

Flow and Heat Transfer in a Viscoelastic Fluid over a non-isothermal Stretching Sheet with viscous dissipation and Internal Heat Generation

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Abstract

Numerical analysis is carried out to study flow and heat transfer characteristics of visco-elastic fluid flow over a non-isothermal stretching sheet with viscous dissipation and internal heat generation. The governing partial differential equations are converted into ordinary differential equations by a similarity transformation. These equations are solved by a numerical method, Quasilinearization technique. The effects of various physical parameters such as Prandtl number (Pr), visco-elasticity (k_1), Eckert number (Ec), internal heat source/sink (α) and temperature parameter(r) on flow and heat transfer are evaluated numerically and presented through graphs.

Keywords: visco-elastic, viscous dissipation, internal heat generation, non-isothermal stretching sheet, boundary layer, heat transfer

INTRODUCTION

In many engineering processes, boundary layer behavior occurs for a flow over a moving continuous solid surface. Manufacturing processes that involve extrusion of a material and heat-treated materials that travel between feed and wind-up rollers or on conveyer belts are examples that exhibit the characteristics of flow over a moving continuous surface. Sakiadis [1] initiated the study of these applications by considering the boundary layer flow over a continuous solid surface moving with constant speed. Erickson et al [2] extended this problem to the case in which the transverse velocity at the moving surface is non zero, with heat and mass transfer in the boundary layer being taken into account.

The above investigations have a definite bearing on the problem of a polymer sheet extruded continuously from a die. It is usually assumed that the sheet is inextensible, but situations may arise in the polymer industry in which it is necessary to deal with a stretching plastic sheet, as noted by Crane [3]. Danberg and Fansler [4] investigated the non similar solution for the flow in the boundary layer past a wall that is stretched with a velocity proportional to the distance along the wall.

Dutta, Roy and Gupta [5] have analyzed the temperature distribution in the flow over a stretching sheet with uniform wall heat flux. Vajravelu and Rollins [6] investigated the heat transfer characteristics in a visco-elastic fluid over a continuous impermeable, linearly stretching sheet with PST and PHF cases. In [6], K. Vajravelu and D. Rollins studied the

heat transfer characteristics in fluid initially at rest and at uniform temperature. As there is an appreciable temperature difference between the surface and the ambient fluid, one needs to consider the temperature dependent heat sources or sinks which may exert strong influence on the heat transfer characteristics.

Foraboschi and Federico [7] have assumed the volumetric rate of heat generation as

$$Q = \begin{cases} Q_0(T - T_0), & T \geq T_0 \\ 0, & T < 0 \end{cases}$$

They studied steady state temperature profiles for linear parabolic and piston flow in circular tubes. The above relation is valid as an approximation of the state of some exothermic process, having T_0 as the onset temperature. When the inlet temperature is not less than T_0 , then $Q = Q_0(T - T_\infty)$ is used and studied its effect on the heat transfer in laminar flow of non-Newtonian heat generating fluids.

Vajravelu and Hadjinicolaou [8] have studied the heat transfer characteristics in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation for both PST and PHF cases. Vajravelu and Roper [9] analyzed heat transfer characteristics in a second grade fluid over stretching sheet with viscous dissipation, internal heat generation, and work due to deformation. Rollins and Vajravelu [10] studied flow and heat transfer characteristics in a second order fluid over a stretching sheet with internal heat generation and absorption for both PST and PHF cases. In all these studies, analytical solutions are obtained using Kummer's function.

In the present paper, an incompressible viscoelastic fluid (Walter's liquid B model) over non-isothermal stretching sheet is considered with Prescribed Surface Temperature. The effect of viscous dissipation and internal heat generation or absorption are included in the energy equation. A numerical approach, Quasilinearization technique is used to study flow and heat transfer characteristics of the fluid. This approach is easily adoptable and it is observed that results are in good agreement with the available literature. The effect of various parameters on flow and heat transfer are analyzed through numerical calculations.

FLOW ANALYSIS:

Consider a laminar steady flow of an incompressible

viscoelastic (Walters' liquid B model) fluid over a semi-infinite, impermeable, non-isothermal stretching sheet coinciding with the plane $y = 0$ is considered, the flow being confined to $y > 0$. Two equal and opposite forces are applied along the x -axis, so that a sheet is stretched with a velocity proportional to the distance from the origin. The resulting motion of the quiescent fluid is thus caused solely by the moving surface. The flow satisfies the rheological equation of state derived by Beard and Walters [11]. The steady two-dimensional boundary layer equations for this fluid, in the usual form are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - k_0 \left\{ \frac{\partial}{\partial x} \left(u \frac{\partial^2 u}{\partial y^2} \right) + \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial y^2} + v \frac{\partial^3 u}{\partial y^3} \right\} \quad (2)$$

where $\nu = \frac{\mu}{\rho}$, $k_0 > 0$

Where u and v are the velocity components along the x and y directions respectively, ν is the kinematic viscosity $k_0 = -\alpha_1 / \rho$ is the co-efficient of elasticity, and ρ is the density. Hence, in the case of second order fluid flow, k_0 takes positive value, as α_1 takes negative value and other quantities have their usual meanings. In deriving (2), it is assumed that the normal stress is of the same order of magnitude as that of the shear stress, in addition to usual boundary layer approximations.

