International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 11 (2018) pp. 9848-9857

Deformation of a Layered Poroelastic Half-Space Subjected to Fluid Extraction

Amit Kumar*, Kuldip Singh, Mukesh Kumar Sharma
Department of Mathematics, Guru Jambheshwar University of Science & Technology, Hisar, Haryana, India.
Corresponding Author

Abstract

The problem of fluid extraction from an internally located circular zone of a poroelastic half-space has been investigated. The objective of the study is to proceed beyond the point sink problem and to develop certain analytic solutions of the ground surface displacements due to fluid extraction at a specified flow rate over a circular disc-shaped region located at the interior of a poroelastic half-space. The solution to the problem is obtained from the rigorous mathematical analysis of the initial/boundary value problem, which, in view of the axial symmetry and time-dependency of the problem, uses Hankel and Laplace transforms techniques.

Keywords: Fluid Extraction, Poroelasticity, Ground Subsidence, Laplace and Hankel Transforms

INTRODUCTION

The phenomenon known as land subsidence is the settlement of the land surface due to any of several factors among which the most important is fluid withdrawal. In geotechnical, hydraulic and petroleum engineering it is sometimes necessary to pump water or some other fluid from the ground. This may be for various reasons, e.g. (a) obtaining supplies of water, oil or gas, (b) reducing pore water pressures in the ground, and (c) to lower down the water table in order to make it suitable for begin construction operations. For the removal of pore fluid from the ground, the fluid pressure is to reduce in the vicinity of the pump, which results an increase in the compressive effective stress state. This increase of effective stress will cause consolidation of the ground and may lead to large scale subsidence. The decrease in pore pressure will not occur immediately. After pumping has commenced, the pore pressures will gradually decrease below their initial values until a steady-state distribution is established. Hence the resultant consolidation and surface subsidence will be time dependent.

Among the various subsidence mechanisms which have been reviewed by Scott (1978), fluid withdrawal from subsurface resources plays an important role. Areas where major land subsidence has occurred have been described by Poland and Davis (1969) and Poland (1972). Gas/oil production in subsiding areas is almost exclusively obtained from geologically recent sediments (post Eocene) at a depth less than 2000 m. The sinking surface takes the shape of a bowl centered above the reservoir and the settlement quickly decreases with the distance from the production wells. In complex geologic environment land subsidence due to fluid removal can be effectively analyzed and predicted by the aid of mathematical models. However developing reliable prediction tools is still a challenge for the researchers working in this field.

The mathematical model of a physical problem requires some assumptions to it implementable. Geertsma (1973) has considered the medium as a homogeneous and isotropic semi-infinite porous medium for the model used to simulate the subsidence over the gas field of Groningen. The subsidence due to fluid extraction has also been reported in the work of Delfranche (1978), Lofgren (1978), Premchitt (1979), and Harada (1983). In addition to the extraction of groundwater; the withdrawal of air and gas can also induce surface subsidence as reported in the work of Bear and Pinder (1978).

The linear theory of poroelasticity introduced by Biot (1941) and its further extensions by Rice and Cleary (1976), Detournay and Cheng (1993), Selvadurai (1996), Wang (2000) has been effectively applied to several areas in the field of geomechanics including ground water withdrawal and re-charge problems related with both homogeneous and stratified fluid-saturated regions. The major applications of the methodology prescribed in the linear theory of poroelasticity regarding the problems of fluid withdrawal have mainly paying attention on the research of the point sink problem. The point sink fluid extraction problem for a poroelastic solid of infinite extent has been investigated by Cleary (1977), Rudnicki (1986), Booker and Carter (1986, and 1987), Kanok-Nukulchai and Chau (1990), Chau (1996), Barry et al. (1997) and later by Chen (2005). However, in a realistic approach, the region over which the fluid withdrawal takes place is in general three-dimensional with an agreement of withdrawal wells configured to manage the subsidence. There is another different approach restricting the state of deformation in the poroelastic medium to only displacements along a specific direction and consequently resulting in a generalization of the general three dimensional equations of poroelasticity to simpler decoupled forms of Gambolati (2006). The key aim of the present paper is to go on ahead of the point sink problem and to build up certain solutions that can validate the dimensions of the region in which fluid is extracted at a specific speed.

FORMULATION OF THE PROBLEM

In the present paper, the problem of the ground surface displacements caused by fluid withdrawal over a circular disc-shaped region placed at the interior of a poroelastic half-space at a flow rate of P (units L^3/T) is considered. P denotes the entire volume of fluid that is extracted per unit time. The soil skeleton is modeled as an isotropic linear elastic material obeying Hooke's law while the pore fluid may be compressible. The flow through the pores is governed by Darcy's law. The physical model of the problem is defined in Fig.1 in which a circular disc-shaped region of finite radius "**a**" and zero thickness placed at a finite depth "**h**" beneath the surface of the poroelastic half-space preserving axial symmetry.

Laplace and Hankel integral transform techniques have been used to obtain the analytical solutions for the time-dependent axisymmetric poroelastic problem.

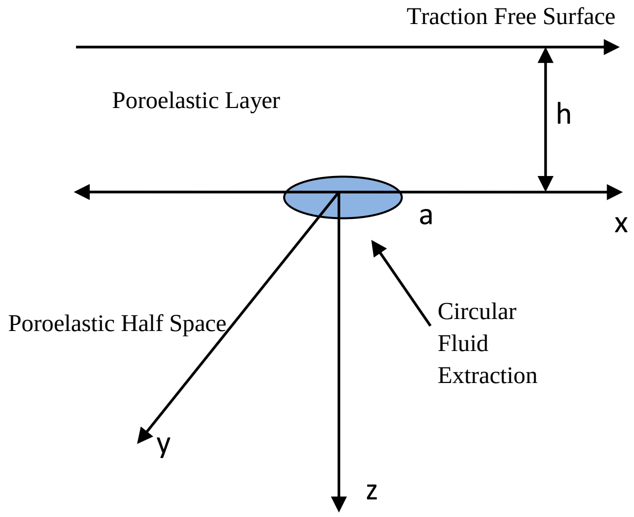

Figure 1

BASIC EQUATIONS

The basic equations of axially symmetric consolidation of a homogeneous elastic porous medium are the storage equation and the equations of equilibrium in the two coordinate directions r and z. For a linearly compressible fluid and linearly compressible solid particles n the case of axially symmetric deformations, the storage equation reduces to

$$\alpha \frac{\partial \varepsilon}{\partial t} + S \frac{\partial p}{\partial t} = \frac{k}{\gamma_f} \left(\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial z^2} \right) \tag{1}$$

where k is the permeability of the porous material, γ_f is the volumetric weight of the fluid, p is the pore pressure, and α is the Biot coefficient, $\alpha = 1 - C_s / C_m$

