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# Displacement and Stress Analysis in Shear Deformable Thick Plate

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## Abstract

This paper presents exact solution of displacement fields of thick rectangular plate which was obtained using traditional third order refined theory for the bending of rectangular plates with different support cases under general boundary conditions. The aim of study is static bending analysis of an isotropic rectangular thick plate using analytical method. Total potential energy equation of a thick plate was formulated from the first principle. This equation was subjected to direct variation to obtain three simultaneous direct governing equations for determination of displacement coefficients. The main assumption here is that the vertical line that is initially normal to the middle surface of the plate before bending is no longer straight nor normal to the middle surface after bending, as a result the shear deformation profile F (z) is used in the place of z. The shear deformation profile equation for vertical shear stress through the thickness of the plate was formulated mathematically in line with Timoshenko work. From this profile equation, the deformation line equation (called function of z or s) was obtained and compared to other four model. These models in comparison includes one polynomial and three trigonometric model. A numerical problem for a rectangular plate simply supported around all the edges was used to test the sufficiency of this study. It was observed that the values of non-dimensional forms of displacements and stresses from the present study agree with the values from previous studies. Also observed is that the values of the in-plane quantities did not vary with span-depth ratio ($\rho$) 6 and above. They are all equal to the values from classical plate theory (CPT) for the values of ($\rho$) 6 and above. However, the out-of-plane quantities varied with span-depth ratio from ($\rho$) equal to 4 up to ($\rho$) equal to 30, after which they become constant and approximately equal to values from CPT. This shows that the present method is reliable and sufficient for thick plate analysis.

**Keywords**: Exact solution, shear deformation, displacement, stress, deflection, total potential energy.

## INTRODUCTION

The use of thick plate materials in engineering is increasing due to their attractive properties such as high strength-to-weight ratio, economy, its ability to withstand heavy loads and ability to tailor the structural properties, etc. Plate structures find numerous applications in the aerospace or aeronautical Engineering, military, structural and mechanical Engineering or automotive industries. In Structural Engineering, plates are widely used in roof and floor slabs, bridge deck slabs, foundation footings, bulkheads, pressure and watertanks, turbine disks, spacecraft panels and ship hulls. Zenkour (2003) in his works on "Exact Mixed-Classical Solution for the Bending Analysis of Shear deformable Rectangular Plates" discovered that thin plate model does not provide a very good analysis of plates in which the thickness-to length ratio is relatively large. The method is very difficult but accurate. This makes analysis of thick plate very imperative.

Refined shear deformation theories based on the power series expansion for displacements with respect to the thickness coordinate, and truncating the series at required order of thickness coordinate are called the higher-order shear deformation theories. This type of series expansion was initially proposed by Basset (1890). Refined plate theories have been characterized by the use of trigonometric displacement function. The refined plate theories; first, second and higher order shear deformation theory – HSDT can be obtained through the analogue means to solve the couples governing differential equations, consequently deduce deformation. Reddy's third-order, and Reissner's higher-order shear deformation plate theory which have two more unknowns' variables in comparison with the classical plate theory. To be applied in this work is the higher order shear deformation theory – HSDT (third order shear deformation theory) using polynomial displacement function.

Many scholars have obtained the closed form solutions and others have obtained approximate solution by use of energy method. However, one thing is common in them all - the use of trigonometric displacement functions to approximate the deformed shapes of the plates. (Chikalthankar et al., 2013; Sayyad, 2011; Akavci, 2007; Sayyad and Ghugal, 2012; Sadrnejad et al., 2009; Daouadji et al., 2013; Hashemi and Arsanjani, 2005; Reddy, 2014; Shimpi and Patel, 2006; Murthy, 1981; Daouadji, Tounsi, Hadji, Henni and El Abbes, 2012; Zhen-qiang, Xiu:xi and Mao-guang, 1994). Others have applied the polynomial displacement functions in numerical methods like finite element method and differential quadrature element methods (Matikainen, Schwab and Mikkola, 2009; Goswami and Becker, 2013, Liu, 2001). In the course of development of refined plate theory, the assumption that the shear deformation line is not varying linear with depth of the plate was introduced. This according to many scholars helps to ensure that the vertical shear stress across the plate section does not remain constant, but varies parabolically with zero values at both the top and bottom surfaces (Kruszewski, 1949; Ambartsumian, 1958 Krishna, 1984; Touratier, 1991; Karama and Mistou, 2003; Sayyad, 2011; Ibearugbulem, 2016). They came up with different shear deformation line functions, here-in-after called F(z). However, there F(z) were not strictly based on the vertical shear stress mathematical formulation. If we follow the work of Timoshenko (Timoshenko and Woinowsky-krieger, 1970), we shall note that maximum shear stress occurs at the mid surface (where z = 0) and the value of maximum shear stress is one and half of vertical shear stress. With most of the F(z) from the literature, we may obtain good profile (curve) for the deformation line and shear stress distribution across the section, but the mid surface value of shear stress may not coincide with that from Timoshenko.

Exact analytical method include simplistic geometry and Integral transform method (Zhong and Li, 2009; Li et al, 2014) and Numerical methods which seek approximate solutions to the plate problem include: variational methods, finite element and finite difference methods and finite strip methods (Ezeh, et al, 2013; Aginam et al, 2012; Aliabadi and Wen, 2011). In this work, stresses, deflection and shear deformation equation will determined using integral direct integration method of exact analytical solution approach.

## DISPLACEMENT FIELD

Figure **3.1** shows a bent elastic plate under lateral loading. Our intention is to obtain the displacement – strain relationships in terms of curvatures. From the assumptions, we have three displacement of thick place which includes the deflection, w(x,y) and the two inplane displacements, u(x,y,z), and v(x,y,z). Ibearugbulem (2015) in his lecture note defined deflection, w.

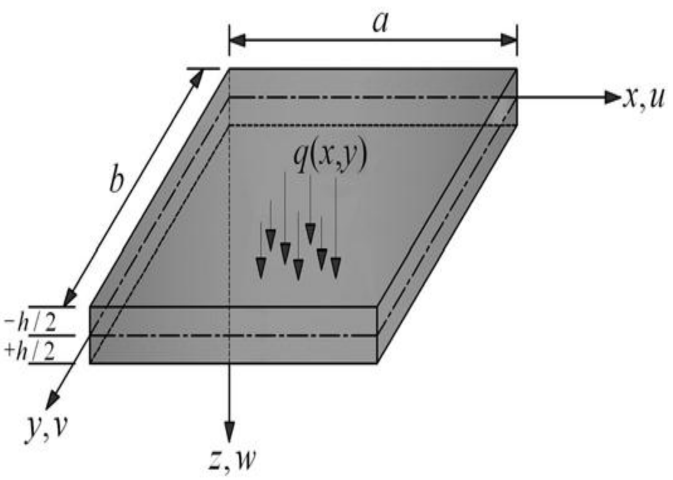
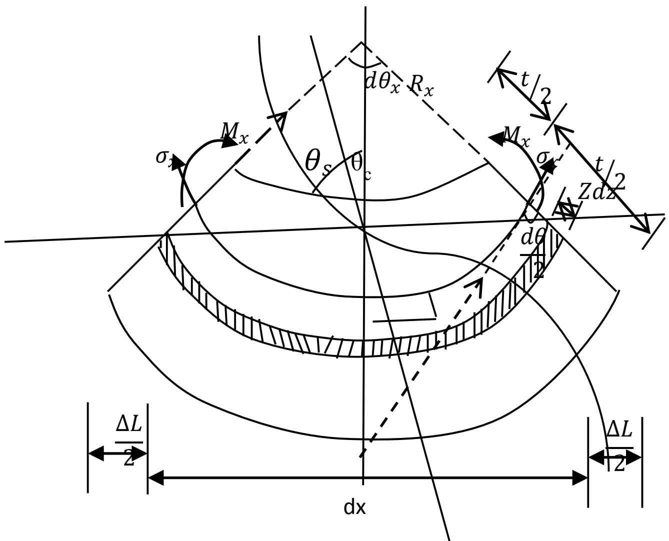



**Figure 1:** Deformation of a section of a thick plate




$Where\ \theta = \theta_c + \theta_s \quad and\ t = h$

**Figure 2:** Geometry of Thick plate

## ENGINEERING STRAIN – DISPLACEMENT RELATIONS

The refined plate theory (RPT) in-plane displacements, u and v as presented on figure1 are defined mathematically as:

$$u = u_c + u_s \quad 1$$

$$v = v_c + v_s \quad 2$$

Where u and v are in-plane displacements in x and y directions respectively. The classical in-plane displacements are commonly defined as:

$$u = \frac{zdw}{dy} + F.\theta_{sX} \quad 3$$

Where $F = F(z)$

Similarly,

$$v = \frac{zdw}{dy} + F.\theta_{sy} \quad 4$$

And

$$w = A_1 h \quad 5$$

From assumptions herein, the strain normal to z axis is zero. This left us with only five engineering strain components $\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{xz}$ and $\gamma_{yz}$.

$$\varepsilon_x = \frac{du}{dy} \equiv \left(-\frac{zd^2w}{dx^2} + \frac{Fd\theta_{Sx}}{dx}\right) \quad 6$$