The boundary conditions for the velocity field are:

$$u = u_w = bx, \quad v = 0 \quad \text{at } y = 0, b > 0$$

$$u \rightarrow 0, \quad \frac{\partial u}{\partial y} \rightarrow 0 \quad \text{as } y \rightarrow \infty \quad (3)$$

The condition $\partial u / \partial y \rightarrow 0$ as $y \rightarrow \infty$ is the augmented condition, since the flow is in an unbounded domain, which has been discussed by Rajgopal [12]. In this case, the flow is caused solely by the stretching of the sheet, since the free stream velocity is zero.

Defining new variables:

$$u = bx f_\eta(\eta), \quad v = -\sqrt{b\nu} f(\eta), \quad \eta = \sqrt{b/\nu} y \quad (4)$$

where $f_\eta(\eta)$ denotes differentiation with respect to η . Clearly u and v defined above satisfy the continuity equation (1), and the equation (2) reduces to

$$f_\eta^2 - f f_{\eta\eta} = f_{\eta\eta\eta} - k_1 \{ 2f_\eta f_{\eta\eta\eta} - f f_{\eta\eta\eta\eta} - f_{\eta\eta}^2 \} \quad (5)$$

where $k_1 = k_0 b / \nu$, is the visco-elastic parameter.

The boundary conditions (3) become

$$f(0) = 0, \quad f_\eta(0) = 1 \quad (6a)$$

$$f_\eta(\eta) \rightarrow 0, \quad f_{\eta\eta}(\eta) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \quad (6b)$$

HEAT TRANSFER ANALYSIS:

The governing boundary layer equation with viscous dissipation and internal heat generation or absorption is given by

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + Q(T - T_\infty) \quad (7)$$

Where k is the thermal diffusivity and c_p is the specific heat of a fluid at constant pressure and Q is the volumetric rate of heat generation.

Here the thermal boundary condition depends on the type of heating process under consideration. Here it is taken as the sheet with Prescribed Surface Temperature (PST case).

Prescribed Surface Temperature (PST case)

For this circumstance, the boundary conditions are

$$T = T_w \left[= T_\infty + A \left(\frac{x}{l} \right)^r \right] \quad \text{at } y=0 \quad (8a)$$

$$T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty \quad (8b)$$

Here l is the characteristic length and r is the temperature parameter.

Define non-dimensional temperature as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (9)$$

Using (4), equation (7) reduces to

$$\theta'' + \text{Pr} f\theta' - \text{Pr}(rf' - \alpha)\theta = -\text{Pr} Ec(f'')^2 \quad (10)$$

where $\text{Pr} = \mu c_p / k$, Prandtl number, where $Ec = bl^2 / c_p A$, Eckert number and $\alpha = Q / (b\rho c_p)$, heat source/sink parameter.

with boundary conditions

$$\theta(0) = 1, \quad \theta(\infty) \rightarrow 0 \quad (11)$$

NUMERICAL SOLUTION OF THE PROBLEM

The flow equation (5) coupled with energy equation (10) constitute a set of highly nonlinear differential equations. So obtaining closed form solution for this set is cumbersome and

time consuming. Hence Quasilinearization method, given by Bellman & Kalaba [13] is used to solve this system. This method is chosen due to the following advantages:

- (i) The method is quadratically convergent, starting from the initial guess value.
- (ii) The solution is valid for a large range of parameters. Even when the required number of initial conditions is not given, this method converges at a fast speed.

For convenience equations (5) and (10) are rearranged as

$$f^{iv} = \frac{1}{k_1 f} \left[(f')^2 - ff'' - f''' + 2k_1 f f''' - k_1 (f'')^2 \right] \quad (12)$$

$$\theta'' = -Pr f\theta' + Pr(rf' - \alpha)\theta - Pr Ec(f'')^2 \quad (13)$$

In order to implement the Quasilinearization method, the equations (12) and (13) are converted to a system of first order differential equations by substituting

$$(f, f', f'', f''', \theta, \theta') = (x_1, x_2, x_3, x_4, x_5, x_6)$$

Then equations (12) and (13) give

$$\begin{aligned} \frac{dx_1}{d\eta} &= x_2 \\ \frac{dx_2}{d\eta} &= x_3 \\ \frac{dx_3}{d\eta} &= x_4 \\ \frac{dx_4}{d\eta} &= \frac{1}{k_1 x_1} \{x_2^2 - x_1 x_3 - x_4 + 2k_1 x_2 x_4 - k_1 x_3^2\} \\ \frac{dx_5}{d\eta} &= x_6 \\ \frac{dx_6}{d\eta} &= -Pr x_1 x_6 - Pr Ec x_3^2 + Pr(rx_2 - \alpha)x_5 \end{aligned} \quad (14)$$

Using Quasilinearization technique, the system (14) can be linearized as

$$\begin{aligned} \frac{dx_1^{r+1}}{d\eta} &= x_2^{r+1} \\ \frac{dx_2^{r+1}}{d\eta} &= x_3^{r+1} \\ \frac{dx_3^{r+1}}{d\eta} &= x_4^{r+1} \end{aligned}$$

$$\begin{aligned} \frac{dx_4^{r+1}}{d\eta} &= \left(\frac{-1}{k_1 (x_1^r)^2} \left((x_2^r)^2 - x_4^r + 2k_1 x_2^r x_4^r - k_1 (x_3^r)^2 \right) \right) x_1^{r+1} \\ &+ \left(\frac{1}{k_1 x_1^r} (2x_2^r + 2k_1 x_4^r) \right) x_2^{r+1} \\ &+ \left(\frac{1}{k_1 x_1^r} (-x_1^r - 2k_1 x_3^r) \right) x_3^{r+1} \\ &+ \left(\frac{1}{k_1 x_1^r} (-1 + 2k_1 x_2^r) \right) x_4^{r+1} + \left(\frac{-x_4^r}{k_1 x_1^r} \right) \end{aligned} \quad (15)$$

$$\begin{aligned} \frac{dx_5^{r+1}}{d\eta} &= x_6^{r+1} \\ \frac{dx_6^{r+1}}{d\eta} &= (-Pr x_6^r) x_1^{r+1} + (r Pr x_5^r) x_2^{r+1} - (2Ec Pr x_3^r) x_3^{r+1} \\ &+ Pr(rx_2^r - \alpha) x_5^{r+1} - Pr x_1^r x_6^{r+1} + \left(Ec Pr (x_3^r)^2 + Pr x_1^r x_6^r - r Pr x_2^r x_5^r \right) \end{aligned}$$

