International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 11 (2018) pp. 9245-9259

Combined Effects on Unsteady MHD Convective flow of Rotating Viscous Fluid through a Porous Medium over a Moving Vertical Plate

B.V.Swarnalathamma[1], M. Veera Krishna[2*] and Prof. J.Prakash[3]

[1]*Department of Science and Humanities, JB institute of Engineering & Technology, Moinabad, Hyderabad, Telangana-500075, India.*

[2]*Department of Mathematics, Rayalaseema University, Kurnool, Andhra Pradesh-518007, India.*

[3]*Dept.of Mathematics, University of Botswana, Privatebag 0022, Gaborone, Botswana.*

([*] - corresponding author)

Abstract

We have discussed unsteady MHD natural convective flow of a viscous, rotating electrically conducting and incompressible fluid over an impulsively moving vertical plate embedded in porous medium with thermal radiation and thermal diffusion. The dimensionless governing coupled boundary layer equations are solved by regular perturbation technique. The effect of pertinent parameters on primary and secondary velocities, temperature and concentration for externally heating and cooling of the plate are shown graphically. Finally, the influence of pertinent parameters on the rate of heat and mass transfer and shear stress coefficient of the wall are equipped through tabular forms.

Keywords: Heat and mass transfer, MHD flows, porous medium, unsteady flows and visco-elastic fluids.

Nomenclature

B_0	Uniform applied magnetic field
x', y', z'	Co-ordinate system
x, y, z	Dimensionless coordinates
u'	Fluid velocity along the x'-axis
v'	Fluid velocity along the y'-axis
u	Non-dimensional fluid velocity along the x'-axis
v	Non-dimensional fluid velocity along the y'-axis
t_0	Characteristic time
Nu	Nusselt number or rate of heat transfer coefficient
Sh	Sherwood number or rate of mass transfer coefficient
C_p	Specific heat at constant pressure
Gr	Thermal Grashof number
Gm	mass Grashof number
$\bar{g}$	Acceleration due to gravity
g	Acceleration due to gravity in magnitude
K	Permeability parameter
k_T	Thermal diffusion ratio
T_m	Mean fluid temperature
C_s	Concentration susceptibility
k^*	Mean absorption coefficient
$\bar{B}$	Magnetic induction vector
M	Magnetic parameter
Pr	Prandtl number
p	Fluid pressure
q_r	Radiative flux
N	Radiation parameter
Sr	Soret number
C'	Species concentration
C'_∞	Species concentration of the fluid far away from the plate
C'_w	Species concentration at the plate
D_m	Molecular mass diffusivity
E	Electric field
Dr	Dufour number
Sc	Schmidt Number
T'_w	Temperature at the plate
T'_∞	Temperature of the fluid far away from the plate
t'	Time

T'	Fluid temperature
U_0	Plate velocity
θ	Non-dimensional temperature
C	Non-dimensional species concentration

Greek symbols:

ρ	Fluid density
k	Thermal conductivity
σ	Electrical conductivity
ν	Kinematic viscosity
β	Coefficient of volume expansion for heat transfer
Ω	Rotation parameter
Ω'	Uniform angular velocity
β^*	Coefficient of volume expansion for mass transfer
τ_x	Skin friction in x' -direction
τ_z	Skin friction in z' -direction
σ^*	Stefan-Boltzmann constant

Superscript

/	Dimensionless property

Subscript

w	Wall conditions
∞	Free stream condition
p	Plate

INTRODUCTION

The problem of MHD laminar flow in the course of a porous medium has become very significant in latest years because of its possible applications in several branches [1-4]. In many processes, in engineering areas, take place at higher temperature and therefore the knowledge of radiation heat transfer becomes a very essential for the design of the related equipment. Also, convective flows with radiation are encountered in many environmental and engineering processes, for example, evaporation from large open reservoirs, heating and cooling chambers, astrophysical flows, solar control technology and space vehicle re-entry. Sharma et al.[5] have found the influence of homogeneous chemical reaction and radiation on unsteady magneto hydrodynamic free convection flow of a viscous incompressible liquid past a heated vertical plate immersed in porous medium in the presence of heat source. Reddy et al.[6] have investigated unsteady MHD radiative and chemically reactive free convection flow near a moving vertical plate in porous medium. Several authors [7-18] have addressed various issues related to thermal radiation. Double-diffusive convection is an essential process in oceanography and plays a role in mantle convection (magma chambers) and some technological applications. Recently, Krishna and Swarnalathamma [19] discussed the peristaltic MHD flow of an incompressible and electrically conducting Williamson fluid in a symmetric planar channel with heat and mass transfer under the effect of inclined magnetic field. Swarnalathamma and Krishna [20] discussed the theoretical and computational study of peristaltic hemodynamic flow of couple stress fluids through a porous medium under the influence of magnetic field with wall slip condition. Veera Krishna and Gangadhar Reddy [21] discussed MHD free convective rotating flow of visco-elastic fluid past an infinite vertical oscillating plate. Veera Krishna and Subba Reddy [22] discussed unsteady MHD convective flow of second grade fluid through a porous medium in a Rotating parallel plate channel with temperature dependent source. Krishna and Gangadhar Reddy [23] discussed the unsteady MHD free convection in a boundary layer flow of an electrically conducting fluid through porous medium subject to uniform transverse magnetic field over a moving infinite vertical plate in the presence of heat source and chemical reaction. Krishna and Subba Reddy [24] have investigated the simulation on the MHD forced convective flow through stumpy permeable porous medium (oil sands, sand) using Lattice Boltzmann method. Krishna and Jyothi [25] discussed the Hall effects on MHD Rotating flow of a visco-elastic fluid through a porous medium over an infinite oscillating porous plate with heat source and chemical reaction. Siva Kumar Reddy et al.[26] investigated MHD flow of viscous incompressible nano-fluid through a saturating porous medium. Veera Krishna et al. [27] discussed heat and mass transfer on unsteady MHD oscillatory flow of blood through porous arteriole. The effects of radiation and Hall current on an unsteady MHD free convective flow in a vertical channel filled with a porous medium have been studied by Veera Krishna et al. [28]. The heat generation/absorption and thermo-diffusion on an unsteady free convective MHD flow of radiating and chemically reactive second grade fluid near an infinite vertical plate through a porous medium and taking the Hall current into account have been studied by Veera Krishna and Chamkha [29]

