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An Analytical, Lagrange's Interpolation Technique for the Use in the Analysis of a 4-Bar Mechanism Coupler Curve Equation

Khalid Nafees

Assistant Professor, Department of Mechanical Engineering,
Inderprastha Engineering College, Ghaziabad, India

Abstract

This paper introduces a methodology to form an equation of a prescribed 4-bar mechanism coupler curve using Lagrange's interpolation polynomial. The dimensions and orientations of all links of the mechanism are prescribed. The actual wooden model of the mechanism is constructed to determine its coordinates for coupler curve. The required equation is obtained by solving the Lagrange's polynomial. The findings are supported by demonstrating, graphically plotting and verifying the analysis of 4-bar mechanism using a numerical example. This concept helps in analyzing performance of the 4-bar mechanism to establish relationship between prescribed link lengths of 4-bar mechanism and its generated coupler curve equations.

Keywords: Coupler curve equation, 4-bar mechanism, Lagrange's polynomial, Kinematic analysis, Path generation

INTRODUCTION

The Kinematics of mechanisms involves study of their motion and techniques adopted to create them. The former part of Kinematics is referred as Kinematic Analysis while the latter is often called as Kinematic Synthesis. In Kinematic Analysis, the mechanism performance is monitored based on properties and motion characteristics. The parameters prescribed are number of links, their lengths and orientations along with type of joints between them. The output of Kinematic Analysis is calculated values of displacements, velocities, accelerations and forces etc. at different locations of mechanism links. It also helps in comparing the actual performance with desired performance of the mechanism.

For performing Kinematic Analysis, the analytical techniques provide major advantage of improved accuracy over graphical techniques that have drawback of limited drawing accuracy. Many researchers have performed Kinematic Analyses of mechanisms using different techniques.

Gurney and Tobias [1] proposed a graphical method for the investigation of regenerative machine tool chatter based on the harmonic response locus of the machine tool structure. The author (s) considered the affect of variation in chip thickness along with the penetration rate on stability. Erdman et al. [2] performed Kineto-elastodynamics analysis and synthesis based on structural analysis flexible approach for those mechanisms which may deflect because of internal body forces or external loads. Also, the author (s) investigated dynamic error caused in the system due to flexural, longitudinal and torsional strain element. Dubowsky and Gardner [3] solved the problem of predicting the dynamic behavior of general planar mechanisms with elastic links and multiple clearance connections. The author (s) emphasised the effects of system elasticity on the high internal impact forces generated by the presence of the clearances at high speeds. Hill and Midha [4] carried out the analysis and design of compliant mechanisms subjected to huge nonlinear deflections corresponding to given set of load. The author (s) applied the Newton-Raphson method to determine the force based on prescribed displacement and force boundary conditions. Uicker et al. [5] proposed an algebraic method using symbolic notation for the displacement analysis of linkages to formulate kinematic relations of a linkage based on matrix equations. The author (s) highlighted the overall solution for closed kinematic mechanisms consisting of higher pairs of prismatic and revolute pairs. Sheth and J. J. Uicker [6] proposed an experimental software system to automate the kinematic, static, and dynamic analyses of arbitrary mechanisms based on network theory and matrix methods. The author (s) made use of interactive capability of the ring data structure SKETCHPAD concepts to develop a general purpose design analysis system. Dobrjanskyj and Freudenstein [7] demonstrated the computer-aided techniques to evaluate feasibility of mechanical system design in preliminary stages. The author (s) applied concepts of graph theory to develop a computerized method for determining structural identity, to sketch the automatic graph of a mechanism based on its incidence matrix and enumerate the systematic general, single-loop constrained spatial mechanisms. Erkaya et al. [8] presented the kinematic as well as dynamic analysis of a slider-cank mechanism having an additional eccentric link between connecting rod and crank pin. Keeping the same stroke and cylinder gas pressure, the author (s) compared the analysis results of modified slider-crank mechanism with conventional slider-crank mechanism and concluded that the modified slider-crank mechanism has greater output torque.