The above system of equations (15) is linear in x_i^{r+1} ($i=1,2,\dots,6$ and general solution can be obtained by using the principle of superposition.

The boundary conditions given by (6) and (11) reduce to

$$x_1^{r+1}(\eta)=0, x_2^{r+1}(\eta)=1, x_5^{r+1}(\eta)=1 \quad \text{at } \eta=0 \quad (16a)$$

$$x_2^{r+1}(\eta) \rightarrow 0, x_3^{r+1}(\eta) \rightarrow 0, x_5^{r+1}(\eta) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \quad (16b)$$

The initial values are chosen as follows:

For homogeneous solution:

$$\begin{aligned} x_i^{h_1}(\eta) &= [0 \ 0 \ 1 \ 0 \ 0 \ 0] \\ x_i^{h_2}(\eta) &= [0 \ 0 \ 0 \ 1 \ 0 \ 0] \\ x_i^{h_3}(\eta) &= [0 \ 0 \ 0 \ 0 \ 0 \ 1] \end{aligned} \quad (17)$$

For particular solution:

$$x_i^p(\eta) = [0 \ 1 \ 0 \ 0 \ 1 \ 0] \quad (18)$$

The general solution for the system (17) is given by

$$\begin{aligned} x_i^{r+1}(\eta) &= C_1 x_i^{h_1}(\eta) + C_2 x_i^{h_2}(\eta) \\ &+ C_3 x_i^{h_3}(\eta) + x_i^p(\eta) \end{aligned} \quad (19)$$

where C_1, C_2, C_3 are the unknown constants, which are to be determined by considering the boundary conditions as $\eta \rightarrow \infty$. This solution ($x_i^{r+1}, i=1,2,\dots,6$) is then compared with solution at the previous step $x_i^r, i=1,2,\dots,6$ and next iteration is performed, if the convergence has not been achieved or greater accuracy is desired.

RESULTS AND DISCUSSIONS:

The flow and heat transfer characteristics of an incompressible visco-elastic fluid over non-isothermal

stretching sheet with viscous dissipation and internal heat generation or absorption have been examined. The governing boundary layer equations are solved using Quasilinearization technique. The computational results of flow and heat transfer characteristics for various parameters are presented in graphs and discussed.

Fig 1 depicts the effect of viscoelastic parameter (k_1) on longitudinal and transverse velocity components. It can be seen, for a fixed value of η , both $f'(\eta)$ and $f(\eta)$ decrease with increasing values of k_1 . This can be explained by the fact that, as the viscoelastic parameter (k_1) increases, the boundary layer adheres strongly to the surface, which in turn retards the flow in longitudinal and transverse directions.

In Fig 2, non-dimensional temperature $\theta(\eta)$ and temperature gradient $\theta'(\eta)$ profiles are plotted for various values internal heat source/sink parameter α . It shows that $\theta(\eta)$ increases with increasing values α . This is due to the fact that heat is generated inside the boundary layer for increasing values of heat source/sink parameter α . The magnitude of $|\theta'(0)|$ decreases with increasing values of α .

In Fig 3, effect of Prandtl number (Pr) on non-dimensional temperature $\theta(\eta)$ and temperature gradient $\theta'(\eta)$ are shown. Temperature $\theta(\eta)$ decreases with increasing values of Prandtl number (Pr). This is consistent with the fact that the thermal boundary thickness decreases with increasing Prandtl number (Pr).

Fig.4 represents that Temperature $\theta(\eta)$ increases with increase in Eckert number (Ec). This is due to the fact that heat energy is stored in the fluid due to frictional heating.