In this paper, we have investigated the analytical study of unsteady magneto hydro dynamic (MHD) natural convective heat and mass transfer of a viscous, rotating fluid, electrically conducting and incompressible fluid flow past an impulsively moving vertical plate embedded in porous medium in the presence of thermal radiation and thermal diffusion.

FORMULATION AND SOLUTION OF THE PROBLEM:

Consider an unsteady MHD natural convective flow with heat and mass transfer of an optically thick radiating, incompressible and electrically conducting viscous fluid past an infinite vertical plate is embedded in a uniform porous medium with a rotating system. Consider x' -axis is along the plate in upward direction and y' -axis is normal to plane of the plate in the fluid. A uniform transverse magnetic field B_0

is applied in a direction which is parallel to y'-axis. The fluid and plate rotate with uniform angular velocity Ω' about the y'-axis. Initially i.e. at time $t' \leq 0$, both the fluid and plate or in rest and these are maintained at a uniform temperature T'_∞. Also species concentration is at the surface of the plate as well as at every point within the fluid and it is maintained at uniform concentration C'_∞. At time $t' \geq 0$, plate starts moving in x'-direction with uniform velocity U_0 in its own plane. The temperature of the plate is raised or lowered to $T'_\infty + \left(T'_w - T'_\infty\right) t' / t_0$ when $0 \leq t' \leq t_0$, and it is maintained at uniform temperature T'_w when $t' \geq t_0$.

Also, at time $t' \geq 0$, species concentration is at the surface of the plate, it is raised to uniform species concentration C'_w and it is maintained thereafter. Geometry of the problem is show in Fig.1. Since plate is an infinite extent in x' and z' directions and it is electrically non-conducting, all physical quantities except pressure depend on y' and t' only. Also, no applied or polarized voltages are assumed to exist, so that the effect of polarization of fluid is negligible. The induced magnetic field also is neglected. The magnetic Reynolds number is very small for liquid metals and partially ionized fluids which are commonly used in industrial applications. Keeping in view of these assumptions and under the Boussinesq's approximation, the governing equations are given by

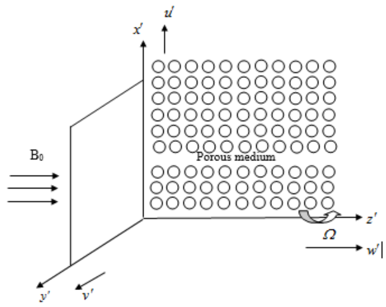


Figure 3. Physical configuration of the Problem

$$\frac{\partial u'}{\partial t'} - 2\Omega' v' = \nu \frac{\partial^2 u'}{\partial z'^2} + \frac{\sigma B_0^{\ 2}}{\rho} u' - \frac{\nu u'}{k_1} + g\beta (T' - T_\infty{}') + g\beta^* (C' - C_\infty{}') \tag{1}$$

$$\frac{\partial v'}{\partial t'} - 2\Omega' u' = \nu \frac{\partial^2 v'}{\partial z'^2} + \frac{\sigma B_0^{\ 2}}{\rho} v' - \frac{\nu}{k_1} v' \tag{2}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p} \frac{\partial^2 T'}{\partial z'^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial z'} + \frac{D_m k_T}{C_s C_p} \frac{\partial^2 C'}{\partial z'^2} \tag{3}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial z'^2} + \frac{D_m k_T}{T_m} \frac{\partial^2 T'}{\partial z'^2} \tag{4}$$

The boundary conditions taken for the problem are

$$U' = u_p{}', \ T = T'_\infty + \varepsilon \left(T'_w - T'_\infty\right) e^{n't'}, \ C = C'_\infty + \varepsilon \left(C'_w - C'_\infty\right) e^{n't'} \quad \text{at} \quad z' = 0,$$

$$U' \to 0, \ \theta \to \theta_\infty, \ \phi \to \phi_\infty \qquad \text{at} \quad z' \to \infty \tag{5}$$

The radiative flux term by using the Rosseland approximation is given by

$$q_r{}' = -\frac{4\sigma^*}{3k^*} \left(\frac{\partial T'^4}{\partial z'} \right)_{z=0} \tag{6}$$

Using the Rosseland approximation, present analysis is limited to optically thick fluids. If temperature differences within the flow are sufficiently very small then eq.(6) can be linearised by expanding T' into the taylor series about T'_∞ which after neglecting higher order terms the form

$$T'^4 \cong T'^3_\infty - 3T'^4_\infty \tag{7}$$

Substituting equation (6) and (7) into equation (3) we obtain

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial z'^2} + \frac{1}{\rho C_p}\frac{16\sigma^* T'^3_\infty}{3k^*}\frac{\partial^2 T'}{\partial z'^2} + \frac{D_m k_T}{C_s C_p}\frac{\partial^2 C'}{\partial z'^2} \tag{8}$$