The parameter S is the storativity, defined as

$$S = nC_f + (\alpha - n) C_s$$

where C_s is the compressibility of the particle material, C_m is the compressibility of the porous medium as a whole and C_f is the compressibility of the pore fluid.

The equilibrium equations in terms of total stresses can be expressed as

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{tt}}{r} + \frac{\partial \sigma_{zr}}{\partial z} = 0 \tag{2}$$

$$\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \sigma_{rz}}{\partial r} + \frac{\sigma_{rz}}{r} = 0 \tag{3}$$

Using Terzaghi's relations, the total stresses can be break up into the effective stresses and the pore pressure by following relations:

$$\sigma_{rr} = \sigma'_{rr} + \alpha p \qquad \sigma_{rz} = \sigma'_{rz} \tag{4}$$

$$\sigma_{zz} = \sigma'_{zz} + \alpha p \qquad \sigma_{zr} = \sigma'_{zr} \tag{5}$$

Using Hooke's law for the case of axial symmetry, the effective stresses can be expressed in terms of the displacement components u_r and u_z by the following relations:

$$\sigma'_{rr} = -\left(K - \tfrac{2}{3} G\right)\varepsilon - 2G \frac{\partial u_r}{\partial r} \tag{6}$$

$$\sigma'_{tt} = -\left(K - \tfrac{2}{3} G\right)\varepsilon - 2G \frac{u_r}{r} \tag{7}$$

$$\sigma'_{zz} = -\left(K - \tfrac{2}{3} G\right)\varepsilon - 2G \frac{\partial u_z}{\partial z} \tag{8}$$

$$\sigma'_{rz} = \sigma'_{zr} = -G\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right) \tag{9}$$

$$\sigma'_{rt} = \sigma'_{tr} = 0 \tag{10}$$

$$\sigma'_{zt} = \sigma'_{tz} = 0 \tag{11}$$

where K and G are the compression modulus and the shear modulus of the porous medium; $u_r(r,z,t)$ and $u_z(r,z,t)$ respectively denote the displacement components in the r and z coordinate directions.

$$\varepsilon = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} \tag{12}$$

is the volume strain.

The sign convention for the stresses is that compressive stresses are considered positive (as for the pore pressure), which is standard practice in soil mechanics. For this reason the expressions for Hooke's law, equations (6) – (9) contain a minus sign.

The equilibrium equations expressed in terms of the displacements and the pore pressure are given by the following relations:

$$G(\nabla^2 u_r - \frac{u_r}{r^2}) - (K + \tfrac{1}{3}G)\frac{\partial \varepsilon}{\partial r} - \alpha \frac{\partial p}{\partial r} = 0 \quad (13)$$

$$G\nabla^2 u_z - (K + \tfrac{1}{3}G)\frac{\partial \varepsilon}{\partial z} - \alpha \frac{\partial p}{\partial z} = 0 \quad (14)$$

which can be re-written in a more convenient form, suggested by McNamee & Gibson (1960),

$$G(\nabla^2 u_r - \frac{u_r}{r^2}) - (2\eta - 1)\frac{\partial \varepsilon}{\partial r} - \alpha \frac{\partial p}{\partial r} = 0 \quad (15)$$

$$G\nabla^2 u_z - (2\eta - 1)\frac{\partial \varepsilon}{\partial z} - \alpha \frac{\partial p}{\partial z} = 0 \quad (16)$$

The coupled partial differential equation governing the pore fluid pressure $p(r,z,t)$ is given as:

$$\beta \frac{\partial p}{\partial t} - \gamma \frac{\partial \varepsilon}{\partial t} = c\nabla^2 p \quad (17)$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$ is the axisymmetric form of the Laplacian operator.

$$c = \frac{2GB^2(1-\nu)(1+\nu_u)^2 k}{9(\nu_u - \nu)(1-\nu_u)\gamma_w} \quad B = \frac{C_m - C_s}{C_m - C_s + n(C_f - C_s)}$$

$$\alpha = \frac{3(\nu_u - \nu)}{B(1-2\nu)(1+\nu_u)}$$

$$\beta = \frac{(1-2\nu_u)(1-\nu)}{(1-2\nu)(1-\nu_u)}$$

$$\gamma = \frac{2GB(1-\nu)(1+\nu_u)}{3(1-2\nu)(1-\nu_u)}$$

$$\eta = \frac{1-\nu}{1-2\nu} = \frac{K + \frac{4}{3}G}{2G} \quad (18)$$

where C_m is the compressibility of the porous skeleton; C_s is the compressibility of the skeletal material; C_f is the compressibility of the pore fluid and n is the porosity; ν_u is the undrained Poisson's ratio of the fluid-saturated medium; k is the hydraulic conductivity; γ_w is the unit weight of pore fluid; B is Skempton's pore pressure parameter; G and ν are, respectively, the shear modulus and Poisson's ratio of the porous skeleton (i.e. the drained elastic parameters).