Similarly reasoning in y direction, we shall obtain:

$$\varepsilon_y = \frac{dv}{dy} \equiv \left(-\frac{zd^2w}{dy^2} + \frac{Fd\theta_{Sy}}{dy}\right) \quad 7$$

$$\gamma_{xy} = 2\frac{z\partial^2 w}{\partial x \partial y} + F\left(\frac{d\theta_{Sx}}{dy} + \frac{Fd\theta_{Sy}}{dx}\right) \quad 8$$

$$\gamma_{xz} = \frac{dF}{dz}.\theta_{Sx} \qquad 9$$

$$\gamma_{yz} = \frac{dF}{dz}.\theta_{Sy} \qquad 10$$

## CONSTITUTIVE RELATIONS

The constitutive equations for five stress components are:

$$\sigma_x = \frac{Ez}{1-\mu^2}\left(\left(\frac{-d^2w}{dx^2} + \frac{Fd\theta_{Sx}}{dx}\right) + \mu\left(\frac{d^2w}{dy^2} + \frac{Fd\theta_{Sx}}{dy}\right)\right) \qquad 11$$

$$\sigma_y = \frac{Ez}{1-\mu^2}\left(\left(-\frac{d^2w}{dy^2} + \frac{Fd\theta_{Sx}}{dx}\right) + \mu\left(\frac{d^2w}{dx^2} + \frac{Fd\theta_{Sx}}{dy}\right)\right) \qquad 12$$

Also, from known state Equation,

$$\tau_{xy} = \frac{E(1-\mu)}{(1-\mu^2)}\left(-\frac{z\partial^2 w}{\partial x\partial y} + F\left(\frac{d\theta_{Sx}}{dy} + \frac{d\theta_{Sy}}{dx}\right)\right) \qquad 13$$

Similarly,

$$\tau_{xz} = \frac{E(1-\mu)}{(1-\mu^2)}\left(\frac{z\partial^2 w}{\partial x\partial z} + F\left(\frac{d\theta_{Sx}}{dz} + \frac{d\theta_{Sz}}{dx}\right)\right) \qquad 14$$

Also,

$$\tau_{yz} = \frac{E(1-\mu)}{(1-\mu^2)}\left(\frac{z\partial^2 w}{\partial y\partial z} + F\left(\frac{d\theta_{Sy}}{dz} + \frac{d\theta_{Sz}}{dy}\right)\right) \qquad 15$$

$$\therefore\ \sigma_x = \frac{E(\varepsilon_x + \mu\varepsilon_y)}{1-\mu^2} \qquad 16$$

Similarly reasoning in y direction, we obtain:

$$\sigma_y = \frac{E(\varepsilon_y + \mu\varepsilon_x)}{1-\mu^2} \qquad 17$$

Similarly reasoning in z direction, we obtain:

$$\sigma_z = \frac{E(\varepsilon_z + \mu\varepsilon_x)}{1-\mu^2} \qquad 18$$

Rearranging equation 13, 14 & 15 gives;

$$\tau_{xy} = G\gamma_{xy} \equiv \frac{E}{2(1+\mu)}.\gamma_{xy} \qquad 18$$

$$\tau_{xz} = G\gamma_{xz} \equiv \frac{E}{2(1+\mu)}.\gamma_{xz} \qquad 19$$

$$\tau_{yz} = G\gamma_{yz} \equiv \frac{E}{2(1+\mu)}.\gamma_{yz} \qquad 20$$

## TOTAL POTENTIAL ENERGY

Total potential energy is the summation of strain energy, U and external work, V. that's

$$\Pi = U + V \qquad 21$$

Let's define external work as:

$$V = -q\ wyxdxdy \qquad 22$$

Therefore, the total potential energy equation for a thick plate of traditional third order shear deformation theory is given as:

$$\Pi = \frac{D}{2}\int_0^a\int_0^b \left\|g_1\left(\frac{\partial^2 w}{\partial x^2}\right)^2 - 2g_2\left(\frac{\partial^2 w}{\partial x^2}.\frac{\partial\theta_{Sx}}{\partial x}\right) + g_3\left(\frac{\partial\theta_{Sx}}{\partial x}\right)^2\right| + \left|2g_1\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2 - 2g_2\left(\frac{\partial^2 w}{\partial x\partial y}.\frac{\partial\theta_{Sx}}{\partial y}\right) - 2g_2\left(\frac{\partial^2 w}{\partial x\partial y}.\frac{\partial\theta_{Sy}}{\partial x}\right)\right| + \left|(1+\mu)g_3\left(\frac{\partial\theta_{Sx}}{\partial y}\right)\left(\frac{\partial\theta_{Sy}}{\partial x}\right)\right| + \frac{(1-\mu)}{2}\left|g_3\left(\frac{\partial\theta_{Sx}}{\partial y}\right)^2 + g_3\left(\frac{\partial\theta_{Sy}}{\partial x}\right)^2\right| + \left|g_1\left(\frac{\partial^2 w}{\partial y^2}\right)^2 - 2g_2\left(\frac{\partial^2 w}{\partial y^2}.\frac{\partial\theta_{Sy}}{\partial y}\right) + g_3\left(\frac{\partial\theta_{Sy}}{\partial y}\right)^2\right| + \left|\frac{(1-\mu)}{2}g_4(\theta_{Sx})^2 + \frac{(1-\mu)}{2}g_4(\theta_{Sy})^2\right\|\partial x\partial y - \int_0^a\int_0^b qw(x,y)\partial x\partial y \qquad 23$$

Let's define the span-depth aspect ratio as:

$$\rho = \frac{a}{t} \qquad 24$$

Where a and t are the primary span (length in x direction, while b is the length in y direction) of the plate and plate thickness respectively. Let define non-dimensional coordinates R and Q and the span-span aspect ratio, $\propto$ as:

Expressing equation 23 in the form of non-dimensional parameters, say R and Q for x and y directions respectively:

Where:

$$R = \frac{x}{a} \equiv x = aR \qquad 25$$

$$Q = \frac{y}{b} \equiv y = bQ \qquad 26$$

$$\propto = \frac{b}{a}, \quad b = \propto a \quad and, \quad a = \frac{b}{\propto} \qquad 27$$

$$w = A_1 h \qquad 28$$

Substituting equation 25, 26, 27, and 28 into 3.108, we shall obtain:

$$\Pi = \frac{D}{2}\int_0^a\int_0^b \Bigg[\Bigg| g_1{A_1}^2\left(\frac{\partial^2 h}{\partial x^2}\right)^2 - 2g_2A_1A_2\left(\frac{\partial^2 h}{\partial x^2}.\frac{\partial^2 h}{\partial x^2}\right) + g_3{A_2}^2\left(\frac{\partial^2 h}{\partial x^2}\right)^2\Bigg| + \Bigg|2g_1\,{A_1}^2\left(\frac{\partial^2 h}{\partial x\partial y}\right)^2 - 2g_2A_1A_2\left(\frac{\partial^2 h}{\partial x\partial y}.\frac{\partial^2 h}{\partial x\partial y}\right) - 2g_2A_1A_3\left(\frac{\partial^2 h}{\partial x\partial y}.\frac{\partial^2 h}{\partial x\partial y}\right)\Bigg| + \Bigg|(1+\mu)g_3A_2A_3\left(\frac{\partial^2 h}{\partial x\partial y}\right)\left(\frac{\partial^2 h}{\partial x\partial y}\right)\Bigg| + \frac{(1-\mu)}{2}\Bigg|g_3{A_2}^2\left(\frac{\partial^2 h}{\partial x\partial y}\right)^2 + g_3{A_3}^2\left(\frac{\partial^2 h}{\partial x\partial y}\right)^2\Bigg| + \Bigg|g_1{A_1}^2\left(\frac{\partial^2 h}{\partial y^2}\right)^2 - 2g_2A_1A_3\left(\frac{\partial^2 h}{\partial y^2}.\frac{\partial^2 h}{\partial y^2}\right) + g_3{A_3}^2\left(\frac{\partial^2 h}{\partial y^2}\right)^2\Bigg| + \Bigg|\frac{(1-\mu)}{2}g_4{A_2}^2\left(\frac{\partial h}{\partial x}\right)^2 + \frac{(1-\mu)}{2}g_4{A_3}^2\left(\frac{\partial h}{\partial y}\right)^2\Bigg|\Bigg]\partial x\partial y - \int_0^a\int_0^b qA_1h\,\partial x\partial y \qquad 29$$

## General Governing Equations

The general variation on the total potential energy will be performed in order to obtain the solution of the resulting three simultaneous governing equations.