Introducing the following non-dimensional quantities are,

$$u = \frac{u'}{U_0},\ v = \frac{v'}{U_0},\ z = \frac{z' U_0^2}{v},\ t = \frac{t' U_0^2}{v},\ \theta^* = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ \phi^* = \frac{C' - C'_\infty}{C'_w - C'_\infty},$$

$$M^2 = \frac{\sigma B_0^2 v}{\rho U_0^2},\ \Omega = \frac{v\Omega'}{U_0^2},\ K = \frac{k_1 U_0^2}{v^2},\ \mathrm{Gr} = \frac{g\beta v(T'_w - T'_\infty)}{U_0^3}, \mathrm{Gm} = \frac{g\beta^* v(C'_w - C'_\infty)}{U_0^3},\quad \mathrm{Pr} = \frac{v_p \phi_p}{k},\ N = \frac{16\sigma^* T_\infty^3}{3kk^*},\ \mathrm{Sc} = \frac{v}{D},\ \mathrm{Sr} = \frac{D_m k_T (T'_w - T'_\infty)}{v T_m (C'_w - C'_\infty)}$$

Then the resultant non-dimensional equations are

$$\frac{\partial u}{\partial t} + 2\Omega v = \frac{\partial^2 u}{\partial z^2} - M^2 u - \frac{u}{k_1} + \mathrm{Gr}\theta + \mathrm{Gm}\phi \tag{9}$$

$$\frac{\partial v}{\partial t} - 2\Omega u = \frac{\partial^2 v}{\partial z^2} - M^2 v - \frac{v}{k_1} \tag{10}$$

$$\frac{\partial \theta}{\partial t} = \left(\frac{1+N}{P_r}\right)\frac{\partial^2 \theta}{\partial z^2} + Ec\left(\frac{\partial q}{\partial z}\right)^2 \tag{11}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{\mathrm{Sc}}\frac{\partial^2 \phi}{\partial z^2} + \mathrm{Sr}\frac{\partial^2 \theta}{\partial z^2} \tag{12}$$

Combining equation (9) and (10), let $q = u + iv$, We obtain

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial z^2} - \lambda q + \mathrm{Gr}\theta + \mathrm{Gm}\phi \tag{13}$$

Where, $\lambda = M^2 + 2iE^{-1} + \frac{1}{K}$

With the following dimensionless boundary conditions

$$q = U_0,\ \theta = 1 + \varepsilon e^{nt},\ \phi = 1 + \varepsilon e^{nt} \quad \text{at} \quad z = 0 \tag{14}$$

$$q \to 0,\ \theta \to 0,\ \phi \to 0 \quad \text{at} \quad z \to \infty \tag{15}$$

We use the linear transformation for low value of ε to solve the equations (11) - (13) subject to the boundary conditions (14) and (15) as follows

$$q(z,t) = q_0(z) + \varepsilon e^{int} q_1(z) + O(\varepsilon^2) \tag{16}$$

$$\theta(z,t) = \theta_0(z) + \varepsilon e^{int} \theta_1(z) + O(\varepsilon^2) \tag{17}$$

$$\phi(z,t) = \phi_0(z) + \varepsilon e^{int} \phi_1(z) + O(\varepsilon^2) \tag{18}$$

After substituting the equation (16) to (18) into (11) to (13) we have

$$\frac{\partial^2 q_0}{\partial z^2} - \lambda q_0 = -\mathrm{Gr}\theta_0 - \mathrm{Gm}\phi_0 \tag{19}$$

$$\frac{\partial^2 q_1}{\partial z^2} - \lambda q_1 - inq_1 = -\mathrm{Gr}\theta_1 - \mathrm{Gm}\phi_1 \tag{20}$$

$$R\frac{\partial^2\theta_0}{\partial z^2} = -Ec\left(\frac{\partial q_0}{\partial z}\right)^2 \qquad (21)$$

$$R\frac{\partial^2\theta_1}{\partial z^2} - in\theta_1 = -2Ec\frac{\partial q_0}{\partial z}\frac{\partial q_1}{\partial z} \qquad (22)$$

$$\frac{1}{\mathrm{Sc}}\frac{\partial^2\phi_0}{\partial z^2} = -\mathrm{Sr}\frac{\partial^2\theta_0}{\partial z^2} \qquad (23)$$

$$\frac{1}{\mathrm{Sc}}\frac{\partial^2\phi_1}{\partial z^2} - in\phi_1 = -\mathrm{Sr}\frac{\partial^2\theta_1}{\partial z^2} \qquad (24)$$

With the following boundary equations

$$q_0 = U_p,\ q_1 = 0,\ \theta_0 = 1,\ \theta_1 = 1,\ \phi_0 = 1,\ \phi_1 = 1 \quad \text{at} \quad z = 0, \qquad (25)$$

$$q_0 = q_1 = \theta_0 = \theta_1 = \phi_0 = \phi_1 = 0 \quad \text{at} \quad z \to \infty \qquad (26)$$

Assuming the viscous dissipation parameter (Ecket number *Ec*) is small, We can solve the non linear coupled equations (19) - (24) writing the asymptotic expansion as follows:

$$q_0(z) = q_{01}(z) + Ecq_{02}(z) + O(Ec^2) \qquad (27)$$

$$q_1(z) = q_{11}(z) + Ecq_{12}(z) + O(Ec^2) \qquad (28)$$

$$\theta_0(z) = \theta_{01}(z) + Ec\theta_{02}(z) + O(Ec^2) \qquad (29)$$

$$\theta_1(z) = \theta_{11}(z) + Ec\theta_{12}(z) + O(Ec^2) \qquad (30)$$

$$\phi_0(z) = \phi_{01}(z) + Ec\phi_{02}(z) + O(Ec^2) \qquad (31)$$

$$\phi_1(z) = \phi_{11}(z) + Ec\phi_{12}(z) + O(Ec^2) \qquad (32)$$

Now substituting equation (27) to (32) into equations (19) to (24), we obtain the following sequence of approximation for $O(Ec)$:

$$\frac{\partial^2 q_{01}}{\partial z^2} - \lambda q_{01} = -\mathrm{Gr}\theta_{01} - \mathrm{Gm}\phi_{01} \qquad (33)$$

$$\frac{\partial^2 q_{02}}{\partial z^2} - \lambda q_{02} = -\mathrm{Gr}\theta_{02} - \mathrm{Gm}\phi_{02} \qquad (34)$$

$$\frac{\partial^2 q_{11}}{\partial z^2} - \lambda q_{11} - inq_{11} = -\mathrm{Gr}\theta_{11} - \mathrm{Gm}\phi_{11} \qquad (35)$$

$$\frac{\partial^2 q_{12}}{\partial z^2} - \lambda q_{12} - inq_{12} = -\mathrm{Gr}\theta_{12} - \mathrm{Gm}\phi_{12} \qquad (36)$$

$$R\frac{\partial^2\theta_{01}}{\partial z^2} = 0 \qquad (37)$$

$$R\frac{\partial^2\theta_{02}}{\partial z^2} = -\left(\frac{\partial q_{01}}{\partial z}\right)^2 \qquad (38)$$

$$R\frac{\partial^2\theta_{11}}{\partial z^2} - in\theta_{11} = 0 \tag{39}$$

$$R\frac{\partial^2\theta_{12}}{\partial z^2} - in\theta_{12} = -2\frac{\partial q_{01}}{\partial z}\frac{\partial q_{11}}{\partial z} \tag{40}$$

$$\frac{1}{\mathrm{Sc}}\frac{\partial^2\phi_{01}}{\partial z^2} = -\mathrm{Sr}\frac{\partial^2\theta_{01}}{\partial z^2} \tag{42}$$

$$\frac{1}{\mathrm{Sc}}\frac{\partial^2\phi_{11}}{\partial z^2} - in\phi_{11} = -\mathrm{Sr}\frac{\partial^2\theta_{11}}{\partial z^2} \tag{43}$$

$$\frac{1}{\mathrm{Sc}}\frac{\partial^2\phi_{12}}{\partial z^2} - in\phi_{12} = -\mathrm{Sr}\frac{\partial^2\theta_{12}}{\partial z^2} \tag{44}$$

Subject to the following boundary conditions

$$q_{01} = U_0, q_{11} = 0, \theta_{01} = \theta_{11} = \phi_{01} = \phi_{11} = 1, q_{02} = q_{12} = \theta_{02} = \theta_{12} = \phi_{02} = \phi_{12} = 0 \text{ at } z = 0 \tag{45}$$

$$q_{01} = q_{11} = \theta_{01} = \theta_{11} = \phi_{01} = \phi_{11} = q_{02} = q_{12} = \theta_{02} = \theta_{12} = \phi_{02} = \phi_{12} = 0 \text{ at } z \to \infty \tag{46}$$

Now solving the linear equations (33) - (44) with the boundary conditions (45) – (46) we get the results as follows

$$q = a_2 e^{-h_1 z} + a_3 + Ec\left(a_{10}e^{-h_1 z} + a_{11}e^{-2h_1 z} + a_{12}\right) + \varepsilon e^{int}\left\{-\left(a_4 + a_5 + a_6\right)e^{-h_2 z} + \left(a_4 + a_6\right)e^{-h_3 z} + \right.$$
$$+ a_5 e^{-h_4 z} + Ec\Big[-\left(m_8 + m_9 + m_{10} + m_{11} + m_{12}\right)e^{-h_2 z} + m_8 e^{-h_3 z} + m_9 e^{-(h_1+h_2)z} +$$
$$\left. + m_{10}e^{-(h_1+h_3)z} + m_{11}e^{-(h_1+h_4)z} + m_{12}e^{-h_4 z}\Big]\right\} \tag{47}$$

$$\theta = 1 + Ec\ a_7(e^{-2h_1 z} - 1) + \varepsilon e^{int}\left\{ e^{-h_3 z} + Ec\Big[(-m_1 + m_2 + m_3)e^{-h_3 z}\right.$$
$$\left. + m_1 e^{-(h_1+h_2)\mathrm{z}} - m_2 e^{-(h_1+h_3)\mathrm{z}} - m_3 e^{-(h_1+h_4)\mathrm{z}}\Big]\right\} \tag{48}$$

$$\phi = 1 + Eca_8(1 - e^{-2h_1 z}) + \varepsilon e^{int}\left\{(1 + a_1)e^{-h_4 z} - a_1 e^{-h_3 z} + Ec\Big[(m_4 + m_5 - m_6 - m_7)e^{-h_4 z}\right.$$
$$\left. - m_4 e^{-h_3 z} - m_5 e^{-(h_1+h_2)\mathrm{z}} + m_6 e^{-(h_1+h_3)\mathrm{z}} + m_7 e^{-(h_1+h_4)\mathrm{z}}\Big]\right\} \tag{49}$$