METHOD OF SOLUTION USING DISPLACEMENT FUNCTIONS

By differentiating equation (15) with respect to r, multiplying equation (15) by 1/r, differentiating equation (16) with respect to z, and then adding the three resulting equations, the following equation is obtained

$$\nabla^2(2\eta G\varepsilon - \alpha p) = 0 \quad (19)$$

It can be re-written as

$$\frac{\alpha p}{2G} = \eta\varepsilon + \frac{\partial F}{\partial z} \quad (20)$$

where F is any harmonic function satisfying the equation

$$\nabla^2 F = 0 \quad (21)$$

Moreover, it is supposed that the two equilibrium equations can be fulfilled by writing

$$u_r = -\frac{\partial D}{\partial r} + z\frac{\partial F}{\partial r} \quad (22)$$

$$u_r = -\frac{\partial D}{\partial z} + z\frac{\partial F}{\partial z} - F \quad (23)$$

The volume strain ε is now expressed as:

$$\varepsilon = \nabla^2 D \quad (24)$$

Using equation (19) it now follows that the pore pressure can be expressed in terms of the displacement functions D and F by the relation

$$\frac{\alpha p}{2G} = -\eta\nabla^2 D + \frac{\partial F}{\partial z} \quad (25)$$

It can easily be demonstrated that the equilibrium equations (15) and (16) are identically satisfied, provided that the function F is definitely harmonic, as specified in equation (21).

Substituting equations (24) and (25) into the equation (17), the final differential equation for the displacement function D is obtained as follows:

$$c\nabla^2 D = \left(\beta + \frac{\alpha\gamma}{2G\eta}\right)\nabla^2 \frac{\partial D}{\partial t} - \frac{\beta}{\eta}\frac{\partial^2 F}{\partial z \partial t} \quad (26)$$

The expressions for the total stresses are now given as

$$\frac{\sigma_{rr}}{2G} = -\nabla^2 D + \frac{\partial^2 D}{\partial r^2} - z\frac{\partial^2 F}{\partial r^2} + \frac{\partial F}{\partial z} \quad (27)$$

$$\frac{\sigma_{zz}}{2G} = -\nabla^2 D + \frac{\partial^2 D}{\partial z^2} - z\frac{\partial^2 F}{\partial z^2} + \frac{\partial F}{\partial z} \quad (28)$$

$$\frac{\sigma_{rz}}{2G} = \frac{\partial^2 D}{\partial r \partial z} - z\frac{\partial^2 F}{\partial r \partial z} \quad (29)$$

The zeroth order Hankel transform $\overline{F}(\xi,z,t)$ of a function $F(r,z,t)$ is given by the relation

$$\overline{F}(\xi,z,t) = \int_0^\infty F(r,z,t) r J_0(\xi r) dr \quad (30)$$

where ξ is the Hankel transform parameter and $J_0(x)$ represents the first kind of Bessel function of order 0

The Laplace transform $\tilde{\bar{F}}(\xi,z,s)$ of a function $\bar{F}(\xi,z,t)$ is given by the relation

$$\tilde{\bar{F}}(\xi,z,s)=\int_0^\infty \bar{F}(\xi,z,t)e^{-st}\,dt \qquad (31)$$

where s is the Laplace transform parameter

Applying Laplace integral transformation with respect to the time variable t and Hankel transformation with respect to radial variable r, the governing partial differential equations (21) and (26) are reduced to the following ordinary differential equations:

$$\left(\frac{d^2}{dz^2}-\xi^2\right)\tilde{\bar{F}}=0 \qquad (32)$$

$$\left(\frac{d^2}{dz^2}-\xi^2\right)\left\{\frac{d^2}{dz^2}-\left[\xi^2+\frac{s}{c}\left(\beta+\frac{\alpha\gamma}{2G\eta}\right)\right]\right\}\tilde{\bar{D}}=-\frac{\beta s}{\eta c}\frac{d\tilde{\bar{F}}}{dz} \qquad (33)$$

INITIAL AND BOUNDARY CONDITIOS

The physical model of the problem is defined in Fig. 1 in which a circular disc-shaped region of finite radius "**a**" and zero thickness placed at a finite depth "**h**" beneath the surface of the poroelastic half-space preserving axial symmetry. The plane circular region is subjected to fluid removal at a constant flow rate of P (units L^3/T)

The boundary and initial conditions governing the fluid withdrawal problem can be described by taking into consideration two types of region

(i) a layer type region given by $r\in(0,\infty)$; $z\in(0,-h)$ and represented by superscript $(\ )^L$

(ii) a half-space region given by $r\in(0,\infty)$; $z\in(0,\infty)$ and represented by superscript

$(\ )^H$

The boundary conditions applicable to the surface $z=-h$ are given by

$$\sigma_{zz}^{(L)}(r,-h,t)=0$$

$$\sigma_{rz}^{(L)}(r,-h,t)=0$$

$$p^{(L)}(r,-h,t)=0 \qquad (34)$$

The continuity conditions applicable to the surface $z=0$ are given by

$$u_r^{(L)}(r,0,t)-u_r^{(H)}(r,0,t)=0$$

$$u_z^{(L)}(r,0,t)-u_z^{(H)}(r,0,t)=0 \qquad (35)$$

$$\sigma_{zz}^{(L)}(r,0,t)-\sigma_{zz}^{(H)}(r,0,t)=0$$

$$\sigma_{rz}^{(L)}(r,0,t)-\sigma_{rz}^{(H)}(r,0,t)=0 \qquad (36)$$

$$p^{(L)}(r,0,t)-p^{(H)}(r,0,t)=0 \qquad (37)$$

The initial conditions are given by

$$\boldsymbol{\sigma}^{(L)}(\mathbf{x},0)=0 \qquad \boldsymbol{\sigma}^{(H)}(\mathbf{x},0)=0 \qquad (38)$$

$$\mathbf{u}^{(L)}(\mathbf{x},0)=0 \qquad \mathbf{u}^{(H)}(\mathbf{x},0)=0 \qquad (39)$$

$$\mathbf{p}^{(L)}(\mathbf{x},0)=0 \qquad \mathbf{p}^{(H)}(\mathbf{x},0)=0 \qquad (40)$$

The pressure gradient discontinuity condition subjected to a fluid withdrawal over the set circular region at the interface is given by

$$\frac{k}{\gamma_w}\left(\frac{\partial p}{\partial z}\right)_{z=0}^{L}-\frac{k}{\gamma_w}\left(\frac{\partial p}{\partial z}\right)_{z=0}^{H}=\frac{P}{\pi a^2}H(t) \qquad 0\le r\le a \qquad (41)$$

$$\frac{k}{\gamma_w}\left(\frac{\partial p}{\partial z}\right)_{z=0}^{L}-\frac{k}{\gamma_w}\left(\frac{\partial p}{\partial z}\right)_{z=0}^{H}=0 \qquad a<r<\infty \qquad (42)$$

where H(t) is the Heaviside step function.