Here, to obtain the non-dimensional equations of equilibrium of forces, equation 29 must be differentiated with respect to w. That is;

$$\frac{d\Pi}{dw} = 0 \qquad 30$$

$$\int_0^1\int_0^1\left[g_1\left(\frac{d^4w}{dR^4} + \frac{2}{\propto^2}\frac{d^4w}{dR^2dQ^2} + \frac{1}{\propto^4}\frac{d^4w}{dQ^4}\right) - ag_2\frac{d^3\theta_{Sx}}{dR^3} - \frac{\mathrm{a}g_2}{\propto^2}\frac{d^3\theta_{Sx}}{dRdQ^2} - \frac{\mathrm{a}g_2}{\propto}\frac{d^3\theta_{Sy}}{dR^2dQ} - \frac{\mathrm{a}g_2}{\propto^3}\frac{d^3\theta_{Sy}}{dQ^3}\right]dRdQ = \frac{qa^4}{D}\int_0^1\int_0^1 1.dRdQ \qquad 31$$

Obtaining values of deflection function $w$ from equations 31 we have:

$$w = \left(a_0 + a_1R + \frac{a_2R^2}{2} + \frac{a_3R^3}{6} + \frac{qa^4}{D}\left(\frac{n_1}{w_3}\right).\frac{R^4}{24}\right)\times\left(b_0 + b_1Q + \frac{b_2Q^2}{2} + \frac{b_3Q^3}{6} + \frac{qa^4}{D}\left(\frac{n_1}{w_3}\right).\frac{Q^4}{24}\right) \qquad 32$$

**DIRECT VARIATION OF TOTAL POTENTIAL ENERGY**

This total potential energy contains three unknown coefficients (A1, A2 and A3) for deflection, rotation in x axis and rotation in y axis. In minimization, when the differentiation is done with respect to the coefficient of the displacement, the result is called the direct governing equation. Here, the total potential energy shall be minimized with respect to the coefficient of the deflection, shear deformation along x axis and shear deformation along y axis; $A_1$, $A_2$, and $A_3$.

Minimizing or differentiating total potential energy equation with respect to $A_1$, $A_2$, and $A_3$ is said to be the direct variation.

Therefore, by differentiating equation 29 with respect to $A_1$, $A_2$, and $A_3$ is said to be the direct variation we have;

$$\frac{\partial \Pi}{\partial A_1} = \frac{\partial \Pi}{\partial A_2} = \frac{\partial \Pi}{\partial A_3} = 0 \qquad 33$$

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \frac{qa^4}{D} \begin{bmatrix} k_6 \\ 0 \\ 0 \end{bmatrix} \qquad 34$$

Where;

$$r_{11} = g_1\left(k_1 + \frac{2}{\propto^2}k_2 + \frac{1}{\propto^4}k_3\right) \qquad 35$$

$$r_{12} = -g_2\left(k_1 + \frac{1}{\propto^2}k_2\right) \qquad 36$$

$$r_{13} = -g_2\left(\frac{1}{\propto^2}k_2 + \frac{1}{\propto^4}k_3\right) \qquad 37$$

$$r_{21} = -g_2 \qquad 38$$

$$r_{22} = \left(g_3 k_1 + \frac{(1-\mu)}{2\propto^2}g_3 k_2 + \frac{(1-\mu)}{2}\rho^2 g_4 k_4\right) \qquad 39$$

$$r_{23} = g_3\frac{(1+\mu)}{2\propto^2}k_2 \qquad 40$$

$$r_{31} = -g_2\left(\frac{1}{\propto^2}k_2 + \frac{1}{\propto^4}k_3\right) \qquad 41$$

$$r_{32} = g_3\frac{(1+\mu)}{2\propto^2}k_2 \qquad 42$$

$$r_{33} = \left(g_3\frac{(1-\mu)}{2}\left(\frac{1}{\propto^2}k_2 + \frac{1}{\propto^4}k_3\right) + g_4\frac{(1-\mu)}{2\propto^2}\rho^2 k_5\right) \qquad 43$$

And;

$$k_1 = \int_0^1\int_0^1\left(\frac{d^2h}{dR^2}\right)^2 dRdQ \qquad 44$$

$$k_2 = \int_0^1\int_0^1\left(\frac{d^2h}{dRdQ}\right)^2 dRdQ \qquad 45$$

$$k_3 = \int_0^1\int_0^1\left(\frac{d^2h}{dQ^2}\right)^2 dRdQ \qquad 46$$

$$k_4 = \int_0^1\int_0^1\left(\frac{dh}{dR}\right)^2 dRdQ \qquad 47$$

$$k_5 = \int_0^1\int_0^1\left(\frac{dh}{dQ}\right)^2 dRdQ \qquad 48$$

$$k_6 = \int_0^1\int_0^1 h.\,dRdQ \qquad 49$$

By solving equation 33, we have;

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \frac{qa^4}{D} \begin{bmatrix} k_6 \\ 0 \\ 0 \end{bmatrix} \qquad 50$$

That is:

$$A_2 = \left(\frac{r_{21}.r_{33} - r_{23}.r_{31}}{r_{22}.r_{33} - r_{23}.r_{32}}\right)A_1 \qquad 51$$

$$A_3 = \left(\frac{r_{21}.r_{32} - r_{22}.r_{31}}{r_{23}.r_{32} - r_{22}.r_{33}}\right)A_1 \qquad 52$$

That is:

$$A_2 = T_2 A_1 \qquad 53$$

$$A_3 = T_3 A_1 \qquad 54$$

Where;

$$T_2 = \frac{r_{21}.r_{33} - r_{23}.r_{31}}{r_{22}.r_{33} - r_{23}.r_{32}} \qquad 55$$

$$T_3 = \frac{r_{21}.r_{32} - r_{22}.r_{31}}{r_{23}.r_{32} - r_{22}.r_{33}} \qquad 56$$

Where $T_1 = 1$

$$A_1 = \frac{qa^4}{D}\left(\frac{k_6}{r_{11}T_1 - r_{12}T_2 - r_{13}T_3}\right) \qquad 57$$

Let:

$$k = \left(\frac{k_6}{r_{11}T_1 - r_{12}T_2 - r_{13}T_3}\right) \qquad 58$$

$$A_1 = \frac{qa^4}{D}.k \qquad 59$$

But:

$$k_T = k_1 + \frac{2}{\propto^2}k_2 + \frac{1}{\propto^4}k_3 \quad 60$$

And,

$$k = \frac{k_6}{k_T} \quad 61$$

## SHEAR DEFORMATION PROFILE IN THE RECTANGULAR SECTION OF PLATE

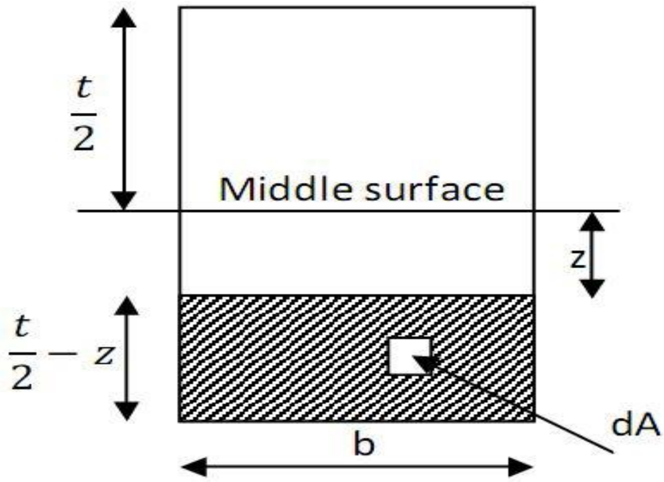



**Figure 3.4:** A rectangular cross section of plate

From strength of materials, the equation shear stress is given as:

$$\tau = \frac{VH}{Ib} \quad 62$$

That is:

$$\tau_{pxz} = \frac{V_x H}{Ib} \quad or \quad \tau_{pxz} = \frac{V_y H}{Ib} \quad 63$$

Where, V, H, I and b are transverse shear force, first moment of area, second moment of inertia and breadth of the section respectively.

Using figure 3.4 and following mathematically principle the first moment of area is obtained:

$$H = \int z dA = \frac{b}{2}\left(\frac{t}{2} - z\right)\left(\frac{t}{2} + z\right) \quad 64$$

That is:

$$H = \frac{b}{2}\left(\frac{t^2}{4} - z^2\right) \quad 65$$

The second moment of inertia for a rectangular section is given as:

$$I = \frac{bt^2}{12} \quad 66$$

Substituting equations (65) and (66) into equation (63) gives:

$$\tau_{pxz} = \frac{V_x H}{Ib}\left(\frac{3}{2} - 6\frac{z^2}{t^2}\right) = \frac{V_x H}{bt}G(z) \quad 67$$

or

$$\tau_{pxy} = \frac{V_x H}{Ib}\left(\frac{3}{2} - 6\frac{z^2}{t^2}\right) = \frac{V_y H}{bt}G(z) \quad 68$$

Where;

$$\tau_{xy} = \frac{V_x H}{Ib} \quad 69$$

And;

$$\tau_{xy} = \frac{V_y H}{Ib} \quad 70$$

Where the shear stress profile $\tau_{pxy} = \mathrm{G(z)}$ is:

$$G(z) = \left(\frac{3}{2} - 6\frac{z^2}{t^2}\right) \quad 71$$

It is assumed here that the shear stress profile, G(z) is related to shear deformation profile, F(z) as:

$$G(z) = \frac{d\mathrm{F(z)}}{dz} \quad 72$$

Using equations (67) or (68) and (71) we obtain:

$$F(z) = \frac{3z}{2}\left(1 - \frac{4}{3}\frac{z^2}{t^2}\right) \equiv \frac{3}{2}\left(z - \frac{4}{3}z\left[\frac{z^2}{t^2}\right]\right) \quad 73$$

$$F(z) = \frac{3}{2}\left(z - \frac{4z^3}{3t^2}\right) \quad 74$$

Where S = z/t (a non-dimensional form of z).