Some important physical quantities related to the heat and mass transfer characteristics are the skin friction coefficient, nusselt number and Sherwood number are given by

$$\tau = \left(\frac{\partial q}{\partial z}\right)_{z=0}$$

$$\tau = -a_2 h_1 - Ec(a_{10}h_1 + 2a_{11}h_1) - \varepsilon e^{int}\left\{-(a_4 + a_5 + a_6)h_2 + (a_4 + a_6)h_3 + a_5 h_4 + Ec\Big[-(m_8 + m_9 + m_{10}\right.$$
$$\left. + m_{11} + m_{12})h_2 + m_8 h_3 + m_9(h_1 + h_2) + m_{10}(h_1 + h_3) + m_{11}(h_1 + h_4) + m_{12}h_4\Big]\right\} \tag{50}$$

$$Nu = \left(\frac{\partial \theta}{\partial z}\right)_{z=0}$$

$$= -2Eca_7 h_1 - \varepsilon e^{int}\left\{ h_3 + Ec\Big[(-m_1 + m_2 + m_3)h_3 + m_1(h_1 + h_2) - m_2(h_1 + h_3) - m_3(h_1 + h_4)\Big]\right\} \tag{51}$$

$$Sh = \left(\frac{\partial \phi}{\partial z}\right)_{z=0}$$

$$Sh = 2Ec\ a_8 h_1 - \varepsilon e^{int}\left\{ (1+a_1)h_4 - a_1 h_3 + Ec\left[(m_4+m_5-m_6-m_7)h_4 - m_4 h_3 - m_5(h_1+h_2) + m_6(h_1+h_3) + m_7(h_1+h_4) \right] \right\} \tag{52}$$

Where,

$$h_1 = \sqrt{\lambda},\ h_2 = \sqrt{\lambda + in},\ h_3 = \sqrt{\frac{in}{R}},\ h_4 = \sqrt{in\mathrm{Sc}},$$

$$a_1 = \frac{\mathrm{Sc}\,\mathrm{Sr}\,\mathrm{i}\,n}{R(h_3^2 - h_4^2)},\ a_2 = (U_0 - \frac{\mathrm{Gr}+\mathrm{Gm}}{h_1^2}),\ a_3 = \frac{\mathrm{Gr}+\mathrm{Gm}}{h_1^2},\ a_4 = \frac{\mathrm{Gr}}{(h_2^2 - h_3^2)},\ a_5 = \frac{\mathrm{Gm}(1+a_1)}{(h_2^2 - h_4^2)}$$

$$a_6 = \frac{\mathrm{Gm}a_1}{(h_3^2 - h_2^2)},\ a_7 = -\frac{a_2^2}{4R},\ a_8 = -\frac{\mathrm{ScSr}a_2^2}{4R},\ a_9 = \left(\frac{\mathrm{Gr} - \mathrm{GmScSr}}{4R}\right)a_2^2,\ a_{10} = -\frac{4a_9}{3h_1^2}$$

$$a_{11} = \frac{a_9}{3h_1^2},\ a_{12} = \frac{a_9}{h_1^2},\ m_1 = \frac{2a_2(a_4+a_5+a_6)h_1h_2}{R(2h_1^2 + in + 2h_1h_2 - h_3^2)},\ m_2 = \frac{2a_2(a_4+a_6)h_1h_3}{R(h_1^2 + 2h_1h_3)}$$

$$m_3 = \frac{2a_2a_5h_1h_4}{h_1^2 + h_4^2 + 2h_1h_4 - h_3^2},\ m_4 = \frac{\mathrm{ScSr}in(-m_1+m_2+m_3)}{R(h_3^2 - h_4^2)},\ m_5 = \frac{\mathrm{ScSr}m_1(h_1+h_2)^2}{(2h_1^2 + in + 2h_1h_2 - h_4^2)}$$

$$m_6 = \frac{\mathrm{ScSr}m_2(h_1+h_3)^2}{(h_1^2 + h_3^2 + 2h_1h_3 - h_4^2)},\ m_7 = \frac{\mathrm{ScSr}m_3(h_1+h_4)^2}{(h_1^2 + 2h_1h_4)},\ m_8 = \frac{-\mathrm{Gr}(-m_1+m_2+m_3) + \mathrm{Gm}m_4}{(h_3^2 - h_2^2)}$$

$$m_9 = \frac{-\mathrm{Gr}m_1 + \mathrm{Gm}m_5}{h_1^2 + 2h_1h_2},\ m_{10} = \frac{\mathrm{Gr}m_2 - \mathrm{Gm}m_6}{h_3^2 + 2h_1h_3 - in}$$

$$m_{11} = \frac{\mathrm{Gr}m_3 - \mathrm{Gm}m_7}{h_4^2 + 2h_1h_4 - in},\ m_{12} = \frac{-\mathrm{Gm}(m_4+m_5-m_6-m_7)}{h_4^2 - h_2^2}$$

RESULTS AND DISCUSSION

We have considered the unsteady free convective flow of a viscous incompressible electrically conducting fluid over an infinite vertical porous plate under the influence of uniform transverse magnate field. The governing equations of the flow field are solved by a regular perturbation method for small Eckert number *Ec*. The closed form solutions for the velocity, temperature and concentration have been derived analytically and also its behavior is computationally discussed with reference to different flow parameters like *M* Hartmann number, *E* Ekman number, *K* porosity parameter, Gr is the thermal Grashof number, Gm is the mass Grashof number, *N* Radiation parameter, Pr is the Prandtl number, Sr Soret parameter, Sc is the Schmidt number and time *t*. Figures (2-11) represent velocity, Figures (12) and Figures (13) represent the temperature and concentration distributions respectively. The stresses, Nusselt number and Sherwood number at the plate are evaluated numerically and discussed with governing parameters and are tabulated in the tables (1-3). Fixing the parameters *n*=1 and $Ec = 0.01$.