Also, the kinematic and dynamic analyses of 4-bar mechanisms were carried out using various techniques, but so far, no research work was carried out to establish relationship between prescribed link lengths of a mechanism and coupler curve equations generated by different points on the coupler. Therefore, in present research work, the analysis of 4-bar mechanism was carried out to formulate an equation of the path followed by coupler. The final expression for this equation is obtained by solving Lagrange's polynomial. A numerical example is demonstrated to reflect the effectiveness of the work.

CONFIGURATION OF 4-BAR MECHANISM

The configuration of the 4-bar mechanism is shown in Fig. 1. It consists of revolute pairs at all of its joints with 4 numbers of links. The prescribed 4-bar mechanism is characterised by 3 binary links with 1 binary offset link. The fixed link, i.e. binary link 1 directly forms revolute pairs at pivots O_1 and O_2 with other links i.e. link 2 (binary crank link) and link 4 (binary rocker link) respectively. These links forms revolute pair with binary offset link 3 at points A and C. The input motion is given to the mechanism through link 2. On rotation of link 2, link 3 generates a coupler curve by tracing point B.

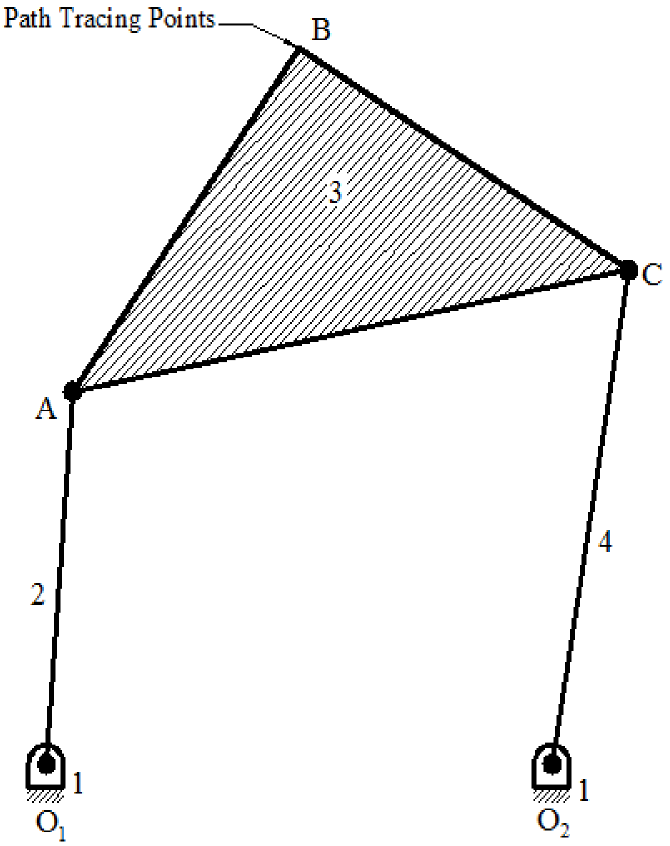

Figure. 1: Configuration of 4-Bar Mechanism

The frame is numbered by 1; the driver is the crank, numbered by 2, that rotates at constant angular speed; the driven is the rocker, numbered by 4, that oscillates about pivot position O_2; the follower is numbered by 3 that generate the coupler curve by tracing point B.

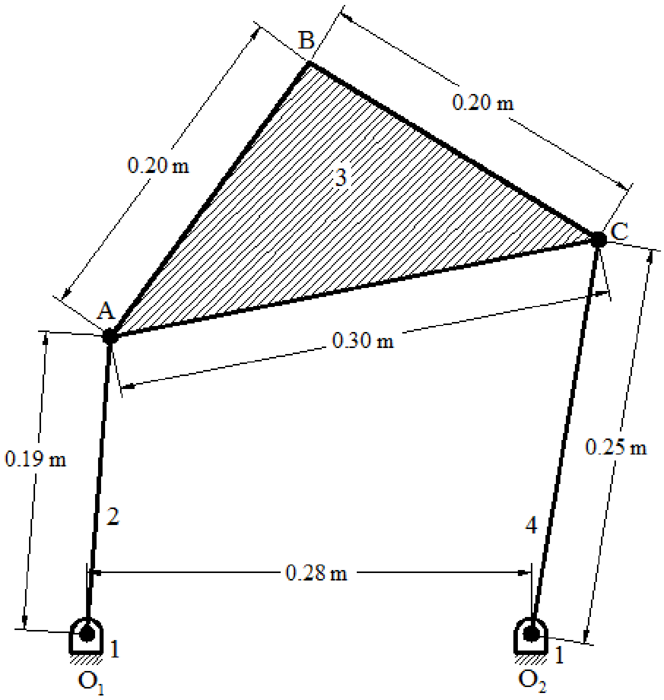

Figure. 2 Dimensional length of 4-Bar Mechanism

LAGRANGE'S POLYNOMIAL [9-10]

