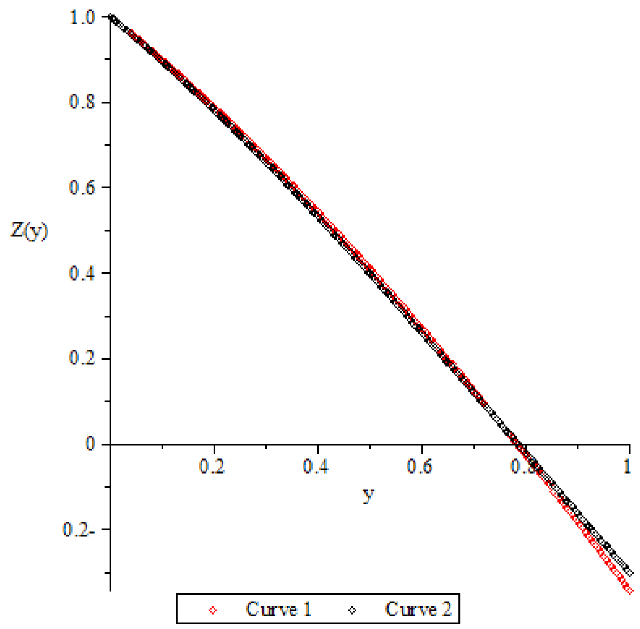

Figure. 1. We are solving the of Sumodu decomposition method (SDM) and compare the approximate solution with the exact solution in [0,1] and help us in this adomian decomposition method (ADM) system where we note that there is close agreement between the approximate solution and the exact solution to be more accurate than those obtained in [14].

For solving Fredholm-Volterra integrodifferential

In the next problem, we apply the combined Sumudu transform-Adomian decomposition method. The standard form of the nonlinear Fredholm-Volterra integrodifferential equation of the first kind is given by [14].

$$\int_0^y K_1(y-r)F(Z(r))dr+\int_0^y K_2(y-r)Z^i(r)dr=\phi(y). \quad (24)$$

On using Sumudu transform of both sides of (24) and using (5), we get

$$S[K_1(y)*F(Z(y))]+S[K_2(y)*Z^{(i)}(y)]=S(\phi(y)) \quad (25)$$

So we have

$$z\ K_1(z)S[F(Z(y))]+z\ K_2(z)S[Z^{(i)}(y)]=F(z) \quad (26)$$

$$S\left[Z\left(y\right)\right]=z^{i-1}\left(\frac{F\left(z\right)+K_2\left(z\right)\zeta\left(z\right)-z\ K_1\left(z\right)S\left[F\left(Z\left(y\right)\right)\right]}{K_2\left(z\right)}\right), \tag{27}$$

Where

$$\zeta\left(z\right)=\frac{1}{z^{i-1}}Z\left(0\right)+\frac{1}{z^{i-2}}Z'\left(0\right)+...+Z^{(i-1)}\left(0\right). \tag{28}$$

We now use the Adomian decomposition method to handle (27). Substituting (10) and (11) into (27),

$$S\left[\sum_{n=0}^{\infty}Z_n\left(y\right)\right]=\frac{z^{i-1}S\left[\phi(y)\right]}{K_2(z)}+Z\left(0\right)+zZ'\left(0\right)+... +z^{i-1}Z^{(i-1)}\left(0\right)-z^{i}\frac{K_1(z)}{K_2(z)}S\left[\left(\sum_{n=0}^{\infty}A_n\right)\right]. \tag{29}$$

The Adomian decomposition method admits the use of the following recursive relation:

$$Z_0\left(y\right)=\frac{z^{i-1}S\left[\varphi(y)\right]}{K_2(z)}+Z\left(0\right)+zZ'\left(0\right)+...+z^{i-1}Z^{(i-1)}\left(0\right),$$
$$Z_{\ell+1}\left(y\right)=-z^{i}\frac{K_1(z)}{K_2(z)}S\left[A_\ell\right],\qquad \ell\geq 0. \tag{30}$$

Example 2.