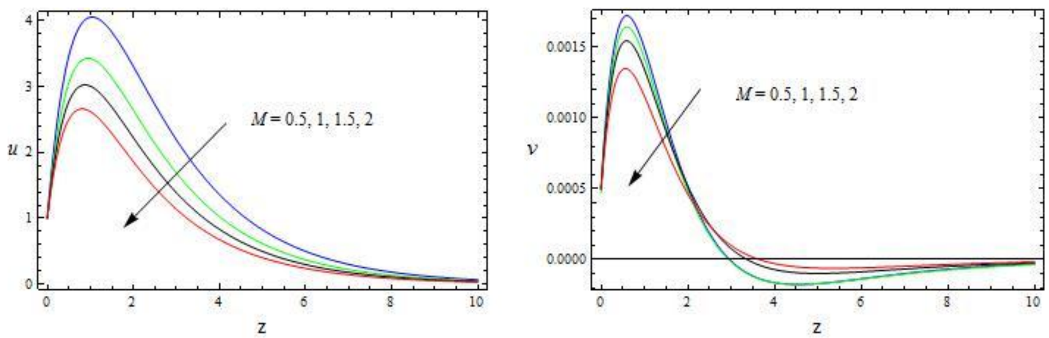
From the Figures (2), we noticed that the magnitude of the velocity components *u* and *v* as well as resultant velocity are reduces with increasing the intensity of the magnetic field *M*. This is because of the reason that effects of a transverse magnetic field on an electrically conducting fluid gives rise to a resistive type force (called Lorentz force) similar to drag force and upon increasing the values of *M* increases the drag force which has tendency to slow down the motion of the fluid. Both the magnitude of the velocity components *u* and *v* are enhances with increasing Ekman number E due to the rotation of the plate. The resultant velocity is also experiences enhancement throughout the fluid region with increasing *E* (Figures 3).

In fluid dynamics, flow through porous media, the Darcy number (Da) represents the relative effect of the permeability of the medium versus its cross-sectional area commonly the diameter squared. The number is named after Henry Darcy and is found from nondimensionalizing the differential form of Darcy's Law. This number should not be confused with the Darcy friction factor which applies to pressure drop in a pipe. The magnitude of both the velocity components enhances with increasing the permeability parameter D. Also we observe that lower the permeability of the porous medium lesser the fluid speed in the entire fluid region. From the figure (4) depicts that the velocity component u increases and v reduces and the resultant velocity also enhances with increasing permeability of the porous medium (Darcy parameter). Lower the permeability of the porous medium lesser the fluid speed in the entire region. We noticed that from the figures (5 & 6), the magnitude of the velocity component u increases and v initially enhances and then gradually reduces throughout the fluid region with increasing thermal Grashof number Gr or mass Grashof number Gm. The resultant velocity is also boost up throughout the fluid medium with increasing Gr or Gm. The primary velocity component u reduces and the secondary velocity v increases with increasing Radiation parameter N. The resultant velocity is increases throughout the fluid medium with increasing Gr or Gm (Figure 7). The primary velocity component u reduces and the secondary velocity v increases with increasing Prandtl number Pr. But the resultant velocity is reduces throughout the fluid region with increasing N (Figure 8). We perceived that from the figures (9 & 11), the magnitude of the velocity components u and v raise throughout the fluid region with increasing Soret parameter Sr or time t. The resultant velocity is also boost up throughout the fluid medium with increasing S or t. Likewise the reversal behaviour is observed in the entire fluid region with increasing Schmidt number Sc. i.e., the velocity component u and v as well as the resultant velocity experiences retardation in the flow field with increasing Schmidt number Sc (Figure 10).

Figures (12) showed the effect of Radiation parameter N and the Prandtl number Pr on the temperature of the flow field. With increasing radiation parameter N reduces the temperature of the flow field. This may happen due the elastic property of the fluid. We also noted that the temperature of the flow field diminishes as the Prandtl number increases. This is consistent with the fact that the thermal boundary layer thickness decreases with increasing Prandtl number.

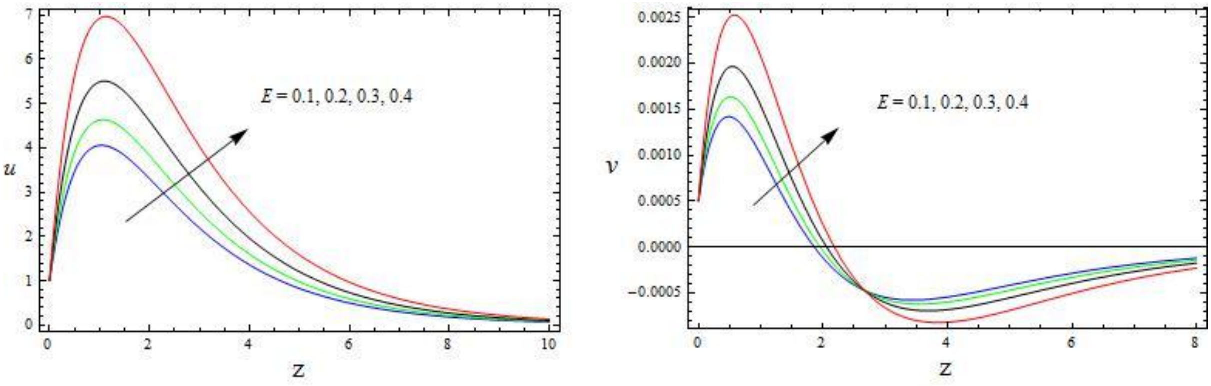
Figures (13) depict the effect of the Soret number Sr and Schmidt number Sc on the concentration distribution. It is observed that presence of the Soret parameter Sr increase the concentration distribution. The concentration distribution linearly decreases at all points of the flow field with the increase in the Schmidt number Sc.