This function of z is exactly Krishna Murty Model (Krishna Murty, 1984) multiplied by 1.5.

However, using the Krishna Murty model will result in underestimating the vertical shear stress by 50%.

Substituting F(z) models into equations (74) gives $g_1$, $g_2$, $g_3$ and $g_4$ values to get the numerical expression of the equations derived above and then compare with previous models in as stated in the literature.

(i) Present Model

$$F(z) = \frac{3}{2}\left(z - \frac{4}{t^2}z^3\right) \equiv 1.5\left(z - \frac{4z^3}{t^2}\right) \quad 75$$

$$g_1 = 1\,; \quad g_2 = 1.2; g_3 = 1.33;\ g_4 = 14.4$$

(ii) Ibearugbulem, model

$$F(S) = \frac{3\mathrm{St}}{2}\left(1 - \frac{4}{3}S^2\right) \equiv \mathrm{tF} \quad 76$$

$$g_1 = 1\,; \quad g_2 = \frac{12}{15}; g_3 = \frac{204}{315};\ g_4 = \frac{96}{15}$$

(iii) Touratier 1991 model

$$F(S) = \frac{\mathrm{t}}{\pi}\sin(\pi S) \quad 77$$

$g_1 = 1; \quad g_2 = 0.774; g_3 = \frac{307}{505}; \; g_4 = 6$

(iv) Karama et al. 2003 model

$$F(S) = St.\exp(-2S^2) \quad 78$$

$g_1 = 1; \quad g_2 = \frac{344}{439}; g_3 = \frac{274}{422}; \; g_4 = 6.18744$

By solving equation 33, we got:

$$k_T = k_1 + \frac{2}{\propto^2}k_2 + \frac{1}{\propto^4}k_3 \quad 79$$

And;

$$k_6 = k_{rq} + k_{rN} + k_{rl} \quad 80$$

Therefore:

$$k_{rq} = \frac{q}{A_1}\int_0^1\int_0^1 h.\,dRdQ \equiv \frac{q}{A_1}k_6 \quad 81$$

$$k_{rq} = \frac{qa^4}{D}k_6 \quad 82$$

But from the matrix above, we got:

$$A_1 = \frac{qa^4}{D}\left(\frac{k_6}{r_{11}T_1 - r_{12}T_2 - r_{13}T_3}\right) \quad 83$$

Let:

$$\overline{A_1} = \left(\frac{k_6}{r_{11}T_1 - r_{12}T_2 - r_{13}T_3}\right) \quad 84$$

That is:

$$A_1 = \overline{A_1}\left(\frac{qa^4}{D}\right) \quad 85$$

Similarly;

$$A_2 = \overline{A_1}\left(\frac{qa^4}{D}\right) \quad 86$$

Similarly;

$$A_3 = \overline{A_3}\left(\frac{qa^4}{D}\right) \quad 87$$

**Definition of Some Quantities**

Expressing equation 3 to 20 in the form of non-dimensional form, therefore the displacements of plate under pure bending then be defined as:

$$w = \overline{A_1}h\left(\frac{qa^4}{D}\right) \quad 88$$

Therefore:

$$u = [-\overline{A_1}s + \overline{A_2}F(s)]\frac{dh}{dR}\left(\frac{tqa^3}{D}\right) \quad 89$$

That is:

$$u = [-\overline{A_1}s + \overline{A_2}F(s)]\frac{dh}{dR}\left(\frac{qa^4}{\rho D}\right) \quad 90$$

Similarly;

That is:

$$v = \frac{1}{\propto}[-\overline{A_1}s + \overline{A_3}F(s)]\frac{dh}{dQ}\left(\frac{tqa^3}{D}\right) \quad 91$$

That is:

$$v = \frac{1}{\propto}[-\overline{A_1}s + \overline{A_3}F(s)]\frac{dh}{dQ}\left(\frac{qa^4}{\rho D}\right) \quad 92$$

Where Cw, Cu, Cv and D, are as defined earlier, q is the uniform distributed load on the plate. Cw, Cu and Cv are extracted from equations (88), (90) and (92) as:

$$c_w = \overline{A_1}h \quad 93$$

$$c_u = [-\overline{A_1}s + \overline{A_2}F(s)]\frac{dh}{dR} \quad 94$$

Similarly;

$$c_v = \frac{1}{\propto}[-\overline{A_1}s + \overline{A_3}F(s)]\frac{dh}{dQ} \quad 95$$

Similarly, let us define the stress components as;

$$\sigma_x = \frac{E}{1-\mu^2}\left(\frac{tqa^2}{D}\right)\left[[-\overline{A_1}s + \overline{A_2}F(s)]\frac{d^2h}{dR^2} + \frac{\mu}{\propto^2}[-\overline{A_1}s + \overline{A_3}F(s)]\frac{d^2h}{dQ^2}\right] 96$$

That is:

$$\sigma_x = 12\left[[-\overline{A_1}s + \overline{A_2}F(s)]\frac{d^2h}{dR^2} + \frac{\mu}{\propto^2}[-\overline{A_1}s + \overline{A_3}F(s)]\frac{d^2h}{dQ^2}\right](q\rho^2) 97$$

Similarly;

$$\sigma_y = \frac{E}{1-\mu^2}\left(\frac{tqa^2}{D}\right)\left[\mu[-\overline{A_1}s + \overline{A_2}F(s)]\frac{d^2h}{dR^2} + \frac{1}{\propto^2}[-\overline{A_1}s + \overline{A_3}F(s)]\frac{d^2h}{dQ^2}\right] 98$$

That is:

$$\sigma_y = q\rho^2\left[12\left[\mu[-\overline{A_1}s + \overline{A_2}F(s)]\frac{d^2h}{dR^2}\right] + \frac{\mu}{\propto^2}[-\overline{A_1}s + \overline{A_3}F(s)]\frac{d^2h}{dQ^2}\right] 99$$

Similarly;

$$\tau_{xy} = \frac{E}{(1+\mu)}\left(\frac{tqa^2}{D}\right)\left[-2\overline{A_1}s + \overline{A_2}F(s) + \overline{A_3}F(s).\frac{1}{\propto}\right]\frac{d^2h}{\partial R\partial Q} 100$$

That is:

$$\tau_{xy} = 6\frac{(1-\mu)}{\propto}\left[-2\overline{A_1}s + \overline{A_2}F(s) + \overline{A_3}F(s).\frac{1}{\propto}\right]\frac{d^2h}{\partial R\partial Q}(q\rho^2) 101$$

Similarly;

$$\tau_{xz} = \frac{E}{2(1+\mu)}\left(\frac{qa^3}{D}\right)\overline{A_2}\frac{dF(z)}{dz}\frac{dh}{dR} \quad 102$$

That is:

$$\tau_{xz} = 6(1-\mu)\overline{A_2}\frac{dF(z)}{dz}\frac{dh}{dR}(q\rho^2) \quad 103$$

Similarly;

$$\tau_{yz} = \frac{(1-\mu)}{2(1-\mu^2)}\overline{A_3}\frac{dF(z)}{dz}\frac{dh}{dQ}\left(\frac{Eqa^3}{D}\right) \quad 104$$

That is:

$$\tau_{yz} = \frac{E}{2(1+\mu)}\left(\frac{qa^3}{D}\right)\overline{A_3}\frac{dF(z)}{dz}\frac{dh}{dQ} \quad 105$$

That is:

$$\tau_{yz} = \frac{6(1-\mu)}{\propto}\overline{A_3}\frac{dF(z)}{dz}\frac{dh}{dQ}(\mathrm{q}\rho^2) \quad 106$$

Where, $c_{\sigma_x}, c_{\sigma_y}, c_{\tau_{xy}}, c_{\tau_{xz}}, c_{\tau_{yz}}$ defined from equations (97) to (106)as:

$$c_{\sigma_x} = \frac{E}{1-\mu^2}\left[[-\overline{A_1}s + \overline{A_2}F(s)]\frac{d^2h}{dR^2} + \frac{\mu}{\propto^2}[-\overline{A_1}s + \overline{A_3}F(s)]\frac{d^2h}{dQ^2}\right] \quad 107$$

Similarly;

$$c_{\sigma_y} = \frac{E}{1-\mu^2}\left[\mu[-\overline{A_1}s + \overline{A_2}F(s)]\frac{d^2h}{dR^2} + \frac{1}{\propto^2}[-\overline{A_1}s + \overline{A_3}F(s)]\frac{d^2h}{dQ^2}\right] \quad 108$$

Similarly;

$$c_{\tau_{xy}} = \frac{E}{(1+\mu)}\left[-2\overline{A_1}s + \overline{A_2}F(s) + \overline{A_3}F(s).\frac{1}{\propto}\right]\frac{d^2h}{\partial R\partial Q} \quad 109$$

Similarly;

$$c_{\tau_{xz}} = \frac{E}{2(1+\mu)}\left(\frac{qa^3}{D}\right)\overline{A_2}\frac{dF(z)}{dz}\frac{dh}{dR} \quad 110$$