Solve the following nonlinear Fredholm-Volterra integrodifferential equation of the first kind by the combined Sumudu transform-Adomian decomposition method ([14], [16]).

$$Z'\left(y\right)+Z\left(y\right)+\tfrac{1}{2}\int_0^y yZ^2\left(r\right)dr-\tfrac{1}{4}\int_0^1 r\,Z^3\left(r\right)dr =2y+y^2+\tfrac{1}{10}y^6-\tfrac{1}{32},\qquad Z\left(0\right)=0 \tag{31}$$

By applying Sumudu transforms to both sides of (31), we have

$$S\left[Z(y)\right]=-\frac{1}{32}z+2z^2+2z^3+72z^7-zS\left[Z\right]-z^4S\left[Z^2(r)\right]+\frac{1}{8}z^2S\left[Z^3(r)\right] \tag{32}$$

Recall that Adomian polynomials for $Z^2\left(r\right)$ and $Z^3\left(r\right)$ are given by

$$A_0=Z_0^2,\qquad A_1=2Z_0Z_1,$$
$$A_2=2Z_0Z_2+Z_1^2,\qquad A_3=2Z_0Z_3+2Z_1Z_2.$$
$$C_0=Z_0^3,\quad C_1=3Z_0^2Z_1,\quad C_2=3Z_0^2Z_2+3Z_0Z_1^2,$$
$$C_3=3Z_0^2Z_3+6Z_0Z_1Z_2+Z_1^3. \tag{33}$$

Substituting the series assumption for $Z(z)$ and Adomian polynomials for $Z^2(r), Z^3(r)$ and using (15) and (17), we obtain

$$S\left[Z_0(y)\right]=-\frac{1}{32}z+2z^2+2z^3+72z^7, \tag{34}$$

$$S\left[Z_{\ell+1}\left(y\right)\right]=-zS\left[Z_\ell\right]-z^4S\left[A_\ell\left(y\right)\right]+\frac{1}{8}z^2S\left[C_\ell\left(y\right)\right]. \tag{35}$$

Taking the inverse Sumudu transform of both sides of (34) and using the recursive relation equation (35) give

$$Z_0\left(y\right)=-0.0312y+y^2+0.3333y^3+0.0142y^7,$$
$$Z_1\left(y\right)=0.0656y^2-0.3333y^3-0.0833y^4-1.951*10^{-7}y^5+9.685*10^{-6}y^6 -2.048*10^{-4}y^7-2.659*10^{-4}y^8+1.496*10^{-3}y^9.... \tag{36}$$

The series solution is given by

$$Z(y)=-0.0312y+1.0656y^2-0.0833y^4-1.951*10^{-7}y^5+9.685*10^{-6}y^6 +0.0139y^7-2.659*10^{-4}y^8+1.496*10^{-3}y^9..., \tag{37}$$

The exact solution of this example is $Z(y) = y^2$ ([14], [16]).

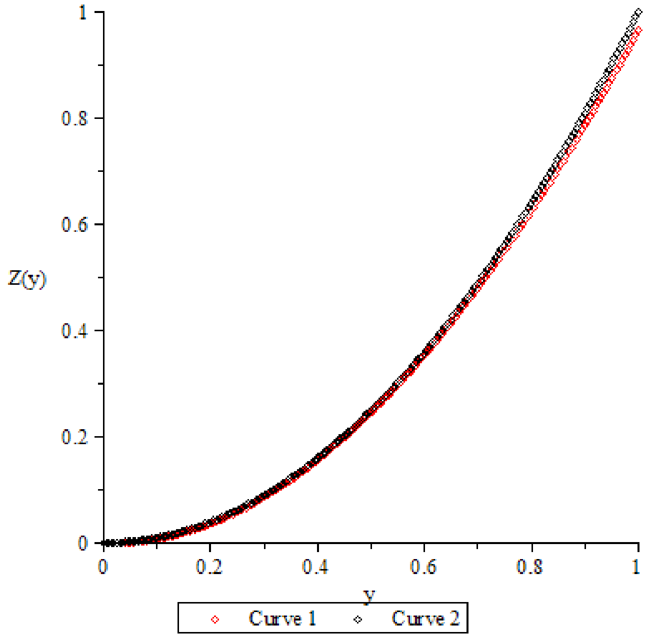

Figure. 2. We are solving the of Sumodu decomposition method (SDM) and compare the approximate solution with the exact solution in [0,1] and help us in this adomian decomposition method (ADM) system where we note that there is close agreement between the approximate solution and the exact solution to be more accurate than those obtained in [14].