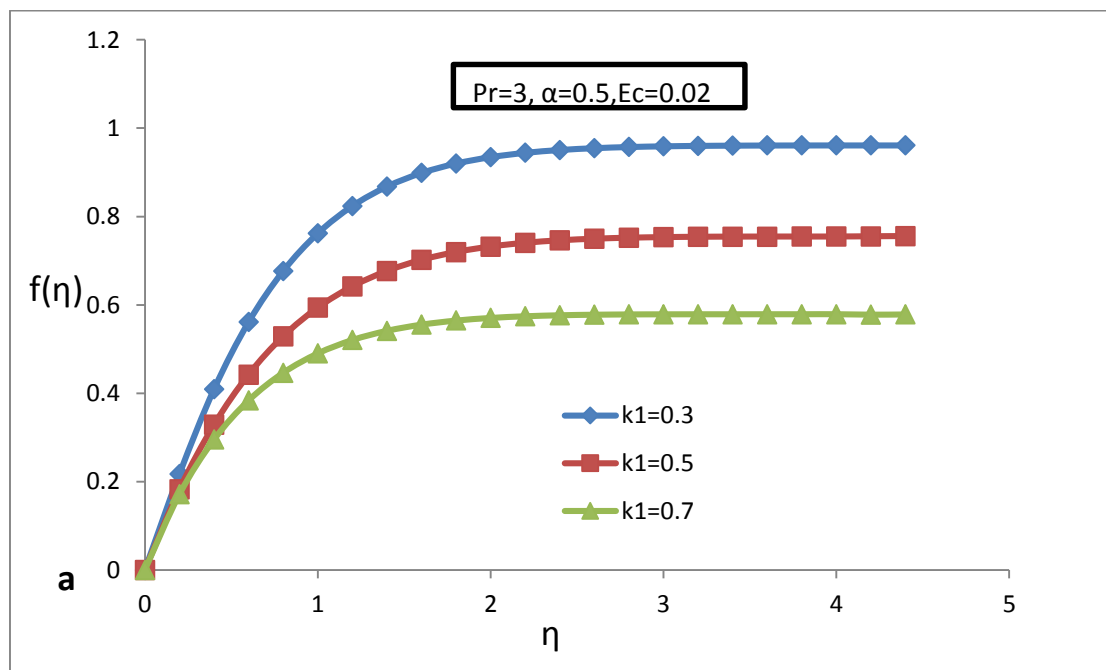
The effect of visco-elastic parameter k_1 on temperature distribution is shown in Fig.5. The effect of increasing values of visco-elastic parameter k_1 is seen to increase the temperature distribution in flow region. This is in conformity with the fact that the increase of non-Newtonian visco-elastic parameter leads to the increase of thermal boundary layer thickness.

In Fig 6, the effect of temperature parameter(r) on the temperature distribution is depicted. When $r>0$, the heat flows from the stretching sheet to the ambient medium. When $r<0$, the temperature gradient is now positive and heat flows into the stretching sheet from the ambient medium.

Finally, Heat transfer characteristics at the wall are given in Table 1. The magnitude of $|\theta'(0)|$ increases with increasing values of Prandtl number (Pr), which implies that more heat is carried out of the sheet, resulting in a decrease of thermal boundary layer thickness and hence decrease in temperature of the fluid. Moreover $|\theta'(0)|$ is negative for all the values of parameters. Physically it means that there is a heat flow only from the sheet

The magnitude of $|\theta'(0)|$ decreases with increasing values of visco-elasticity (k_1), internal heat source/sink (α) and Eckert number (Ec), which results in increase the temperature $\theta(\eta)$.

From our numerical results, it can be concluded that temperature of the fluid increases with increasing values of viscoelastic parameter k_1 , heat source/sink parameter α , Eckert number Ec and decreases with increasing values of Prandtl number Pr.