Similarly;

$$c_{\tau_{yz}} = \frac{E}{2(1+\mu)}\left(\frac{qa^3}{D}\right)\overline{A_3}\frac{dF(z)}{dz}\frac{1}{\propto}\frac{dh}{dQ} \quad 111$$

Substituting values of $D$ $and$ ρ into equations (88), (90) and (92) gives:

$$\mathrm{w} = 12\mathrm{q}\rho^3(1-\mu^2)c_w \quad 112$$

$$\mathrm{u} = 12\mathrm{q}\rho^3(1-\mu^2)c_u \quad 113$$

$$\mathrm{v} = 12\mathrm{q}\rho^3(1-\mu^2)c_v \quad 114$$

Similarly, substituting values of $D$ $and$ ρ into equations into equations (97) (99), (101), (103) and (105) gives:

$$\sigma_x = 12\mathrm{q}\rho^2 c_{\sigma_x} \quad 115$$

$$\sigma_y = 12\mathrm{q}\rho^2 c_{\sigma_y} \quad 116$$

$$\tau_{xy} = 12\mathrm{q}\rho^2(1-\mu)c_{\tau_{xy}} \quad 117$$

$$\tau_{xz} = 12\mathrm{q}\rho^3(1-\mu)c_{\tau_{xz}} \quad 118$$

$$\tau_{yz} = 12\mathrm{q}\rho^3(1-\mu)c_{\tau_{yz}} \quad 119$$

Let us define non-dimensional form of the displacements and stress components according to

Sayyad et al. (2012) as:

$$\overline{w} = \frac{100\mathrm{Ew}}{qt\rho^4} \quad 120$$

$$\bar{u} = \frac{\mathrm{uE}}{qt\rho^3} \quad 121$$

$$\bar{v} = \frac{\mathrm{uE}}{qt\rho^3} \quad 122$$

That is:

$$\overline{\sigma_x} = \frac{\sigma_x}{\mathrm{q}\rho^2} \quad 123$$

Similarly;

$$\overline{\sigma_y} = \frac{\sigma_y}{\mathrm{q}\rho^2} \quad 124$$

$$\tau_{xy} = \frac{\tau_{xy}}{\mathrm{q}\rho^2} \quad 125$$

$$\tau_{xzp} = \frac{\tau_{xzp}}{\mathrm{q}\rho} \quad 126$$

Similarly;

$$\tau_{yz} = \frac{\tau_{yzp}}{\mathrm{q}\rho} \quad 127$$

Using equations (120) to (127), we define the non-dimensional form of the displacements and stress components as:

$$\mathrm{w} = 1200(1-\mu^2)c_w \quad 128$$

$$\mathrm{u} = 12(1-\mu^2)c_u \quad 129$$

$$\mathrm{v} = 12(1-\mu^2)c_v \quad 130$$

$$\sigma_x = 12c_{\sigma_x} \quad 131$$

$$\sigma_y = 12c_{\sigma_y} \quad 132$$

$$\tau_{xy} = 12(1-\mu)c_{\tau_{xy}} \quad 133$$

$$\tau_{xz} = 12(1-\mu)c_{\tau_{xz}} \quad 134$$

$$\tau_{yz} = 12(1-\mu)c_{\tau_{yz}} \quad 135$$

## NUMERICAL PROBLEM

Determine the deflection at the center of SSSS thick plate. Determine also the in-plane stresses and the vertical shear stresses at the edges of the plate. Polynomial displacement function shall be used. The actual deflection equation derived using polynomial displacement function is given as: $w = \frac{F_{a4}.F_{b4}}{576}(R - 2R^3 + R^4) \times (Q - 2Q^3 + Q^4)$. The polynomial displacement function, h is given as: $h = (R - 2R^3 + R^4) \times (Q - 2Q^3 + Q^4)$. The k values herein are given as:

$$k_1 = 0.2361904761,\; k_2 = 0.2359183673 k_3 = 0.2361904761, k_4 = 0.0239002267,$$

$$k_5 = 0.0239002267, k_6 = 0.04.$$

## RESULTS AND DISCUSSIONS

Results obtained for displacements and stresses are compared and discussed with the corresponding results of classical plate theory (CPT), first order shear deformation theory (FSDT), higher order shear deformation theories (HSDTs) in both trigonometric and polynomial displacement model, of various authors and the exact elasticity solution of plate. For the purpose of comparison, results were specially generated according to the exact elasticity solution.

A close look at tables 1 to 6 reveals that the values of in-plane quantities, u and v, and that of out-of-plane quantities, w decrease as the span-depth ratio increases. The values of these quantities for all the span-depth ratios are close but unequal to the values from CPT. However, the inplane quantities are functions of x, y and z as it vary with the plate thickness. But the outplane displacement is a function of only x and y only, so did not vary linearly with the plate thickness As in the case of dimensional quantities (tables 1 to 3) the variation of the out-of-plane quantities decreases as the span-depth ratio increases and becomes insignificant from span-depth ratio of 30. The values of non-dimensional form of deflection for span-depth ratios of 100 and above are equal to the value from CPT. The result from the present study was compared with the values from previous studies. This is shown on tables 7, 8, 9 and 10. It is observed that values of out plane displacement using this exact approach are slightly higher, showing some level of accuracy and safety of the analysis. Table 7, 8, 9, 10 and 11 showed that at span-depth ratio above 100, the values obtained from the models used herein coincide exactly with values from CPT. This is quite expected since we assumed in CPT analyses that at span-depth ratios from 100 and above, a plate can be taking as being thin. Also the values obtained from the analyses showed that at span-depth ratios of 30 up to 100, a plate can be taking as moderately thick. Therefore, it can be concluded that from this research that thick plate is the one whose span-depth ratios value is 4 up to 30. However, a critical observation of the values reveals that the values from different models are very good approximation of one another. Hence, the model derived herein is quite sufficient for thick plate (Traditional third order refined plate theory) analyses. The values obtained from work done in using trigonometric function also coincide with the one obtained using polynomial shape functions. It shall also be deduced that polynomial shape functions are also adequate for thick plate analyses.

**Table 1:** Displacement and stresses of SSSS plate for b/a = 1.0

| $\rho = \frac{a}{t}$ | $w$<br>$\frac{qb^4}{D}$ | $u$<br>$\frac{tqb^3}{D}$ | $v$<br>$\frac{tqb^3}{D}$ | $\sigma_x$<br>$\frac{tqb^2}{D}$ | $\sigma_y$<br>$\frac{tqb^2}{D}$ | $\tau_{xy}$<br>$\frac{tqb^2}{D}$ | $\tau_{xz}$<br>$\frac{tqb^2}{D}$ |
|---|---|---|---|---|---|---|---|
| 4 | 0.005637 | -0.007686 | -0.007686 | 0.032939 | 0.032939 | -0.01892 | 0.003578 |
| 5 | 0.005087 | -0.007295 | -0.007295 | 0.031265 | 0.031265 | -0.01796 | 0.003542 |
| 6 | 0.004794 | -0.007086 | -0.007086 | 0.03037 | 0.03037 | -0.01744 | 0.003523 |
| 7 | 0.004618 | -0.006961 | -0.006961 | 0.029834 | 0.029834 | -0.01714 | 0.003512 |
| 8 | 0.004504 | -0.006881 | -0.006881 | 0.029488 | 0.029488 | -0.01694 | 0.003505 |
| 9 | 0.004427 | -0.006826 | -0.006826 | 0.029252 | 0.029252 | -0.0168 | 0.0035 |
| 10 | 0.004372 | -0.006786 | -0.006786 | 0.029084 | 0.029084 | -0.0167 | 0.003496 |
| 15 | 0.004241 | -0.006693 | -0.006693 | 0.028686 | 0.028686 | -0.01648 | 0.003488 |
| 20 | 0.004196 | -0.006661 | -0.006661 | 0.028547 | 0.028547 | -0.0164 | 0.003485 |
| 25 | 0.004175 | -0.006646 | -0.006646 | 0.028483 | 0.028483 | -0.01636 | 0.003484 |
| 30 | 0.004163 | -0.006638 | -0.006638 | 0.028448 | 0.028448 | -0.01634 | 0.003483 |
| 35 | 0.004156 | -0.006633 | -0.006633 | 0.028427 | 0.028427 | -0.01633 | 0.003482 |
| 40 | 0.004152 | -0.00663 | -0.00663 | 0.028413 | 0.028413 | -0.01632 | 0.003482 |
| 45 | 0.004149 | -0.006628 | -0.006628 | 0.028404 | 0.028404 | -0.01631 | 0.003482 |
| 50 | 0.004146 | -0.006626 | -0.006626 | 0.028397 | 0.028397 | -0.01631 | 0.003482 |
| 55 | 0.004145 | -0.006625 | -0.006625 | 0.028392 | 0.028392 | -0.01631 | 0.003482 |
| 60 | 0.004144 | -0.006624 | -0.006624 | 0.028389 | 0.028389 | -0.01631 | 0.003482 |
| 65 | 0.004143 | -0.006623 | -0.006623 | 0.028386 | 0.028386 | -0.0163 | 0.003481 |
| 70 | 0.004142 | -0.006623 | -0.006623 | 0.028383 | 0.028383 | -0.0163 | 0.003481 |
| 75 | 0.004141 | -0.006622 | -0.006622 | 0.02838 | 0.02838 | -0.0163 | 0.00306 |
| 80 | 0.004141 | -0.006622 | -0.006622 | 0.02838 | 0.02838 | -0.0163 | 0.003481 |
| 85 | 0.00414 | -0.006622 | -0.006622 | 0.028379 | 0.028379 | -0.0163 | 0.003481 |
| 90 | 0.00414 | -0.006621 | -0.006621 | 0.028378 | 0.028378 | -0.0163 | 0.003481 |