SYSTEM OF NONLINEAR FREDHOLM AND VOLTERRA INTEGRODIFFERENTIAL EQUATIONS

In this section, we will study systems of nonlinear Fredholm-Volterra integrodifferential equations of the second kind by combining Sumudu transform-Adomian decomposition method.

System of nonlinear Fredholm-Volterra integrodifferential equations of the second kind

Consider a system of nonlinear Fredholm-Volterra integrodifferential equations of the second kind as follows [14]:

$$Z^{(i)} = \phi_1(y) + \int_0^y \big(K_1(y-r)F_1(z(r)) + B_1(y-r)G_1(w(r))\big)dr,$$
$$W^{(i)} = \phi_2(y) + \int_0^y \big(K_2(y-r)F_2(z(r)) + B_2(y-r)G_2(w(r))\big)dr. \quad (38)$$

Applying Sumudu transforms to both sides of (38), we have

$$\frac{1}{z^i}S[Z(y)] - \frac{1}{z^i}Z(0) - \frac{1}{z^{i-1}}Z'(0) - \ldots - \frac{Z^{i-1}(0)}{z} = S[\phi_1(y)]$$
$$+S\big[\big(K_1(y)*F_1(z(y)) + B_1(y)*G_1(w(y))\big)\big],$$
$$\frac{1}{z^i}S[W(y)] - \frac{1}{z^i}W(0) - \frac{1}{z^{i-1}}W'(0) - \ldots - \frac{W^{i-1}(0)}{z} = S[\phi_2(y)]$$
$$+S\big[\big(K_2(y)*F_2(z(y)) + B_2(y)*G_2(w(y))\big)\big]. \quad (39)$$

After rearrangement, we get

$$S[Z(y)] = Z(0) + zZ'(0) + \ldots + z^{i-1}Z^{(i-1)}(0) + z^i S[\phi_1(y)]$$
$$+ z^i S[(K_1(y)*F_1(z(y)) + B_1(y)*G_1(w(y)))], \quad (40)$$

$$S[W(y)] = W(0) + zW'(0) + \ldots + z^{i-1}W^{(i-1)}(0) + z^i S[\phi_2(y)]$$
$$+ z^i S[(K_2(y)*F_2(z(y)) + B_2(y)*G_2(w(y)))] \quad (41)$$

To overcome the difficulty of the nonlinear terms $F_n(z(y)),\ n = 1,2,$ we apply the Adomian decomposition method for handling (40) and (41). To achieve this goal, we first represent the linear terms $z(y)$ and $w(y)$ at the left side by an infinite series of components given by

$$Z(y) = \sum_{n=0}^{\infty} Z_n(y), \qquad W(y) = \sum_{n=0}^{\infty} W_n(y) \quad (42)$$

And the nonlinear terms $F_n(z(y))$ at the right side of (40) and (41) by

$$F(z(r)) = \sum_{i=0}^{\infty} A_i, \qquad A_i = \frac{1}{i!}\frac{d^i}{d\lambda^i}\left[F\left(\sum_{n=0}^{i} \lambda^n u_n\right)\right]_{\lambda=0}, \quad i = 0,1,2,\ldots, \quad (43)$$