To verify mechanism coupler curve continuity in order to model relationship between its prescribed parameters, the set of discrete data points obtained through mechanism wooden model are needed to convert into equation. This process is called data or curve fitting. It is carried out using Lagrange's Polynomial for the purpose of interpolation.

In the polynomial interpolation, the problem is to determine a polynomial equation that passes through (n + 1) data points defined by $(x_0, y_0), (x_1, y_1), \ldots\ldots, (x_n, y_n)$.

To obtain solution, few basic polynomials are formed based on following constraint condition:

$$L_{n,k}(x_j) = \begin{cases} 1 \text{ when } j = k \\ 1 \text{ when } j \neq k \end{cases} \quad (1)$$

The n[th] degree Lagrange interpolating polynomial is given by

$$L(x) = \sum_{k=0}^{n} y_k L_{n,k}(x) \quad (2)$$

E.g., the three basis polynomials for a set of x values $\{x_0, x_1, x_2\}$ are given by

$$L_{2,0}(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} \quad (3a)$$

$$L_{2,1}(x) = \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} \quad (3b)$$

$$L_{2,2}(x) = \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} \quad (3c)$$

Therefore, the 2nd degree Lagrange interpolating polynomial is given by

$$L(x) = y_0 L_{2,0}(x) + y_1 L_{2,1}(x) + y_2 L_{2,2}(x) \qquad (4)$$

NUMERICAL PROBLEM BASED ON GENERATION OF COUPLER CURVE EQUATIONS FOR 4-BAR MECHANISM

Problem Statement: It is required to generate coupler curve equation through coupler tracing point of prescribed 4-bar mechanism. The dimensional lengths of each link of 4-bar mechanism, shown in Fig. 2, are:

O_1O_2 = 0.28 m; O_1A = 0.19 m;

AC = 0.30 m; O_2C = 0.25 m;

AB = BC = 0.20 m

Observations: On 5 successive rotations of binary crank link O_1A by 72° (Refer Fig. 3), the coordinates of coupler tracing point B are recorded as given in table 1.

Table 1 Coordinates of Tracing point B of the Coupler of Prescribed 4-bar Mechanism

Coordinates	B_1	B_2	B_3	B_4	B_5
X	32.8	20.2	6.0	- 3.0	- 6.0
f(X)	31.7	35.2	29.6	20.0	13.5

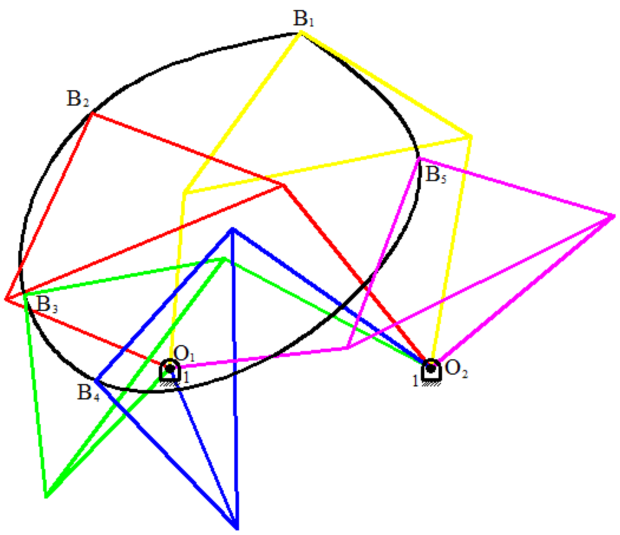

Figure. 3. Successive Rotation of Crank link O_1A of Prescribed 4-Bar Mechanism

METHODOLOGY TO GENERATE COUPLER CURVE EQUATIONS FOR 4-BAR MECHANISM

The Lagrange's interpolation polynomial is applied to determine the coupler curve equations of a prescribed 4-bar mechanism. In order to generate coupler curve equation, the coordinates of intermediate points are needed. For this purpose, a wooden model of the prescribed 4-bar mechanism is developed. The successive rotation of crank link of the model yields the required coordinates of intermediate points. These coordinates are fed into Lagrange's interpolation polynomial to generate the coupler curve equation.