Besides, the solution be supposed to fulfill the regularity conditions such that

$$\mathbf{p}^{(i)}(\mathbf{x},t),\ \mathbf{u}^{(i)}(\mathbf{x},t),\ \boldsymbol{\sigma}^{(i)}(\mathbf{x},t)\to 0 \quad \text{as } |x|\to\infty,\quad t\in(0,\infty),\quad (i=L,H) \qquad (43)$$

SOLUTION OF THE PROBLEM

The general solutions of ordinary differential equations (32) and (33) in case of poroelastic layer are given by

$$\tilde{\bar{F}}^{(L)}(\xi,z,s)=A_1e^{-\xi z}+B_1e^{\xi z} \qquad (44)$$

$$\tilde{\bar{D}}^{(L)}(\xi,z,s)=C_1e^{-\xi z}+D_1e^{\xi z}+E_1e^{-\varphi z}+F_1e^{\varphi z}+\Psi A_1ze^{-\xi z}+\Psi B_1ze^{\xi z} \qquad (45)$$

The general solutions of ordinary differential equations (32) and (33) in case of poroelastic half-space region are given by

$$\tilde{\bar{F}}^{(H)}(\xi,z,s)=A_2e^{-\xi z} \qquad (46)$$

$$\tilde{\bar{D}}^{(H)}(\xi,z,s)=C_2e^{-\xi z}+E_2e^{-\phi z}+\Psi A_2ze^{-\xi z} \qquad (47)$$

Where $\phi = \sqrt{\xi^2 + \frac{s}{c}\left(\beta + \frac{\alpha\gamma}{2G\eta}\right)}$

$$\Psi = \frac{\beta G}{2G\beta\eta + \alpha\gamma} \tag{48}$$

Altogether 9 arbitrary functions of ξ and s are encountered and these can be determined by satisfying the boundary conditions (34) – (42). The final expressions for the displacements, $u_r(r,z,t)$ and $u_z(r,z,t)$, pore water pressure $p(r,z,t)$ and the stresses $\sigma_{rr}(r,z,t)$, $\sigma_{zz}(r,z,t)$, $\sigma_{rz}(r,z,t)$ caused by the fluid extraction take the following forms:

$$\frac{u_z^{(L)}}{P^*} = \int_{\zeta-i\infty}^{\zeta+i\infty}\int_o^{\infty} K_0 \left\{ \begin{array}{l} \left(\dfrac{e^{-\xi h} - e^{-\phi h}}{K_1}\right) \\ \left[\begin{array}{l} K_2 e^{-\xi z - \xi h} + \\ K_3(\phi e^{-\phi z - \phi h} - \xi e^{-\xi z - \xi h}) \end{array}\right] \\ + \xi(e^{-\xi z - \xi h} + e^{\xi z + \xi h})\dfrac{e^{-\xi h}}{K_4 \xi} \\ + \phi(e^{-\phi z - \phi h} + e^{\phi z + \phi h})\dfrac{e^{-\phi h}}{K_4 \phi} \end{array} \right\} \frac{e^{st}}{s}\, d\xi\, ds \tag{49}$$

$$\frac{u_z^{(H)}}{P^*} = \int_{\zeta-i\infty}^{\zeta+i\infty}\int_o^{\infty} K_0 \left\{ \begin{array}{l} \left(\dfrac{e^{-\xi h} - e^{-\phi h}}{K_1}\right) \\ \left[K_2 e^{-\xi z - \xi h} + K_3(\phi e^{-\phi z - \phi h} - \xi e^{-\xi z - \xi h})\right] \\ + \xi(e^{-\xi z - \xi h} + e^{-\xi z + \xi h})\dfrac{e^{-\xi h}}{K_4 \xi} \\ + \phi(e^{-\phi z - \phi h} + e^{-\phi z + \phi h})\dfrac{-e^{-\phi h}}{K_4 \phi} \end{array} \right\} \frac{e^{st}}{s}\, d\xi\, ds \tag{50}$$

$$\frac{u_r^{(L)}}{P^*} = \int_{\zeta-i\infty}^{\zeta+i\infty}\int_o^{\infty} K_0 \left\{ \begin{array}{l} \left(\dfrac{e^{-\xi h} - e^{-\phi h}}{K_1}\right) \\ \left[K_5 e^{-\xi z - \xi h} + K_3 \xi(e^{-\phi z - \phi h} - e^{-\xi z - \xi h})\right] \\ - \xi(e^{\xi z + \xi h} - e^{-\xi z - \xi h})\dfrac{e^{-\xi h}}{K_4 \xi} \\ - \xi(e^{\phi z + \phi h} - e^{-\phi z - \phi h})\dfrac{-e^{-\phi h}}{K_4 \phi} \end{array} \right\} \frac{e^{st}}{s}\, d\xi\, ds \tag{51}$$