| $\rho = \frac{a}{t}$ | $w$ | $u$ | $v$ | $\sigma_x$ | $\sigma_y$ | $\tau_{xy}$ | $\tau_{xz}$ |
|---|---|---|---|---|---|---|---|
| | $\frac{qb^4}{D}$ | $\frac{tqb^3}{D}$ | $\frac{tqb^3}{D}$ | $\frac{tqb^2}{D}$ | $\frac{tqb^2}{D}$ | $\frac{tqb^2}{D}$ | $\frac{tqb^2}{D}$ |
| 95 | 0.00414 | -0.006621 | -0.006621 | 0.028377 | 0.028377 | -0.0163 | 0.003481 |
| 100 | 0.004139 | -0.006621 | -0.006621 | 0.028376 | 0.028376 | -0.0163 | 0.003481 |
| 1000 | 0.004139 | -0.006621 | -0.006621 | 0.028375 | 0.028375 | -0.0163 | 0.003481 |

**Table 2:** displacement and stresses of SSSS plate for b/a = 1.5

| $\rho = \frac{a}{t}$ | $w$ | $u$ | $v$ | $\sigma_x$ | $\sigma_y$ | $\tau_{xy}$ | $\tau_{xz}$ |
|---|---|---|---|---|---|---|---|
| | $\frac{qb^4}{D}$ | $\frac{tqb^3}{D}$ | $\frac{tqb^3}{D}$ | $\frac{tqb^2}{D}$ | $\frac{tqb^2}{D}$ | $\frac{tqb^2}{D}$ | $\frac{tqb^2}{D}$ |
| 4 | 0.009992 | -0.014154 | -0.009437 | 0.052885 | 0.034739 | -0.02323 | 0.004916 |
| 5 | 0.00924 | -0.01362 | -0.009081 | 0.05089 | 0.033428 | -0.02235 | 0.004881 |
| 6 | 0.008837 | -0.013333 | -0.008889 | 0.049817 | 0.032724 | -0.02188 | 0.004863 |
| 7 | 0.008595 | -0.013161 | -0.008774 | 0.049175 | 0.032301 | -0.0216 | 0.004852 |
| 8 | 0.008438 | -0.01305 | -0.0087 | 0.048759 | 0.032028 | -0.02142 | 0.004844 |
| 9 | 0.008332 | -0.012974 | -0.00865 | 0.048475 | 0.031842 | -0.02129 | 0.004839 |
| 10 | 0.008255 | -0.01292 | -0.008613 | 0.048272 | 0.031708 | -0.0212 | 0.004836 |
| 15 | 0.008075 | -0.012792 | -0.008528 | 0.047793 | 0.031394 | -0.02099 | 0.004827 |
| 20 | 0.008012 | -0.012747 | -0.008498 | 0.047626 | 0.031284 | -0.02092 | 0.004824 |
| 25 | 0.007983 | -0.012726 | -0.008484 | 0.047548 | 0.031233 | -0.02088 | 0.004823 |
| 30 | 0.007967 | -0.012715 | -0.008477 | 0.047506 | 0.031205 | -0.02087 | 0.004822 |
| 35 | 0.007957 | -0.012708 | -0.008472 | 0.047481 | 0.031188 | -0.02085 | 0.004822 |
| 40 | 0.007951 | -0.012704 | -0.008469 | 0.047464 | 0.031178 | -0.02085 | 0.004822 |
| 45 | 0.007947 | -0.012701 | -0.008467 | 0.047453 | 0.03117 | -0.02084 | 0.004821 |
| 50 | 0.007944 | -0.012699 | -0.008466 | 0.047445 | 0.031165 | -0.02084 | 0.004821 |
| 55 | 0.007942 | -0.012697 | -0.008465 | 0.047439 | 0.031161 | -0.02084 | 0.004821 |
| 60 | 0.00794 | -0.012696 | -0.008464 | 0.047435 | 0.031158 | -0.02083 | 0.004821 |
| 65 | 0.007939 | -0.012695 | -0.008463 | 0.047431 | 0.031156 | -0.02083 | 0.004821 |
| 70 | 0.007937 | -0.012694 | -0.008463 | 0.047428 | 0.031154 | -0.02083 | 0.004821 |
| 75 | 0.007936 | -0.012693 | -0.008462 | 0.047424 | 0.031151 | -0.02083 | 0.004237 |
| 80 | 0.007936 | -0.012693 | -0.008462 | 0.047424 | 0.031151 | -0.02083 | 0.004821 |
| 85 | 0.007935 | -0.012693 | -0.008462 | 0.047423 | 0.03115 | -0.02083 | 0.004821 |
| 90 | 0.007935 | -0.012692 | -0.008461 | 0.047421 | 0.031149 | -0.02083 | 0.004821 |
| 95 | 0.007934 | -0.012692 | -0.008461 | 0.04742 | 0.031149 | -0.02083 | 0.004821 |
| 100 | 0.007934 | -0.012692 | -0.008461 | 0.047419 | 0.031148 | -0.02083 | 0.004821 |
| 1000 | 0.007934 | -0.012691 | -0.008461 | 0.047419 | 0.031148 | -0.02083 | 0.004821 |

**Table 3:** displacement and stresses of SSSS plate for b/a = 2.0

| $\rho = \frac{a}{t}$ | $w$ | $u$ | $v$ | $\sigma_x$ | $\sigma_y$ | $\tau_{xy}$ | $\tau_{xz}$ |
|---|---|---|---|---|---|---|---|
| | $\frac{qb^4}{D}$ | $\frac{tqb^3}{D}$ | $\frac{tqb^3}{D}$ | $\frac{tqb^2}{D}$ | $\frac{tqb^2}{D}$ | $\frac{tqb^2}{D}$ | $\frac{tqb^2}{D}$ |
| 4 | 0.012963 | -0.01863 | -0.009316 | 0.066025 | 0.033781 | -0.02293 | 0.005666 |
| 5 | 0.012099 | -0.018016 | -0.009008 | 0.063848 | 0.032667 | -0.02217 | 0.005631 |
| 6 | 0.011634 | -0.017685 | -0.008843 | 0.062676 | 0.032067 | -0.02177 | 0.005613 |
| 7 | 0.011355 | -0.017487 | -0.008744 | 0.061974 | 0.031708 | -0.02152 | 0.005601 |
| 8 | 0.011175 | -0.017359 | -0.00868 | 0.061519 | 0.031475 | -0.02136 | 0.005594 |
| 9 | 0.011052 | -0.017271 | -0.008636 | 0.061208 | 0.031316 | -0.02126 | 0.005589 |
| 10 | 0.010963 | -0.017208 | -0.008604 | 0.060986 | 0.031202 | -0.02118 | 0.005586 |

| $\rho = \frac{a}{t}$ | $w$ $\frac{qb^4}{D}$ | $u$ $\frac{tqb^3}{D}$ | $v$ $\frac{tqb^3}{D}$ | $\sigma_x$ $\frac{tqb^2}{D}$ | $\sigma_y$ $\frac{tqb^2}{D}$ | $\tau_{xy}$ $\frac{tqb^2}{D}$ | $\tau_{xz}$ $\frac{tqb^2}{D}$ |
|---|---|---|---|---|---|---|---|
| 15 | 0.010755 | -0.01706 | -0.00853 | 0.060461 | 0.030933 | -0.021 | 0.005577 |
| 20 | 0.010682 | -0.017008 | -0.008504 | 0.060277 | 0.03084 | -0.02093 | 0.005574 |
| 25 | 0.010649 | -0.016985 | -0.008492 | 0.060193 | 0.030796 | -0.0209 | 0.005573 |
| 30 | 0.01063 | -0.016972 | -0.008486 | 0.060146 | 0.030773 | -0.02089 | 0.005572 |
| 35 | 0.010619 | -0.016964 | -0.008482 | 0.060119 | 0.030758 | -0.02088 | 0.005572 |
| 40 | 0.010612 | -0.016959 | -0.008479 | 0.060101 | 0.030749 | -0.02087 | 0.005572 |
| 45 | 0.010607 | -0.016955 | -0.008478 | 0.060088 | 0.030743 | -0.02087 | 0.005571 |
| 50 | 0.010604 | -0.016953 | -0.008476 | 0.06008 | 0.030738 | -0.02086 | 0.005571 |
| 55 | 0.010601 | -0.016951 | -0.008475 | 0.060073 | 0.030735 | -0.02086 | 0.005571 |
| 60 | 0.010599 | -0.016949 | -0.008475 | 0.060068 | 0.030732 | -0.02086 | 0.005571 |
| 65 | 0.010598 | -0.016948 | -0.008474 | 0.060064 | 0.03073 | -0.02086 | 0.005571 |
| 70 | 0.010596 | -0.016947 | -0.008474 | 0.060061 | 0.030729 | -0.02086 | 0.005571 |
| 75 | 0.010595 | -0.016946 | -0.008473 | 0.060057 | 0.030727 | -0.02086 | 0.004896 |
| 80 | 0.010595 | -0.016946 | -0.008473 | 0.060057 | 0.030727 | -0.02086 | 0.005571 |
| 85 | 0.010594 | -0.016946 | -0.008473 | 0.060055 | 0.030726 | -0.02086 | 0.005571 |
| 90 | 0.010593 | -0.016945 | -0.008473 | 0.060053 | 0.030725 | -0.02086 | 0.005571 |
| 95 | 0.010593 | -0.016945 | -0.008472 | 0.060052 | 0.030724 | -0.02086 | 0.005571 |
| 100 | 0.010593 | -0.016945 | -0.008472 | 0.060051 | 0.030724 | -0.02086 | 0.005571 |
| 1000 | 0.010592 | -0.016944 | -0.008472 | 0.06005 | 0.030723 | -0.02085 | 0.005571 |