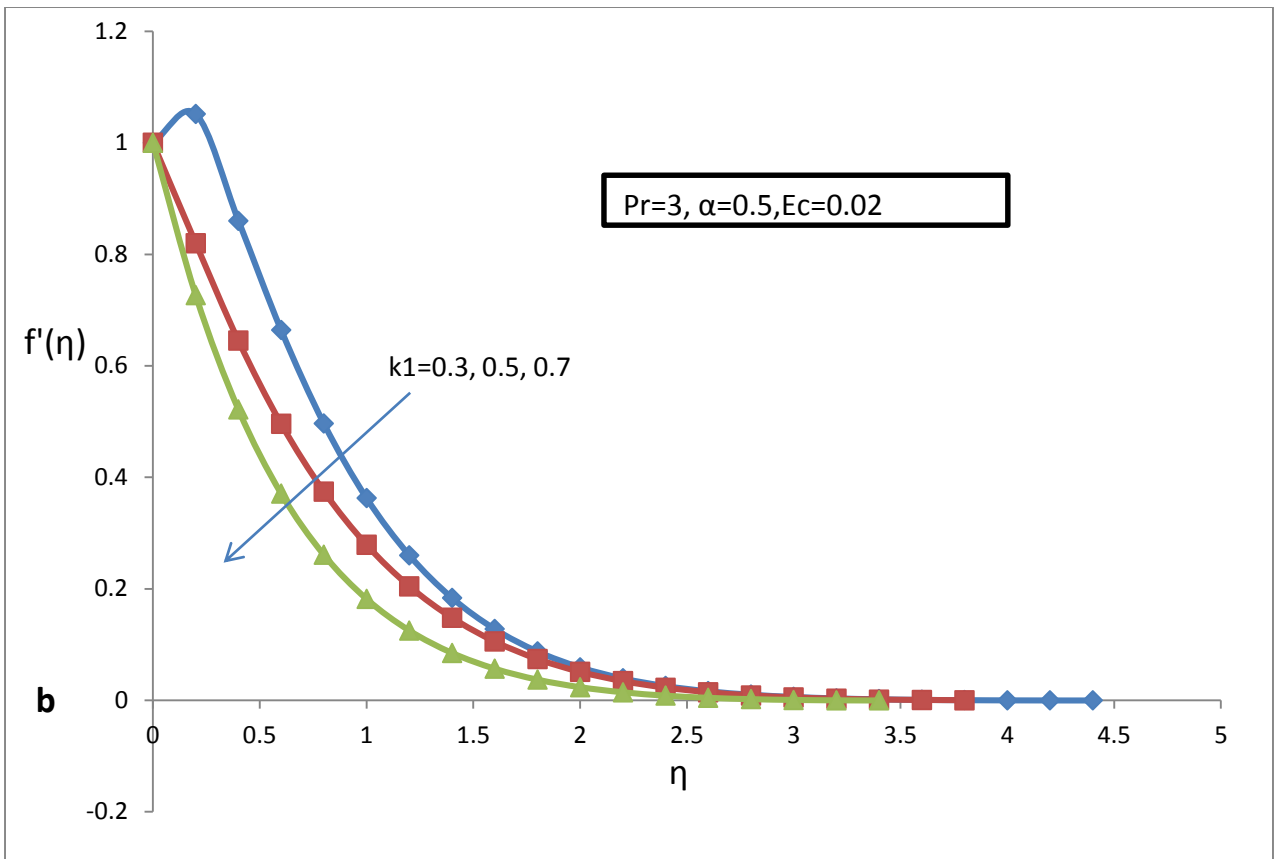
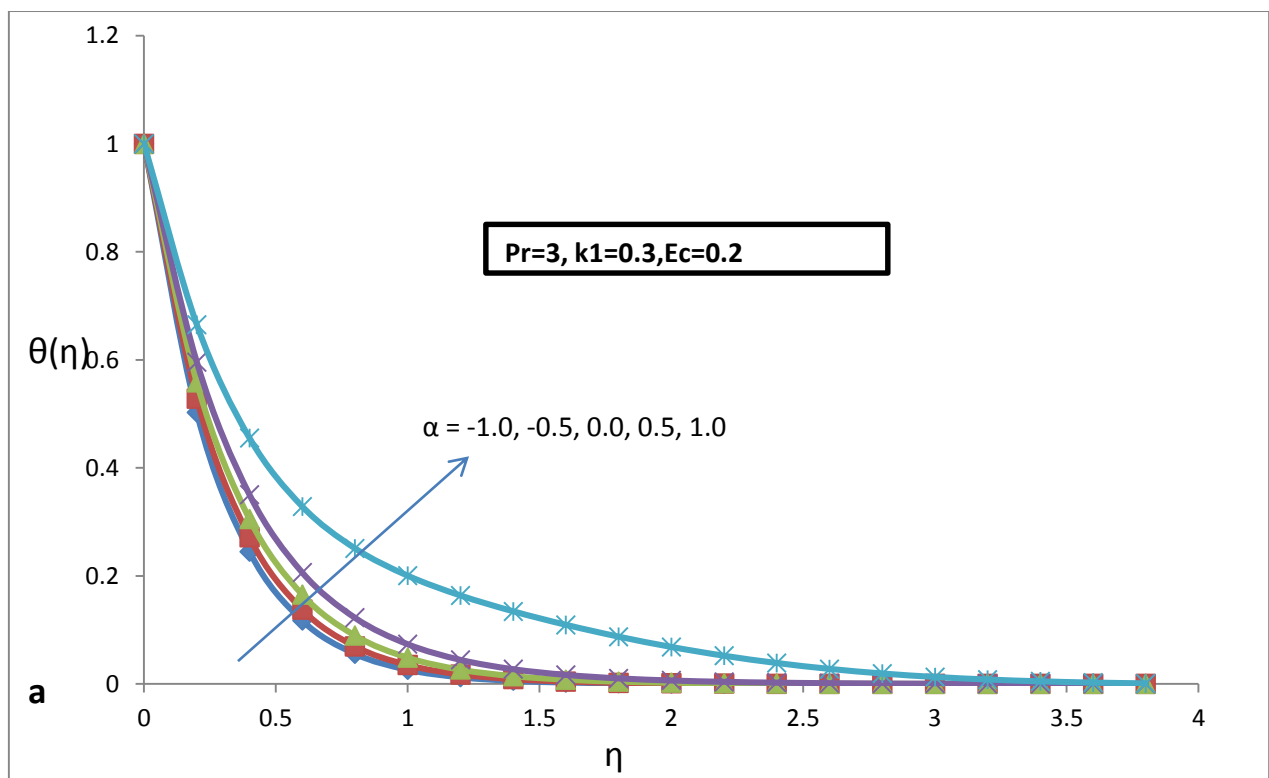


Figure 1. Effect of viscoelastic parameter (k_1) on (a) transverse velocity profile, (b) longitudinal velocity profile.