The skin friction, Nusselt number and Sherwood number are calculated numerically and tabulated in the tables (1-3). The skin friction coefficients both τ_{xy} and τ_{xy} increase with the increasing in E, Gr and Sr and also reduce with increasing Porous parameter K and mass Grashof number Gm. The component τ_{xy} enhances and c reduces with increasing M and Sc. The reversal behaviour is observed with increasing Radiation parameter N or Prandtl number Pr (Table. 1). Nusselt number (Nu) at the surface of the plate reduces with increase radiation parameter N. Also enhance the rate of heat transfer with increase Prandtl number Pr or the frequency of oscillations n (Table. 2). Table (3) shows that the rate of mass transfer increases with increasing of Schmidt number and in the presence of Soret number decreases Sherwood number.



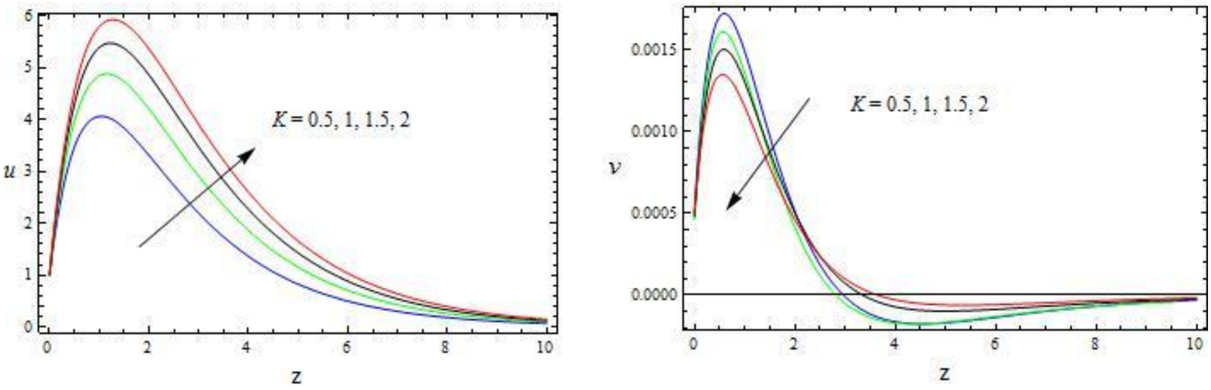
Figures 2. The velocity Profiles for u and v with M

$$E = 0.1, K = 0.5, \mathrm{Gr} = 3, \mathrm{Gm} = 1, \mathrm{Pr} = 0.71, N = 0.3, \mathrm{Sr} = 0.5, \mathrm{Sc} = 0.22, t = 0.1$$



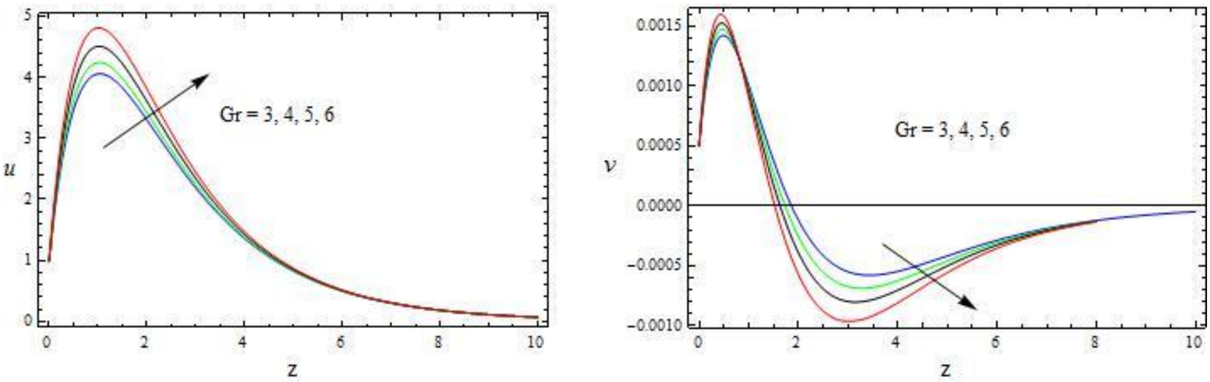
Figures 3. The velocity Profiles for *u* and *v* with *E*

$$M = 0.5, K = 0.5, \mathrm{Gr} = 3, \mathrm{Gm} = 1, \mathrm{Pr} = 0.71, N = 0.3, \mathrm{Sr} = 0.5, \mathrm{Sc} = 0.22, t = 0.1$$



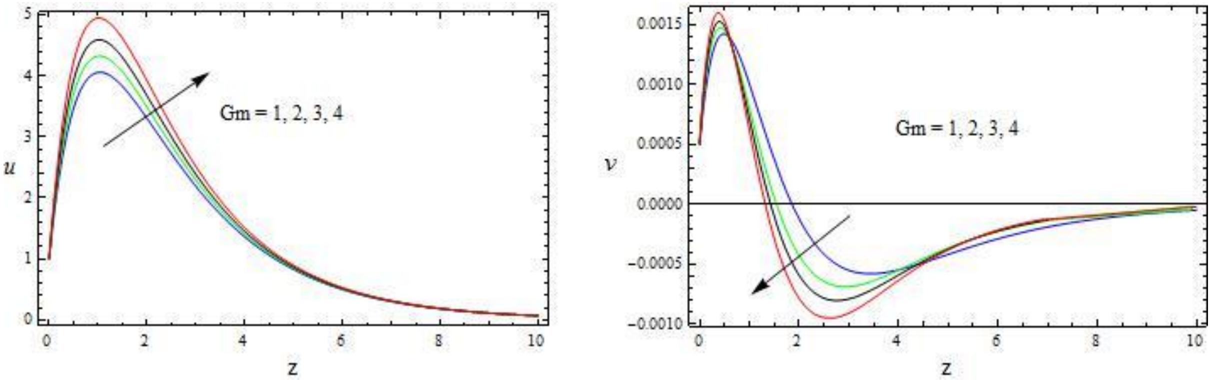
Figures 4. The velocity Profiles for *u* and *v* with *K*

$$M = 0.5, E = 0.1, \mathrm{Gr} = 3, \mathrm{Gm} = 1, \mathrm{Pr} = 0.71, N = 0.3, \mathrm{Sr} = 0.5, \mathrm{Sc} = 0.22, t = 0.1$$