Where Adomian polynomials $A_i, i \geq 0,$ can be obtained from all forms of nonlinearity. Substituting (42) and (43) into (40) and (41) leads to

$$S\left[\sum_{i=0}^{\infty} Z_i(y)\right] = Z(0) + zZ'(0) + \ldots + z^{i-1}Z^{(i-1)}(0) + z^i S[\phi_1(y)]$$
$$+ z^i S[K_1(y)]S\left[\sum_{i=0}^{\infty} A_i\right] + z^i S[B_1(y)]S\left[\left(\sum_{i=0}^{\infty} \tilde{A}_i\right)\right], \quad (44)$$

$$S\left[\sum_{i=0}^{\infty} W_i(y)\right] = W(0) + zW'(0) + \ldots + z^{i-1}W^{(i-1)}(0) + z^i S\left[\phi_2(y)\right]$$
$$+ z^i S\left[K_2(y)\right] S\left[\left(\sum_{i=0}^{\infty} C_i\right)\right] + z^i S\left[B_2(y)\right] S\left[\left(\sum_{i=0}^{\infty} C_i\right)\right]. \quad (45)$$

Adomian decomposition method admits the use of the following recursive relations:

$$\begin{aligned} S\left[Z_0(y)\right] &= Z(0) + zZ'(0) + \ldots + z^{i-1}Z^{(i-1)}(0) + z^i S\left[\varphi_1(y)\right], \\ S\left[Z_{\ell+1}(y)\right] &= z^i S\left[K_1(y)\right] S\left[A_\ell\right] + z^i S\left[B_1(y)\right] S\left[A_\ell\right], \\ S\left[W_0(y)\right] &= W(0) + z\,W'(0) + \ldots + z^{i-1}W^{(i-1)}(0) + z^i S\left[\varphi_2(y)\right], \\ S\left[W_{\ell+1}(y)\right] &= z^i S\left[K_2(y)\right] S\left[C_\ell\right] + z^i S\left[B_1(y)\right] S\left[C_\ell\right]. \end{aligned} \quad (46)$$

The combined Sumudu transform-Adomian decomposition method for solving systems of nonlinear Volterra integrodifferential equations of the second kind will be illustrated by studying the following example.

Example 3.

Solve the system of nonlinear Fredholm-Volterra integrodifferential equation by using the combined Sumudu transform-Adomian decomposition method ([1], [14]).

$$\begin{aligned} Z'(y) &= 2y + \tfrac{149}{64} + \tfrac{1}{64}\int_0^1 \left(Z^2(r) + W^2(r)\right) dr, \\ W'(y) &= 2y - \tfrac{67}{64} + \tfrac{1}{64}\int_0^1 \left(Z^2(r) - W^2(r)\right) dr, \\ Z(0) &= 1, \qquad W(0) = 1. \end{aligned} \quad (47)$$

Taking Sumudu transforms of both sides of (47), we obtain

$$\begin{aligned} Z(z) &= 1 + 2z^2 + \tfrac{149}{64} z + \tfrac{1}{64} z^2 S\left[Z^2(y) + W^2(y)\right], \\ W(z) &= 1 + 2z^2 - \tfrac{67}{64} z + \tfrac{1}{64} z^2 \left[Z^2(y) - W^2(y)\right]. \end{aligned} \quad (48)$$

By using (46), we have

$$\begin{aligned} Z_0(z) &= 1 + \tfrac{149}{64} z + 2z^2, \quad Z_{\ell+1} = \tfrac{1}{64} z^2 S\left[A_\ell(y) + C_\ell(y)\right], \\ W_0(z) &= 1 - \tfrac{67}{64} z + 2z^2, \quad W_{\ell+1} = \tfrac{1}{64} z^2 S\left[A_\ell(y) - C_\ell(y)\right]. \end{aligned} \quad (49)$$

Recall that Adomian polynomials for $Z^2(y)$ and $W^2(y)$ are given by

$$\begin{aligned} &A_0(y) = Z_0^2, \quad A_1(y) = 2Z_0Z_1, \quad A_2(y) = 2Z_0Z_2 + Z_1^2, \\ &A_3(y) = 2Z_0Z_3 + 2Z_1Z_2, \\ &C_0(y) = W_0^2, \quad C_1(y) = 2W_0W_1, \quad C_2(y) = 2W_0W_2 + W_1^2, \\ &C_3(y) = 2W_0W_3 + 2W_1W_2. \end{aligned} \quad (50)$$