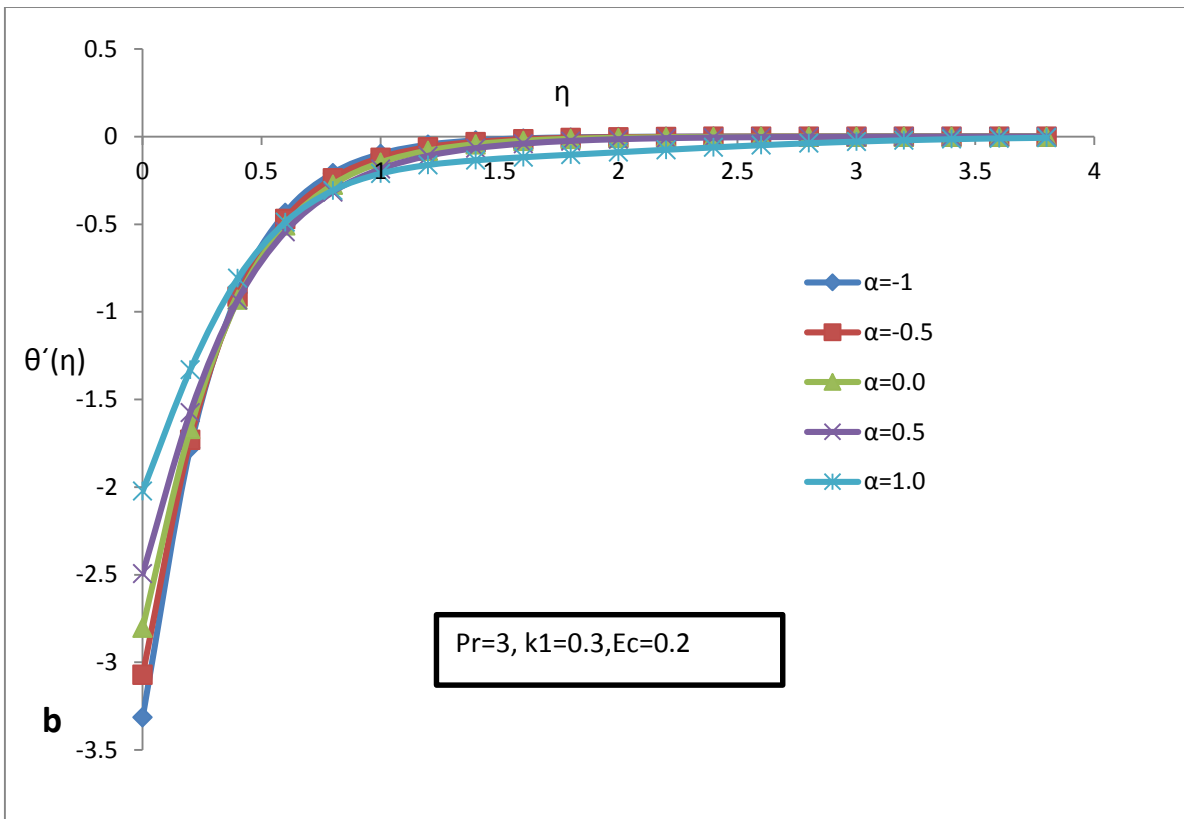
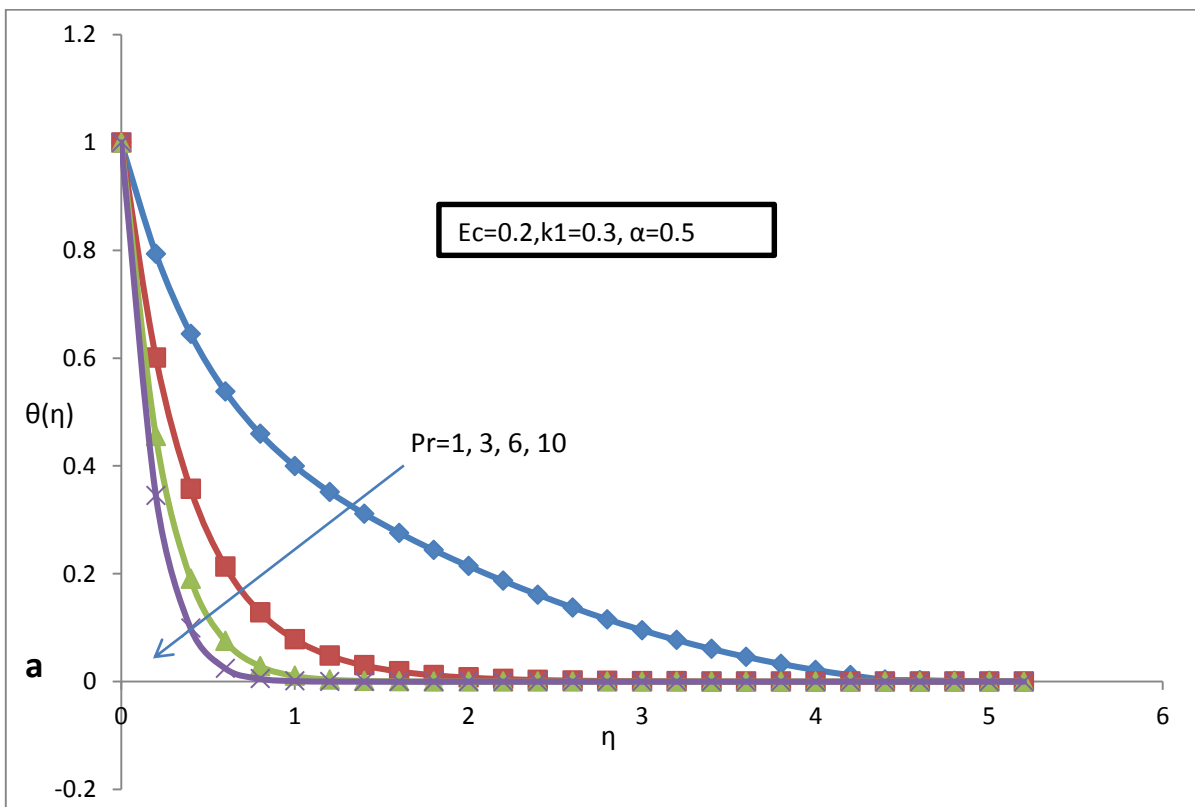


Figure 2. Effect of heat source/sink parameter (α) on (a). Temperature $\theta(\eta)$, (b). Temperature gradient $\theta'(\eta)$