$$\frac{u_r^{(H)}}{P^*} = \int_{\zeta-i\infty}^{\zeta+i\infty}\int_o^{\infty} K_0 \left\{ \begin{array}{l} \left(\dfrac{e^{-\xi h} - e^{-\phi h}}{K_1}\right) \\ \left[K_5 e^{-\xi z - \xi h} + K_3 \xi(e^{-\phi z - \phi h} - e^{-\xi z - \xi h})\right] \\ - \xi(e^{-\xi z + \xi h} - e^{-\xi z - \xi h})\dfrac{e^{-\xi h}}{K_4 \xi} \\ - \xi(e^{-\phi z + \phi h} - e^{-\phi z - \phi h})\dfrac{-e^{-\phi h}}{K_4 \phi} \end{array} \right\} \frac{e^{st}}{s}\, d\xi\, ds \tag{52}$$

$$\frac{p^{(L)}}{P^{**}} = \int_{\zeta-i\infty}^{\zeta+i\infty}\int_o^{\infty} K_0 \left\{ \begin{array}{l} -\left(\dfrac{e^{-\phi z - 2\phi h} - e^{\phi z}}{2\phi}\right) \\ -\left[\dfrac{(K_6)(e^{-\xi z - \xi h} - e^{-\phi z - \phi h})(e^{-\xi h} - e^{-\phi h})}{K_1}\right] \end{array} \right\} \frac{e^{st}}{s}\, d\xi\, ds \tag{53}$$

$$\frac{p^{(H)}}{P^{**}} = \int_{\zeta-i\infty}^{\zeta+i\infty}\int_o^{\infty} K_0 \left\{ \begin{array}{l} -\left(\dfrac{e^{-\phi z - 2\phi h} - e^{-\phi z}}{2\phi}\right) \\ -\left[\dfrac{(K_6)(e^{-\xi z - \xi h} - e^{-\phi z - \phi h})(e^{-\xi h} - e^{-\phi h})}{K_1}\right] \end{array} \right\} \frac{e^{st}}{s}\, d\xi\, ds \tag{54}$$

$$\frac{\sigma_{rr}^{(L)}}{P^{**}} = \int_{\zeta-i\infty}^{\zeta+i\infty}\int_o^{\infty} K_0 \times \left\{ \begin{array}{l} K_7\left(\dfrac{e^{-\xi h} - e^{-\phi h}}{K_1}\right)e^{-\xi z - \xi h} \\ + K_3(\xi^2 e^{-\xi z - \xi h} - \phi^2 e^{-\phi z - \phi h})\left(\dfrac{e^{-\xi h} - e^{-\phi h}}{K_1}\right) \\ + \dfrac{\phi(e^{-\phi z - 2\phi h} - e^{\phi z})}{K_4} - \dfrac{\xi(e^{-\xi z - 2\xi h} - e^{\xi z})}{K_4} \end{array} \right\} \frac{e^{st}}{s}\, d\xi\, ds \tag{55}$$

$$\frac{\sigma_{rr}^{(H)}}{P^{**}} = \int_{\zeta-i\infty}^{\zeta+i\infty}\int_o^{\infty} K_0 \times \left\{ \begin{array}{l} K_7\left(\dfrac{e^{-\xi h} - e^{-\phi h}}{K_1}\right)e^{-\xi z - \xi h} \\ + K_3(\xi^2 e^{-\xi z - \xi h} - \phi^2 e^{-\phi z - \phi h})\left(\dfrac{e^{-\xi h} - e^{-\phi h}}{K_1}\right) \\ + \dfrac{\phi(e^{-\phi z - 2\phi h} - e^{-\phi z})}{K_4} - \dfrac{\xi(e^{-\xi z - 2\xi h} - e^{-\xi z})}{K_4} \end{array} \right\} \frac{e^{st}}{s}\, d\xi\, ds \tag{56}$$

$$\frac{\sigma_{z}^{(L)}}{P^{**}}=\int_{\zeta-i\infty}^{\zeta+i\infty}\int_{o}^{\infty}K_0\times\left\{\begin{array}{l}K_8\left(\dfrac{e^{-\xi h}-e^{-\phi h}}{K_1}\right)e^{-\xi z-\xi h}\\+K_9(e^{-\phi z-\phi h}-e^{-\xi z-\xi h})\left(\dfrac{e^{-\xi h}-e^{-\phi h}}{K_1}\right)\\+\dfrac{\xi(e^{-\xi z-2\xi h}-e^{\xi z})}{K_4}+\dfrac{\xi^2(e^{\phi z}-e^{-\phi z-2\phi h}-)}{K_4\phi}\end{array}\right\}\frac{e^{st}}{s}\,d\xi\,ds \quad (57)$$

$$\frac{\sigma_{z}^{(H)}}{P^{**}}=\int_{\zeta-i\infty}^{\zeta+i\infty}\int_{o}^{\infty}K_0\times\left\{\begin{array}{l}K_8\left(\dfrac{e^{-\xi h}-e^{-\phi h}}{K_1}\right)e^{-\xi z-\xi h}\\+K_9(e^{-\phi z-\phi h}-e^{-\xi z-\xi h})\left(\dfrac{e^{-\xi h}-e^{-\phi h}}{K_1}\right)\\+\dfrac{\xi(e^{-\xi z-2\xi h}-e^{-\xi z})}{K_4}+\dfrac{\xi^2(e^{-\phi z}-e^{-\phi z-2\phi h}-)}{K_4\phi}\end{array}\right\}\frac{e^{st}}{s}\,d\xi\,ds \quad (58)$$

$$\frac{\sigma_{rz}^{(L)}}{P^{**}}=\int_{\zeta-i\infty}^{\zeta+i\infty}\int_{o}^{\infty}K_0\times\left\{\begin{array}{l}K_{10}\left(\dfrac{e^{-\xi h}-e^{-\phi h}}{K_1}\right)e^{-\xi z-\xi h}\\+K_3(\xi e^{-\phi z-\phi h}-\xi^2e^{-\xi z-\xi h})\left(\dfrac{e^{-\xi h}-e^{-\phi h}}{K_1}\right)\\+\dfrac{\xi(e^{-\xi z-2\xi h}-e^{\xi z})}{K_4}+\dfrac{\xi(e^{\phi z}-e^{-\phi z-2\phi h})}{K_4}\end{array}\right\}\frac{e^{st}}{s}\,d\xi\,ds \quad (59)$$