**Table 4:** Non dimensional forms of displacement and stresses of SSSS plate for b/a = 1. 0

| $\rho = \frac{a}{t}$ | $\bar{w}$ | $\bar{u}$ | $\bar{v}$ | $\overline{\sigma_x}$ | $\overline{\sigma_y}$ | $\overline{\tau_{xy}}$ | $\overline{\tau_{xz}}$ |
|---|---|---|---|---|---|---|---|
| 4 | 6.155294867 | -0.08393 | -0.08393 | 0.35969 | 0.35969 | -0.206591 | 0.390666 |
| 5 | 5.555519414 | -0.07966 | -0.07966 | 0.341416 | 0.341416 | -0.196095 | 0.386809 |
| 6 | 5.234582628 | -0.07738 | -0.07738 | 0.331635 | 0.331635 | -0.190478 | 0.384747 |
| 7 | 5.042716855 | -0.07602 | -0.07602 | 0.325788 | 0.325788 | -0.187119 | 0.383513 |
| 8 | 4.918845442 | -0.07514 | -0.07514 | 0.322013 | 0.322013 | -0.184951 | 0.382717 |
| 9 | 4.834216376 | -0.07453 | -0.07453 | 0.319434 | 0.319434 | -0.18347 | 0.382173 |
| 10 | 4.773829136 | -0.07411 | -0.07411 | 0.317593 | 0.317593 | -0.182413 | 0.381785 |
| 15 | 4.631294839 | -0.07309 | -0.07309 | 0.313249 | 0.313249 | -0.179918 | 0.380869 |
| 20 | 4.58156931 | -0.07274 | -0.07274 | 0.311734 | 0.311734 | -0.179047 | 0.380549 |
| 25 | 4.558581738 | -0.07257 | -0.07257 | 0.311033 | 0.311033 | -0.178645 | 0.380402 |
| 30 | 4.546102141 | -0.07249 | -0.07249 | 0.310653 | 0.310653 | -0.178426 | 0.380321 |
| 35 | 4.538579882 | -0.07243 | -0.07243 | 0.310424 | 0.310424 | -0.178295 | 0.380273 |
| 40 | 4.533698669 | -0.0724 | -0.0724 | 0.310275 | 0.310275 | -0.178209 | 0.380242 |
| 45 | 4.530352591 | -0.07237 | -0.07237 | 0.310173 | 0.310173 | -0.178151 | 0.38022 |
| 50 | 4.527959394 | -0.07236 | -0.07236 | 0.3101 | 0.3101 | -0.178109 | 0.380205 |
| 55 | 4.526188823 | -0.07234 | -0.07234 | 0.310046 | 0.310046 | -0.178078 | 0.380193 |
| 60 | 4.524842229 | -0.07233 | -0.07233 | 0.310005 | 0.310005 | -0.178054 | 0.380185 |
| 65 | 4.523794306 | -0.07233 | -0.07233 | 0.309973 | 0.309973 | -0.178036 | 0.380178 |
| 70 | 4.522962836 | -0.07232 | -0.07232 | 0.309948 | 0.309948 | -0.178021 | 0.380173 |
| 75 | 4.521743102 | -0.07231 | -0.07231 | 0.309911 | 0.309911 | -0.178 | 0.334129 |

| $\rho = \frac{a}{t}$ | $\bar{w}$ | $\bar{u}$ | $\bar{v}$ | $\overline{\sigma_x}$ | $\overline{\sigma_y}$ | $\overline{\tau_{xy}}$ | $\overline{\tau_{xz}}$ |
|---|---|---|---|---|---|---|---|
| 80 | 4.521743102 | -0.07231 | -0.07231 | 0.309911 | 0.309911 | -0.178 | 0.380165 |
| 85 | 4.521288141 | -0.07231 | -0.07231 | 0.309897 | 0.309897 | -0.177992 | 0.380162 |
| 90 | 4.520906885 | -0.07231 | -0.07231 | 0.309885 | 0.309885 | -0.177985 | 0.380159 |
| 95 | 4.520584232 | -0.0723 | -0.0723 | 0.309875 | 0.309875 | -0.17798 | 0.380157 |
| 100 | 4.520308759 | -0.0723 | -0.0723 | 0.309867 | 0.309867 | -0.177975 | 0.380156 |
| 1000 | 4.520071696 | -0.0723 | -0.0723 | 0.30986 | 0.30986 | -0.177971 | 0.380154 |

**Table 5:** Non dimensional forms of displacement and stresses of SSSS plate for b/a = 1.5

| $\rho = \frac{a}{t}$ | $\bar{w}$ | $\bar{u}$ | $\bar{v}$ | $\overline{\sigma_x}$ | $\overline{\sigma_y}$ | $\overline{\tau_{xy}}$ | $\overline{\tau_{xz}}$ |
|---|---|---|---|---|---|---|---|
| 4 | 10.91072 | -0.15457 | -0.10305 | 0.577506 | 0.379352 | -0.253655 | 0.536864 |
| 5 | 10.09042 | -0.14873 | -0.09916 | 0.555714 | 0.365034 | -0.244082 | 0.533058 |
| 6 | 9.649686 | -0.1456 | -0.09707 | 0.544005 | 0.357341 | -0.238938 | 0.531012 |
| 7 | 9.385568 | -0.14372 | -0.09582 | 0.536989 | 0.352731 | -0.235856 | 0.529787 |
| 8 | 9.214797 | -0.14251 | -0.09501 | 0.532452 | 0.34975 | -0.233863 | 0.528994 |
| 9 | 9.098012 | -0.14168 | -0.09445 | 0.529349 | 0.347712 | -0.2325 | 0.528452 |
| 10 | 9.014623 | -0.14109 | -0.09406 | 0.527134 | 0.346256 | -0.231526 | 0.528065 |
| 15 | 8.81761 | -0.13968 | -0.09312 | 0.5219 | 0.342817 | -0.229227 | 0.527151 |
| 20 | 8.748817 | -0.1392 | -0.0928 | 0.520073 | 0.341617 | -0.228424 | 0.526832 |
| 25 | 8.717003 | -0.13897 | -0.09265 | 0.519227 | 0.341061 | -0.228053 | 0.526684 |
| 30 | 8.69973 | -0.13885 | -0.09256 | 0.518768 | 0.34076 | -0.227851 | 0.526604 |
| 35 | 8.689317 | -0.13877 | -0.09252 | 0.518492 | 0.340578 | -0.22773 | 0.526556 |
| 40 | 8.682559 | -0.13872 | -0.09248 | 0.518312 | 0.34046 | -0.227651 | 0.526524 |
| 45 | 8.677927 | -0.13869 | -0.09246 | 0.518189 | 0.340379 | -0.227597 | 0.526503 |
| 50 | 8.674613 | -0.13867 | -0.09245 | 0.518101 | 0.340321 | -0.227558 | 0.526487 |
| 55 | 8.672162 | -0.13865 | -0.09243 | 0.518036 | 0.340279 | -0.22753 | 0.526476 |
| 60 | 8.670298 | -0.13864 | -0.09243 | 0.517987 | 0.340246 | -0.227508 | 0.526467 |
| 65 | 8.668847 | -0.13863 | -0.09242 | 0.517948 | 0.340221 | -0.227491 | 0.526461 |
| 70 | 8.667696 | -0.13862 | -0.09241 | 0.517917 | 0.340201 | -0.227477 | 0.526455 |
| 75 | 8.666007 | -0.13861 | -0.0924 | 0.517873 | 0.340171 | -0.227458 | 0.462698 |
| 80 | 8.666007 | -0.13861 | -0.0924 | 0.517873 | 0.340171 | -0.227458 | 0.526447 |
| 85 | 8.665377 | -0.1386 | -0.0924 | 0.517856 | 0.34016 | -0.22745 | 0.526445 |
| 90 | 8.664849 | -0.1386 | -0.0924 | 0.517842 | 0.340151 | -0.227444 | 0.526442 |
| 95 | 8.664402 | -0.1386 | -0.0924 | 0.51783 | 0.340143 | -0.227439 | 0.52644 |
| 100 | 8.664021 | -0.13859 | -0.0924 | 0.51782 | 0.340137 | -0.227435 | 0.526438 |
| 1000 | 8.663693 | -0.13859 | -0.09239 | 0.517811 | 0.340131 | -0.227431 | 0.526437 |