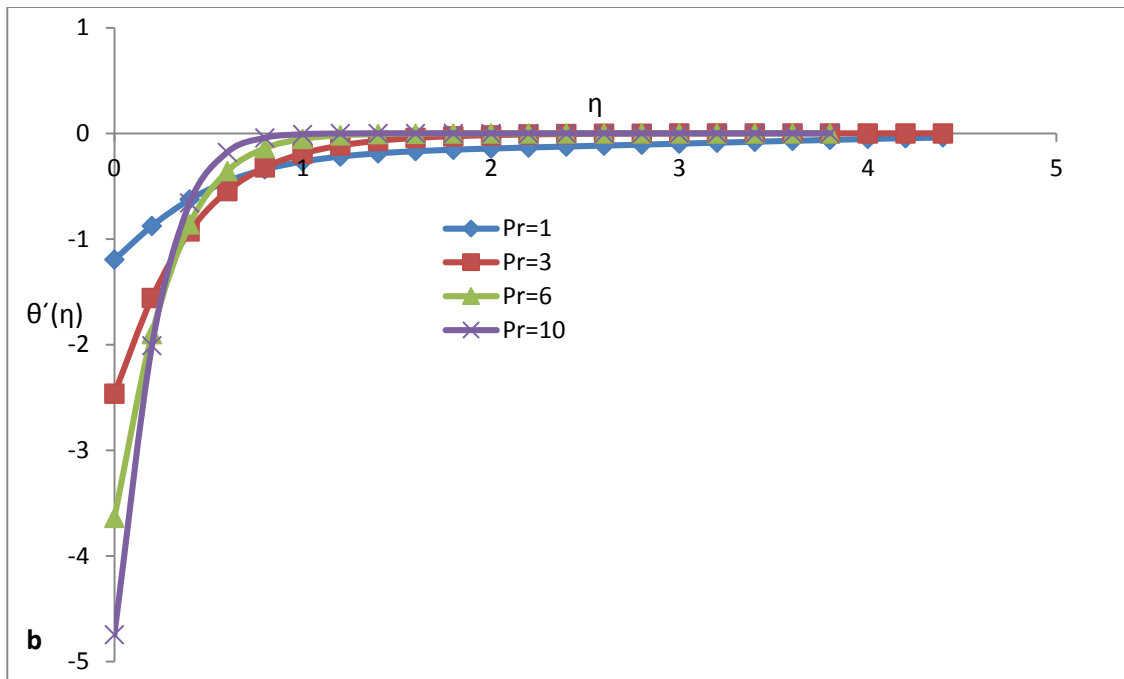


Figure 3. Effect of Prandtl number (Pr) on (a). Temperature $\theta(\eta)$, (b). Temperature gradient $\theta'(\eta)$

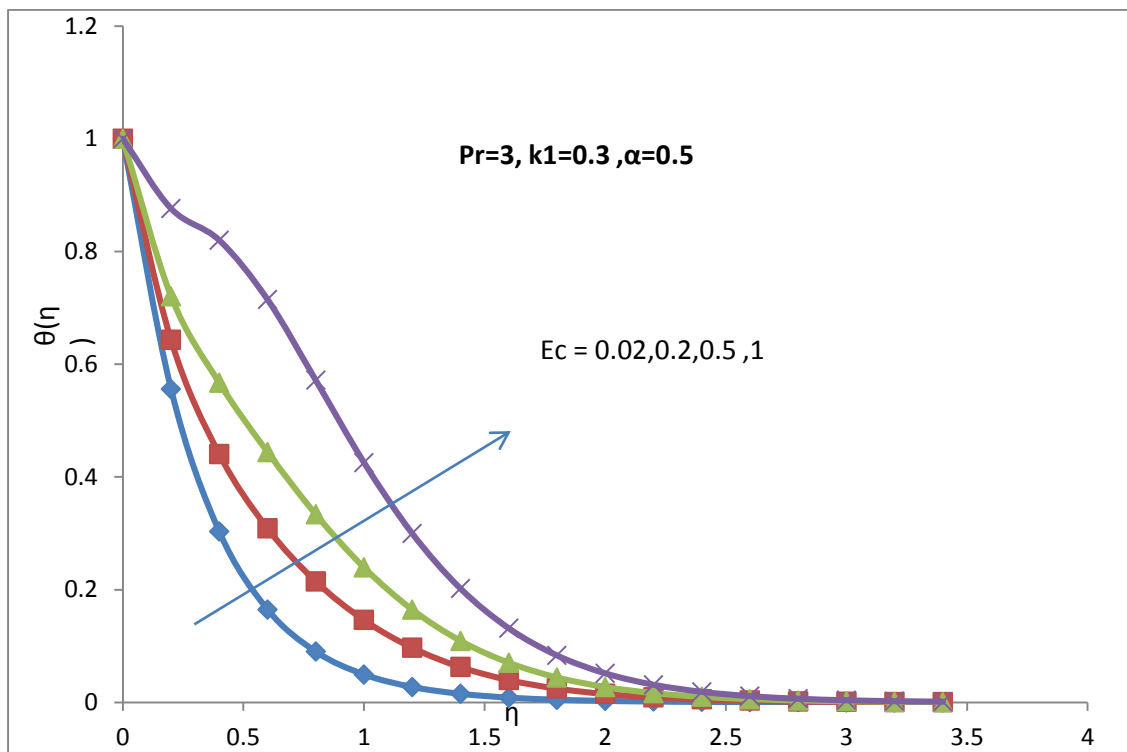


Figure4. Variation of non-dimensional temperature $\theta(\eta)$ Vs η for different values of Eckert number (Ec).