$$\frac{\sigma_{rz}^{(H)}}{P^{**}}=\int_{\zeta-i\infty}^{\zeta+i\infty}\int_{o}^{\infty}K_0\times\left\{\begin{array}{l}K_{10}\left(\dfrac{e^{-\xi h}-e^{-\phi h}}{K_1}\right)e^{-\xi z-\xi h}\\+K_3(\xi e^{-\phi z-\phi h}-\xi^2e^{-\xi z-\xi h})\left(\dfrac{e^{-\xi h}-e^{-\phi h}}{K_1}\right)\\+\dfrac{\xi(e^{-\xi z-2\xi h}-e^{\xi z})}{K_4}+\dfrac{\xi(e^{\phi z}-e^{-\phi z-2\phi h})}{K_4}\end{array}\right\}\frac{e^{st}}{s}\,d\xi\,ds \quad (60)$$

Where

$$K_0=J_1(a\xi)J_0(r\xi)\;\; K_1=\eta\Psi(\phi-\xi)^2-\eta(\phi^2-\xi^2)+\xi(\phi-\xi)$$

$$K_2=\xi\Psi(z+h)-\Psi-\xi(z+h)$$

$$K_3=\frac{(2\eta\xi\Psi-\xi)}{\eta(\phi^2-\xi^2)}$$

$$K_4=2\eta(\phi^2-\xi^2)$$

$$K_5=\xi\Psi(z+h)+1-\xi(z+h)$$

$$K_6=(2\eta\xi\Psi-\xi)$$

$$K_7=(2\xi\Psi-\xi^2\Psi(z+h)+\xi^2(z+h)-2\xi)$$

$$K_8=(\xi^2\Psi-\xi^2)(z+h)$$

$$K_9=\frac{(2\eta\xi^3\Psi-\xi^3)}{\eta(\phi^2-\xi^2)}$$

$$K_{10}=(\xi^2\Psi-\xi^2)(z+h)-\Psi+\xi)$$

$$P^*=P\gamma_w\alpha/2G\pi ak$$

$$P^{**}=P\gamma_w\alpha/\pi ak$$

and ζ is a real number connected to the Bromwich contour integration linked with Laplace transform inversion as given by Salvadurai (2000).

To produce expressions for the displacements and stresses that can be expressed in terms of r, z and t, both the operations concerning Hankel transform inversion and Laplace transform inversion have to be performed. But these transformed expressions of the type (49)–(60) cannot be inverted precisely to produce unambiguous analytical results for the displacements, pore fluid pressures and total stresses. Therefore numerical techniques have to be used to execute Hankel and Laplace transforms inversions.

RESULTS

The results are presented for the non-dimensional time, $t^*=\dfrac{ct}{h^2}$

The various material parameters used for the numerical computations are given as follows: $\gamma_w=9.81\ \mathrm{kN/m^3};\ \ \nu=0.25;\ \ \nu_u=0.5;$

$k=10^{-15}\ \mathrm{m/s};\quad G=20.0\ \mathrm{GPa}$. The fluid extraction rate P is fixed.

In figure 2, the normalized surface displacement at $z=-h$, normalized with respect to volume flow rate P has been plotted against normalized radius $\frac{r}{h}$ for different values of non-dimensional time t^*. It is observed that the surface displacement and the volume rates demonstrate a near-linear association. If the volume rate doubles, the surface displacement also doubles.

In figure 3, the surface displacement normalized with respect to steady-state response at $r = 0$, has been plotted against normalized radius $\frac{r}{h}$ for different values of non-dimensional time t^*.

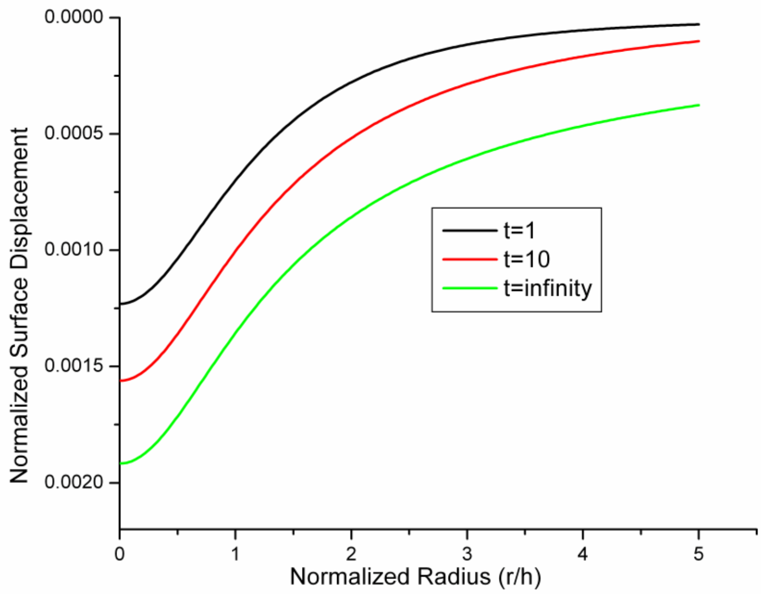

Figure 2. Profile of non-dimensinalized surface displacement at different time t^*