**Table 7:** Non dimensional forms of displacement and stresses of SSSS plate for b/a = 1. 0 and $\frac{a}{t} = 4$

| ρ=4 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Different studies | $\bar{w}$ | $\bar{w}$ Percentage difference | $\bar{u}$ | $\overline{\sigma_x}$ | $\overline{\sigma_y}$ | $\overline{\tau_{xy}}$ | $\overline{\tau_{xz}}$ | Plate theory |
| Present study | 6.1553 | | -0.0839 | 0.3597 | 0.3597 | -0.2066 | 0.3907 | TRDT |
| Ibearugbulem et al | 6.1044 | 0.833825 | -0.0765 | 0.3279 | 0.3279 | -0.1883 | 0.3906 | TRDT |
| Karama | 6.1035 | 0.848693 | -0.0769 | 0.3452 | 0.3452 | -0.1859 | 0.3849 | TSDT |
| Krishna | 6.1483 | 0.113853 | -0.0756 | 0.3392 | 0.3392 | -0.1826 | 0.3878 | TSDT |
| Reddy | 5.6800 | 8.367958 | 0.0790 | 0.3180 | - | 0.2080 | 0.4830 | HSDT |
| Gugal $ Power | 5.8580 | 5.075111 | 0.0790 | 0.2971 | - | 0.1850 | 0.4770 | HPSDT |
| Mindlin | 5.6330 | 9.272146 | 0.0741 | 0.2870 | - | 0.1951 | 0.3331 | FSDT |
| Touratier | 6.1342 | 0.343973 | -0.0758 | 0.3402 | 0.3402 | -0.1832 | 0.3973 | TSDT |
| Kirchhoff | 4.5201 | 36.17619 | -0.0723 | 0.3099 | 0.3099 | -0.1790 | 0.0000 | CPT |
| Average | | 7.628969 | | | | | | |

**Table 8:** Non dimensional forms of displacement and stresses of SSSS plate for b/a = 1. 0 and $\frac{a}{t} = 10$

| ρ = 10 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Different studies** | $\bar{w}$ | $\bar{w}$ **Percentage difference** | $\bar{u}$ | $\overline{\sigma_x}$ | $\overline{\sigma_y}$ | $\overline{\tau_{xy}}$ | $\overline{\tau_{xz}}$ | **Plate theory** |
| **Present study** | 4.7738 | | -0.0741 | 0.3176 | 0.3176 | -0.1824 | 0.3818 | TRDT |
| **Ibearugbulem et al** | 4.7723 | 0.031431 | -0.0730 | 0.3127 | 0.3127 | -0.1796 | 0.3920 | TRDT |
| **Karama** | 4.7970 | 0.483636 | -0.0723 | 0.3243 | 0.3243 | -0.1746 | 0.3909 | TSDT |
| **Krishna** | 4.8011 | 0.56862 | -0.0720 | 0.3233 | 0.3233 | -0.1741 | 0.3892 | TSDT |
| **Reddy** | 4.6660 | 2.31033 | 0.0750 | 0.2890 | - | 0.2031 | 0.4920 | HSDT |
| **Gugal $ Power** | 4.6650 | 2.332262 | 0.0740 | 0.2890 | - | -0.1780 | 0.3920 | HPSDT |
| **Mindlin** | 4.6701 | 2.220509 | 0.0741 | 0.2870 | - | 0.1951 | 0.3331 | FSDT |
| **Touratier** | 4.7991 | 0.527182 | -0.0721 | 0.3235 | 0.3235 | -0.1742 | 0.3991 | TSDT |
| **Sayaad el al** | 4.665 | 2.332262 | - | 0.289 | 0.289 | 0.199 | 0.507 | TSDT |
| **Ghugal $ Sayaad** | 4.625 | 3.217297 | - | 0.307 | 0.307 | 0.195 | 0.504 | HSDT |
| **Krishnamurty** | 4.639 | 2.905799 | - | 0.290 | 0.290 | 0.195 | 0.490 | HSDT |
| **Kirchhoff** | 4.436 | 7.614968 | -0.0723 | 0.287 | 0.287 | 0.195 | 0.330 | CPT |
| **Average** | | 2.2313 | | | | | | |

**Table 9:** Non dimensional forms of displacement and stresses of SSSS plate for b/a = 1. 0 and $\frac{a}{t} = 100$

| ρ = 100 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Different studies** | $\bar{w}$ | $\bar{w}$ **Percentage difference** | $\bar{u}$ | $\overline{\sigma_x}$ | $\overline{\sigma_y}$ | $\overline{\tau_{xy}}$ | $\overline{\tau_{xz}}$ | **Plate theory** |
| **Present study** | 4.5203 | | -0.0723 | 0.3099 | 0.3099 | -0.1780 | 0.3802 | TRDT |
| **Ibearugbulem et al** | 4.5201 | 0.004425 | -0.0723 | 0.3098 | 0.3098 | -0.1779 | 0.3920 | TRDT |
| **Karama** | 4.5460 | 0.565332 | -0.0714 | 0.3203 | 0.3203 | -0.1725 | 0.3909 | TSDT |
| **Krishna** | 4.5460 | 0.565332 | -0.0714 | 0.3203 | 0.3203 | -0.1725 | 0.3892 | TSDT |
| **Reddy** | - | - | - | - | - | - | - | HSDT |
| **Gugal $ Power** | - | - | - | - | - | - | - | HPSDT |
| **Mindlin** | - | - | - | - | - | - | - | FSDT |
| **Touratier** | 4.5460 | 0.565332 | 0.0714 | 0.3203 | 0.3203 | -0.1725 | 0.3991 | TSDT |
| **Kirchhoff** | 4.5201 | 0.004425 | -0.0723 | 0.3099 | 0.3099 | -0.1790 | 0.0000 | CPT |
| **Average** | | 0.425105 | | | | | | |

**Table 10:** Non dimensional forms of displacement and stresses of SSSS plate for b/a = 1. 0 and $\frac{a}{t} = 1000$

| ρ = 1000 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Different studies** | $\bar{w}$ | $\bar{w}$ **Percentage difference** | $\bar{u}$ | $\overline{\sigma_x}$ | $\overline{\sigma_y}$ | $\overline{\tau_{xy}}$ | $\overline{\tau_{xz}}$ | **Plate theory** |
| **Present study** | 4.5201 | | -0.0723 | 0.3099 | 0.3099 | -0.1780 | 0.3802 | TRDT |
| **Ibearugbulem et al** | 4.5175 | 0.057554 | -0.0723 | 0.3098 | 0.3098 | -0.1779 | 0.3923 | TRDT |
| **Karama** | 4.5435 | 0.515021 | -0.0714 | 0.3203 | 0.3203 | -0.1725 | 0.3921 | TSDT |
| **Krishna** | 4.5435 | 0.515021 | -0.0714 | 0.3203 | 0.3203 | -0.1725 | 0.3895 | TSDT |
| **Reddy** | - | - | - | - | - | - | - | HSDT |
| **Gugal $ Power** | - | - | - | - | - | - | - | HPSDT |
| **Mindlin** | - | - | - | - | - | - | - | FSDT |
| **Touratier** | 4.5435 | 0.515021 | -0.0714 | 0.3203 | 0.3203 | -0.1725 | 0.3994 | TSDT |
| **Kirchhoff** | 4.5201 | 0 | -0.0723 | 0.3099 | 0.3099 | -0.1790 | 0.0000 | CPT |
| **Average** | | 0.400655 | | | | | | |

**Table 11:** Non dimensional forms of displacement and stresses of SSSS plate for b/a = 2. 0 and $\frac{a}{t} = 1.0$

| ρ=1. 0 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Different studies** | $\bar{w}$ | $\bar{w}$ **Percentage difference** | $\bar{u}$ | $\overline{\sigma_x}$ | $\overline{\sigma_y}$ | $\overline{\tau_{xy}}$ | $\overline{\tau_{xz}}$ | **Plate theory** |
| **Present study** | **11.972** | | **-0.1879** | **0.666** | **0.341** | **-0.231** | **0.610** | **TRDT** |
| **Sayaad el al** | **11.415** | **4.879544** | **-** | **0.613** | **0.279** | **0.325** | **0.705** | **TSDT** |
| **Ghugal $ Sayaad** | **11.340** | **5.573192** | **-** | **0.638** | **0.245** | **0.277** | **0.701** | **HSDT** |
| **Krishnamurty** | **11.310** | **5.853227** | **-** | **0.613** | **0.310** | **0.278** | **0.682** | **HSDT** |
| **Reddy** | **11.420** | **4.833625** | **0.0790** | **0.612** | **0.278** | **0.280** | **0.679** | **HSDT** |
| **Mindlin** | **11.420** | **4.833625** | **0.0741** | **0.610** | **0.277** | **0.276** | **0.545** | **FSDT** |
| **Kirchhoff** | **11.060** | **8.245931** | **-0.0723** | **0.610** | **0.278** | **0.277** | **0.686** | **CPT** |
| **Average** | | **5.703191** | | | | | | |

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