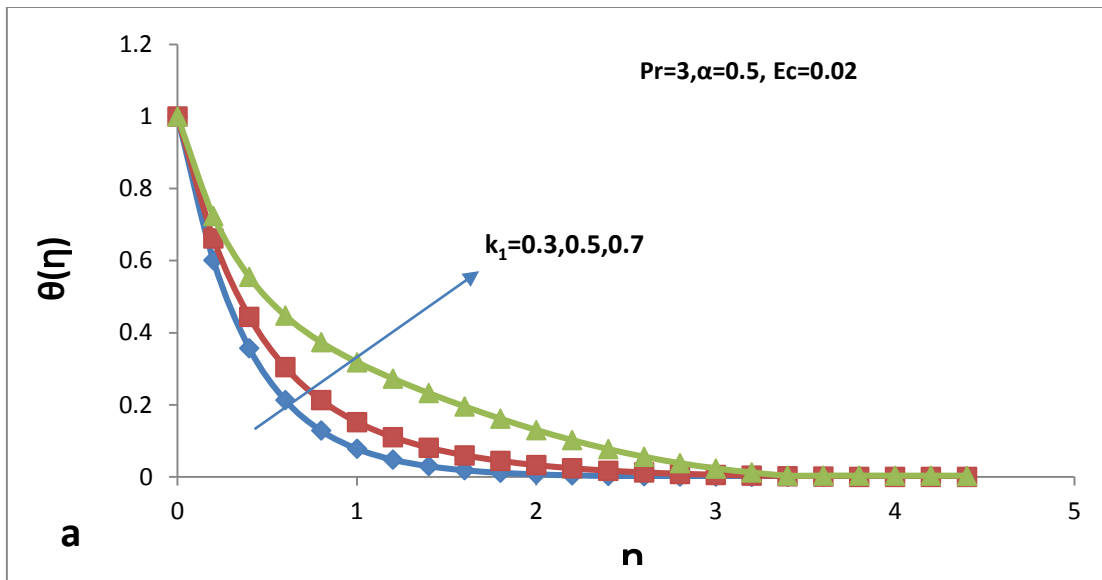


Figure 5. Variation of non-dimensional temperature $\theta(\eta)$ Vs η for different values of visco-elasticity (k_1).

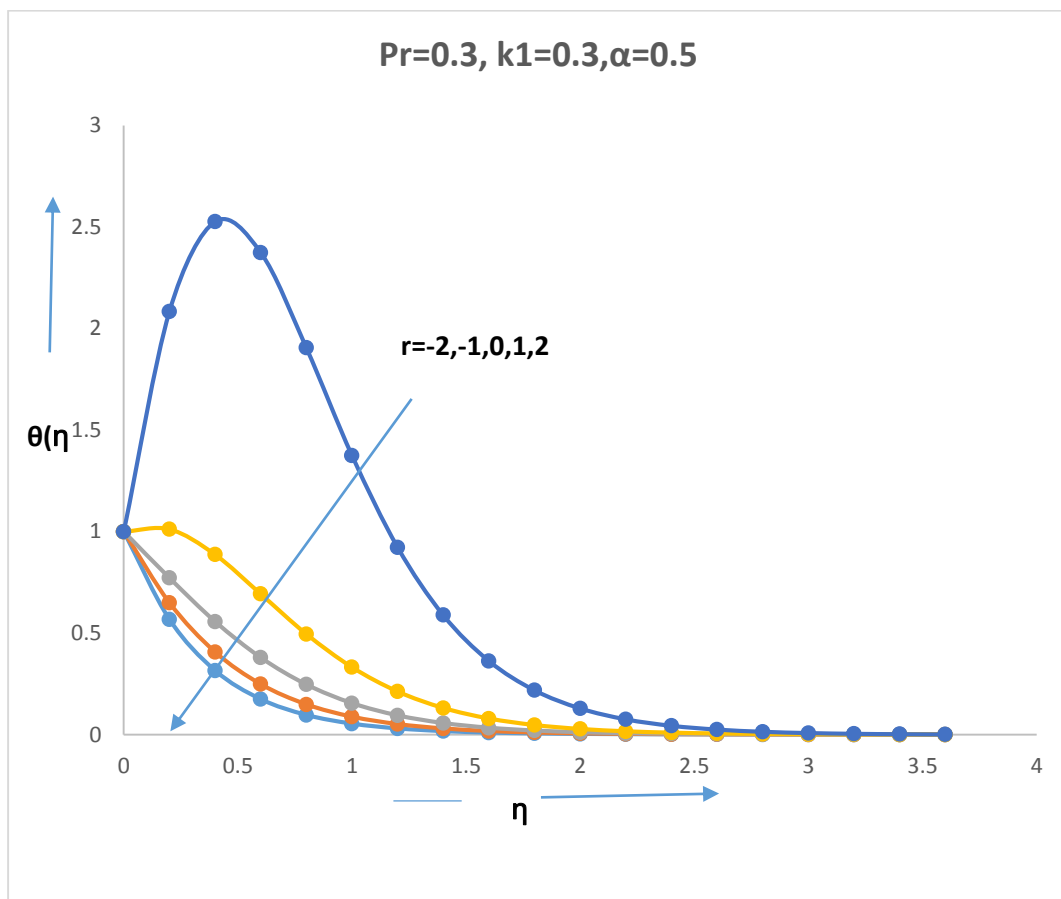


Figure6. Variation of $\theta(\eta)$ vs η for different values of wall temperature parameter(r)

Table 1. Heat Transfer characteristics at the wall

Visco-elasticity(k_1)	Prandtl number(Pr)	Internal heat source/sink (α)	Eckert number $Ec=0.0$	Eckert number $Ec=0.5$
0.3	3	0.5	-2.76053	-1.97008
0.5			-2.46793	-1.96787
0.7			-2.33234	-1.52267
0.3	1	0.5	-1.46771	-1.12829
	3		-2.76053	-1.97008
	6		-3.98965	-2.66140
		-0.5	-3.07090	-2.52390
		0.0	-2.80242	-2.47628
		0.5	-2.49414	-2.06475

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