Taking the inverse Sumudu transform of both sides of (46) and using the recursive relation equations (46), we obtain the solution as follows:

$$\begin{aligned} Z(y) &= 1 + 1.0281y + 1.0156y^2 + 6.65*10^{-3}y^3 + 0.0136y^4 \\ &\quad + 1.998*10^{-3}y^5 + 1.04*10^{-3}y^6 + \ldots, \\ W(y) &= 1 - 1.0068y + y^2 + 0.0175y^3 + 5.6208*10^{-3}y^4 \\ &\quad + 5.264*10^{-3}y^5 + \ldots\ . \end{aligned} \quad (51)$$

The exact solution of this example of a system is given by $Z(y) = 1 + y + y^2, \quad W(y) = 1 - y + y^2$ ([1], [14]).

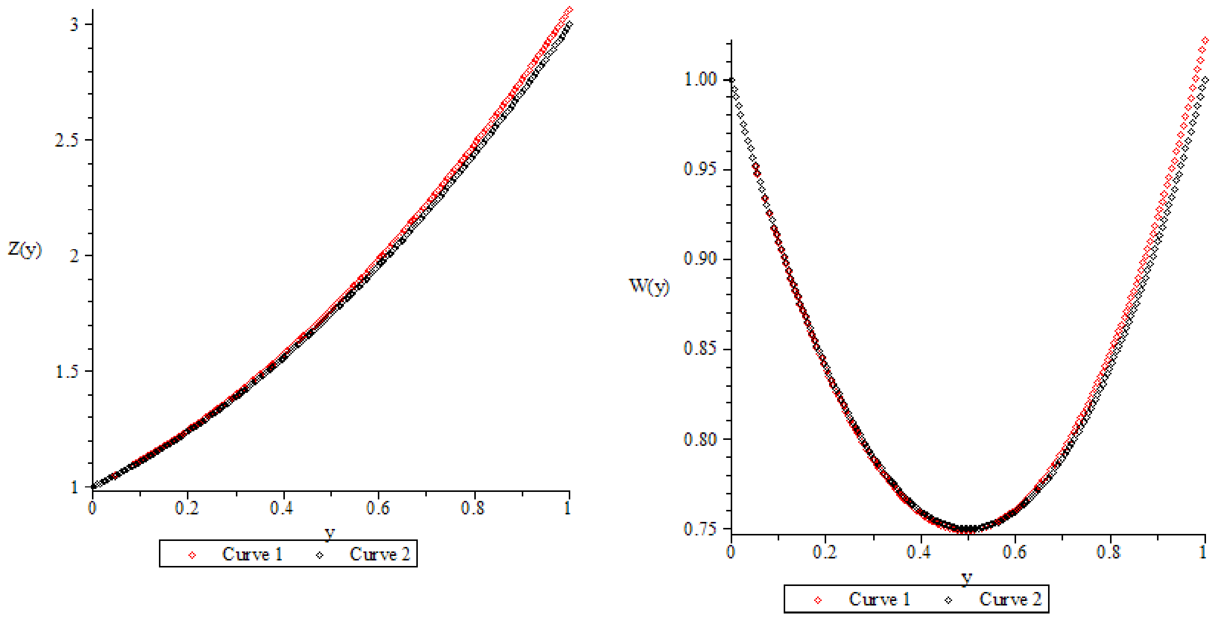

Figure. 3. We are solving the of Sumodu decomposition method (SDM) and compare the approximate solution with the exact solution in [0,1] and help us in this Adomian decomposition method (ADM) system where we note that there is close agreement between the approximate solution and the exact solution to be more accurate than those obtained in [14].

CONCLUSION

In this article, we approximate method for the solution of nonlinear Fredholm- Volterra integral equations using Sumudu transform-Adomian decomposition method This method was applied to the system of nonlinear equations of Fredholm- Volterra integral equations by providing three examples where we found that this method is useful for any type of Fredholm- Volterra integral equation system Therefore this method can even be applied to many complicated linear and nonlinear Fredholm-Volterra integrodifferential equations.

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