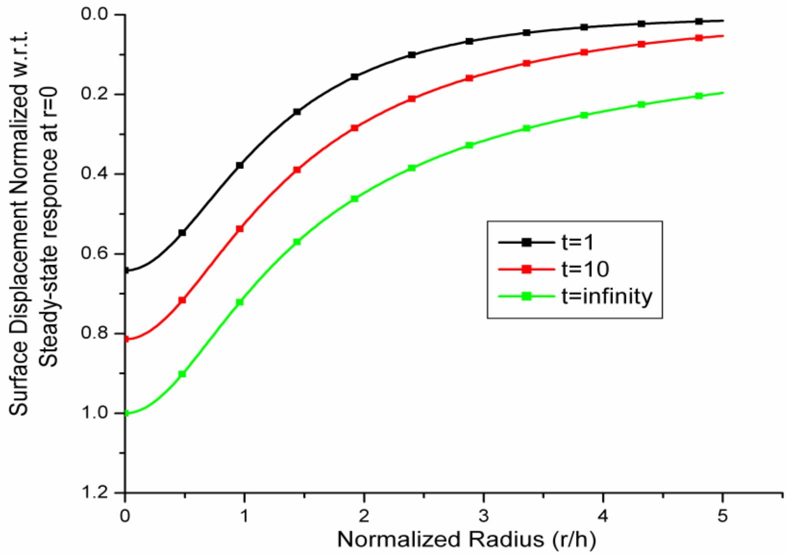

Figure 3. Profile of Surface Displacement Normalized w.r.t. Steady-state responce at r=0 at different time t^*

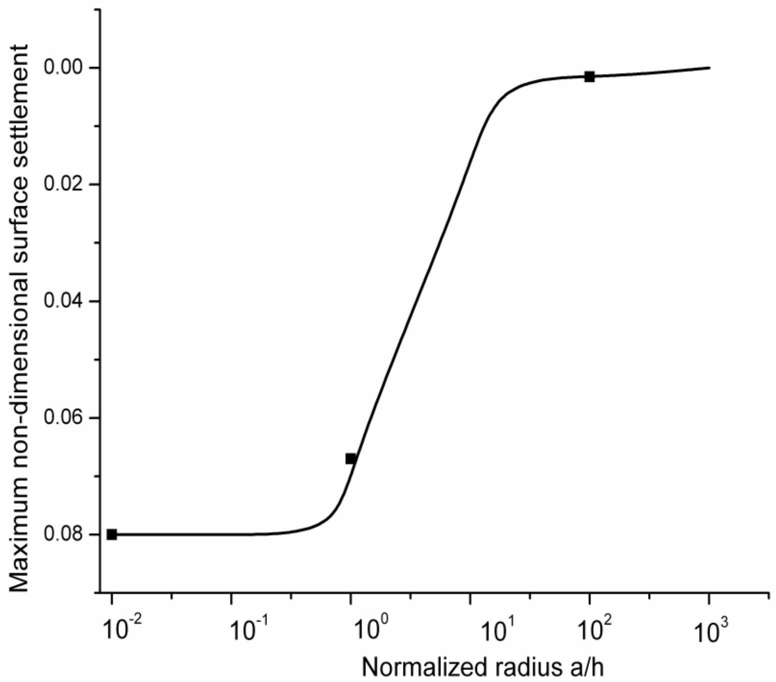

Figure 4. Profile of maximum non-dimensionalized surface settlement for different values of a/h

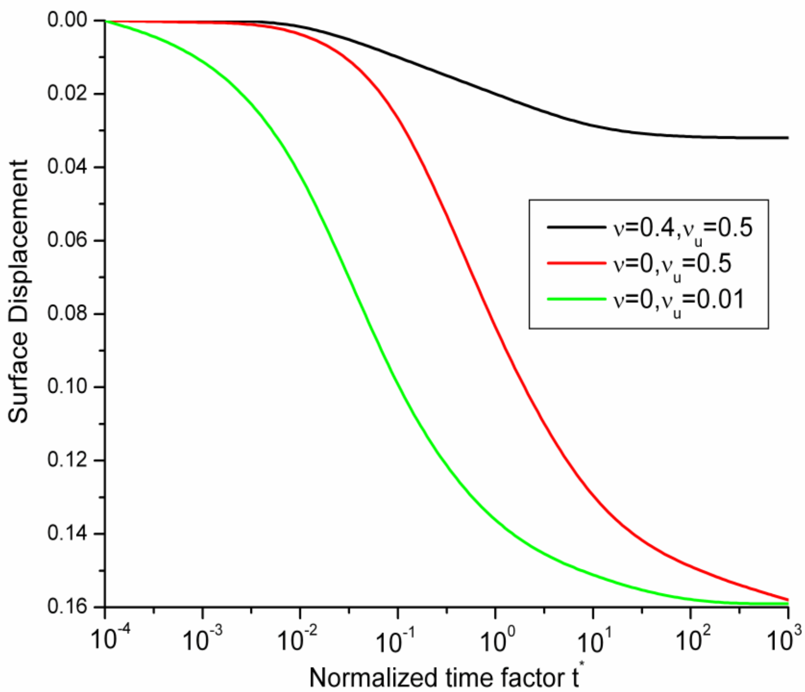

Figure 5. Profile of surface displacement with normalized time factor t^*

In figure 4, the maximum values of non-dimensional surface settlement have been plotted for different values of $\frac{a}{h}$.It is observed that the surface displacement remains unaltered until it reaches up to $\frac{a}{h}=10^{-1}$The surface settlement starts to diminish as $\frac{a}{h}$ exceeds 10^{-1} and converges to zero when $\frac{a}{h}>10^{2}$

In figure 5, the surface displacement at $z=-h$ has been plotted against non-dimensional time t^{*} for dissimilar values of Poisson's ratio. For the case $\nu_u=0.5$, it is observed that surface settlement is large when Poisson's ratio ν is small. One farthest case is also exhibited in this figure when $\nu=0$ *and* $\nu_u=10^{-2}$. It is observed that surface settlement is smaller for non-zero values of Poisson's ratio ν than those for $\nu=0$

DISCUSSIONS

This paper presents mathematical solutions to the problem of withdrawal of fluids from a fluid saturated porous medium with a deformable skeleton. The Biot's classical theory of linear poroelasticity has been successfully applied for Hookean elastic deformations of the porous skeleton and Darcy flow through the available pore space. The major applications of the methodology prescribed in the linear theory of poroelasticity regarding the problems of fluid withdrawal have mainly paying attention on the research of the point sink problem. The point sink fluid extraction problem for a poroelastic solid of infinite extent has been investigated by several researchers. However, in a realistic approach, the region over which the fluid withdrawal takes place is in general three-dimensional with an agreement of withdrawal wells configured to manage the subsidence. There is another different approach restricting the state of deformation in the poroelastic medium to only displacements along a specific direction and consequently resulting in a generalization of the general three dimensional equations of poroelasticity to simpler decoupled forms. The effects of widespread distributions of injections of changeable strength that are distributed in both space and time can be always obtained due to the linear behavior of the he governing equations of classical poroelasticity theory .