To generate coupler curve equations, following steps are followed:

Step1. Put the graph paper between the wooden board and model of prescribed 4-bar mechanism.

Step2. Put the pencil at tracing point B of the coupler.

Step3. Record the coordinate of tracing point B at position 1 of the coupler as X_0, $f(X_0)$.

Step4. Rotate the crank (Link 2) through 72° and record the coordinate of tracing point B at position 2 of the coupler as X_1, $f(X_1)$.

Step5. Rotate the crank (Link 2) through 144° and record the coordinate of tracing point B at position 3 of the coupler as X_2, $f(X_2)$.

Step6. Rotate the crank (Link 2) through 216° and record the coordinate of tracing point B at position 4 of the coupler as X_3, $f(X_3)$.

Step7. Rotate the crank (Link 2) through 288° and record the coordinate of tracing point B at position 5 of the coupler as X_4, $f(X_4)$.

Step8. To generate final coupler curve equation, substitute the above recorded coordinates X_i, $f(X_i)$ into the Lagrange's Polynomial Eqn. (4) and solve it. (where i = 0, 2...4)

RESULTS AND DISCUSSION

The final coupler curve equation determined for prescribed 4-bar mechanism by solving Lagrange's polynomial equation is:

$$f(X) = -0.002238\,X^4 + 0.003432\,X^3 - 0.0786791\,X^2 + 1.2181\,X + 24.4549$$

The curve of above equation is graphically shown in Fig. 4. The generation of coupler curve equation for the prescribed 4-bar mechanism helps to carry out its actual path generation performance analysis and compare it with desired path generation performance. Also, it supplements the precise and accurate requirements of automation industry.

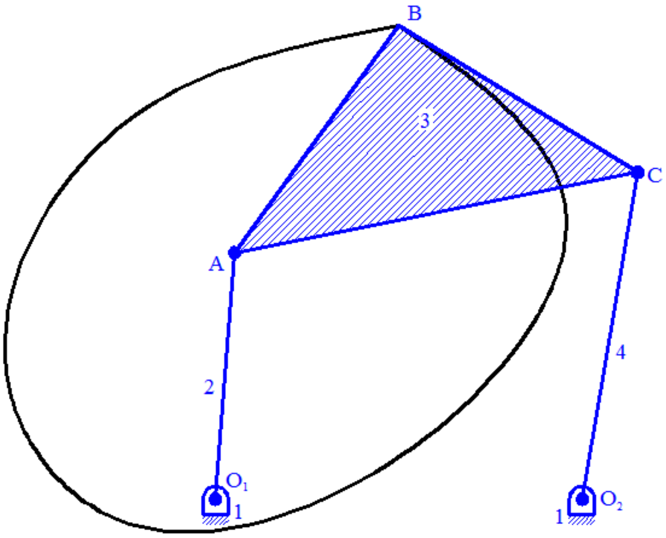

Figure. 4 Coupler Curve Equation generated by prescribed 4-Bar Mechanism

CONCLUSIONS

The present research work suggests a methodology to generate coupler curve equation for a single degree of freedom, four-bar mechanism with prescribed dimensions. To generate the equation, the coordinates of coupler tracing point is obtained with the help of actual model of the four-bar mechanism with prescribed dimensions. The final equation is formed and generated by solving the Lagrange's polynomial. The whole generation procedure for prescribed 4-bar mechanism is demonstrated, graphically plotted and verified by a numerical example. The present research work is beneficial in minimising the error between overall actual performance and desired performance of the mechanism. Also, it is useful in analyzing the path generation performance of the four-bar mechanism which further helps to augment the accurate requirements of automation industry.

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