The key aim of the present paper is to go on ahead of the problem of point sink and to build up certain solutions that can validate the extent of the province in which fluid is removed at a specific speed. The canonical problem allied with the removal from a porous elastic half-space is to regard as the position where these processes happen over a circular disc-shaped region of finite radius "**a**" and zero thickness positioned at a finite depth "**h"** beneath the surface of the poroelastic half-space conserving axial symmetry. The problem of this nature has not been considered so far in the literature in the earth sciences or geomechanics. The solution to the problem is obtained from the thorough mathematical study of the initial-boundary value problem using Hankel and Laplace transforms techniques. This course of action can be further used to build up numerical results for the problem of withdrawal over the circular disc-shaped region and assessment of results from a disc-shaped region of withdrawal with the point sink problem.

REFERENCES

[1] Barry, S. I., Mercer, G. N. and Zoppou, C. Deformation and fluid flow due to a source in a poro-elastic layer. *Applied Mathematical Modelling 1997*; 21(11): 681–689.

[2] Bear, J. and Pinder, G. F. Porous medium deformation in multiphase flow. J. Eng. Mech. Dio, Proc. A.S.C.E. 1978; 104 (EM4): 881-894.

[3] Biot, M. A. General theory of three dimensional consolidations. J Appl. Phys. 1941a; Vol. 12: 155-164.

[4] Biot, M. A. Consolidation settlement under a rectangular load distribution. J. Appl. Phys. 1941b; Vol. 12: 426–430.

[5] Booker, J. R. and Carter, J. P. Elastic consolidation around a point sink embedded in a half-space with anisotropic permeability. Int. J. Numer. Anal. Methods Geomech. 1987a; Vol. 11(1): 61-77.

[6] Booker, J. R. and Carter, J. P. Withdrawal of a compressible pore fluid from a point sink in an isotropic elastic half-space with anisotropic permeability. Int. J. Solids Struct. 1987b; Vol. 23(3): 369-385.

[7] Booker, J. R., and Carter, J. P. Analysis of a point sink embedded in a porous elastic half-space, Int. J. Numer. Anal. Methods Geomech. 1986a; Vol. 10(2): 137-150.

[8] Booker, J. R., and Carter, J. P. Long term subsidence due to fluid extraction from a saturated, anisotropic elastic soil mass. Q. J. Mech. Appl. Math. 1986b; Vol. 39(1): 85-97.

[9] Chau, K. T. Fluid point source and point forces in linear elastic diffusive half-spaces. *Mech. Mater.* 1996;, Vol. 23: 241–253.

[10] Chen, G. J. Steady-state solutions of multilayered and cross-anisotropic poroelastic half-space due to a point sink. Int. J. Geomech. 2005; Vol. 5: 45-57.

[11] Cleary, M. P. Fundamental solutions for a fluid-saturated porous solid. Int. J. Solids Struct. 1977; Vol. 13: 785-806.

[12] Delfranche, A. P. Land subsidence versus head decline in Texas. Evaluation and Prediction of

subsidence. 1978; **S.** K. Saxena ed., A.S.C.E., New York: 320-331.

[13] Detournay, E. and Cheng, A. H. D. Fundamentals of poroelasticity in Comprehensive Rock Engineering: Principles. 1993; Practice and Projects, Pergamon Press, Oxford.

[14] Geertsma, J. A basic theory of subsidence due to reservoir compaction: the homogeneous case, from "The analysis of surface subsidence resulting from gas production in the Groningen area, the Netherlands". 1973; Ed. Ned. Aard. Maatschappij B.V: 43-61.

[15] Harada, Y. and Yamonouchi, T. Land subsidence in saga plain, Japan and its analysis by the quasi three-dimensional aquifer model. Geotech. Eng. 1983; Vol. 14: 23-54.

[16] Kanok-Nukulchai, W. and Chau, K. T. Point sink fundamental solutions for subsidence prediction. *Journal of Engineering Mechanics*. 1990; 116(5): 1176–1182.

[17] Lofgren, B. E. Changes in aquifer-system properties with ground Water depletion. Evaluation and Prediction of Subsidence. 1978; ASCE New York: 26-46.

[18] Poland, J. F. and Davis, G. H. Land subsidence due to withdrawal of fluids, from "Reviews in Engineering Geology 2" Geological Society of America. 1969: 187-269.

[19] Poland, J.F. Subsidence and its control in underground waste management and environmental implications, T.D. Cook éd., Memoir no. 18. American Association of Petroleum Geologists. 1972: 50-71.

[20] Premchitt, J. Land subsidence in Bangkok, Thailand results of initial investigation. Geotech. Eng. 1978; Vol. 10(1): 49-76

[21] Rice, J. R. and Clearly, M. P. Some basic stress diffusion solutions for fluid-saturated elastic porous media with compressible constituents. *Rev. Geophys. Space Phys.* 1976; Vol. 14: 227–241.

[22] Rudnicki, J. W. Fluid mass sources and point forces in linear elastic diffusive solids. *Mech. Mater.* 1986; Vol. 5: 383–393.

[23] Scott, R. F. Subsidence-a review; Evaluation and Prediction of Subsidence. Saxena S. K. ed. 1978; ASCE New York: 1-25.

[24] Selvadurai A.P.S. (ed.). Mechanics of poroelastic media. Dordrecht, The Netherlands; Kluwer Academic; 1996.

[25] Wang, H. F. Theory of Linear Poroelasticity with applications to Geomechanics and Hydrogeology. Princeton University press, Princeton; 2000.