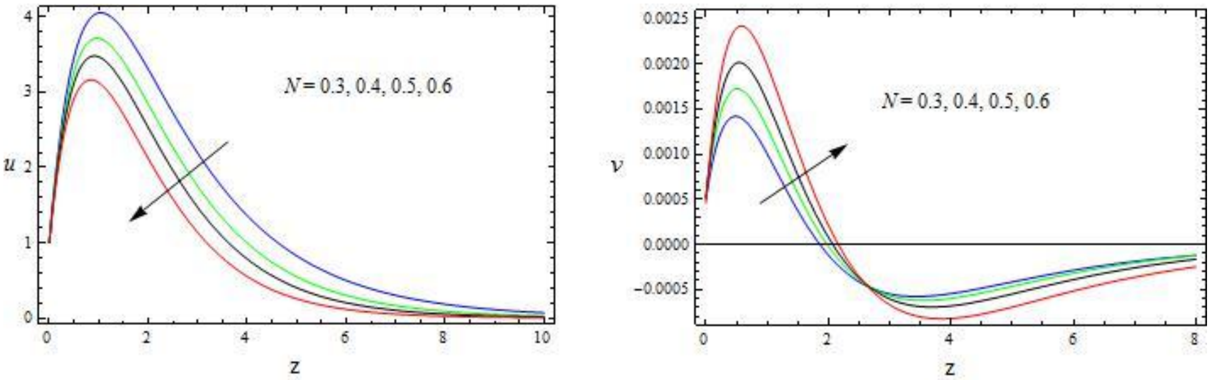
Figures 5. The velocity Profiles for *u* and *v* with Gr

$$M = 0.5, E = 0.1, K = 0.5, \mathrm{Gm} = 1, \mathrm{Pr} = 0.71, N = 0.3, \mathrm{Sr} = 0.5, \mathrm{Sc} = 0.22, t = 0.1$$



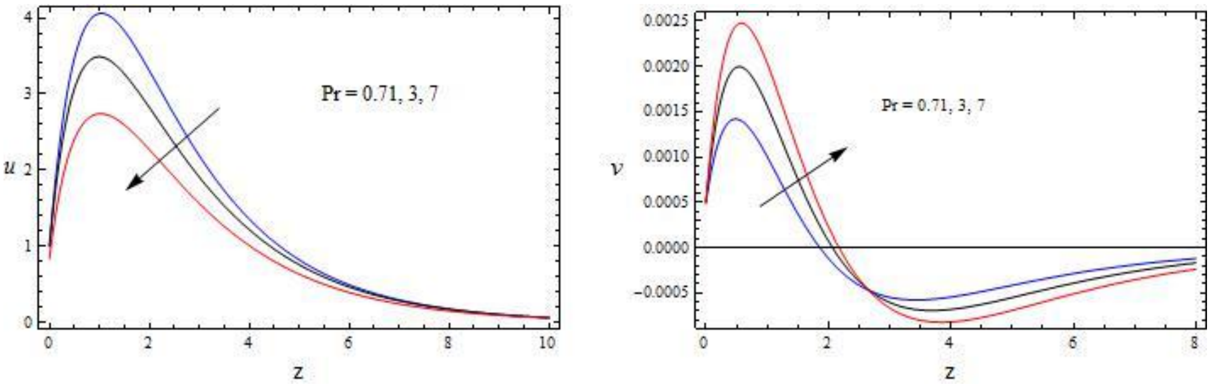
Figures 6. The velocity Profiles for *u* and *v* with Gm

$$M = 0.5, E = 0.1, K = 0.5, \mathrm{Gr} = 3, \mathrm{Pr} = 0.71, N = 0.3, \mathrm{Sr} = 0.5, \mathrm{Sc} = 0.22, t = 0.1$$



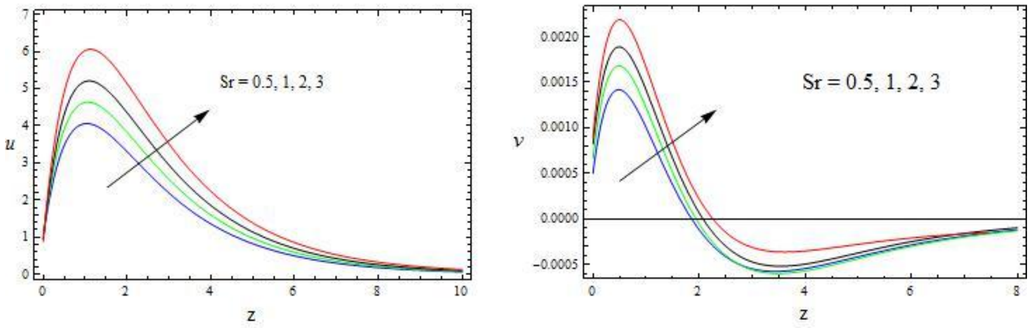
Figures 7. The velocity Profiles for *u* and *v* with *N*

$$M = 0.5, E = 0.1, K = 0.5, \mathrm{Gr} = 3, \mathrm{Gm} = 1, \mathrm{Pr} = 0.71, \mathrm{Sr} = 0.5, \mathrm{Sc} = 0.22, t = 0.1$$



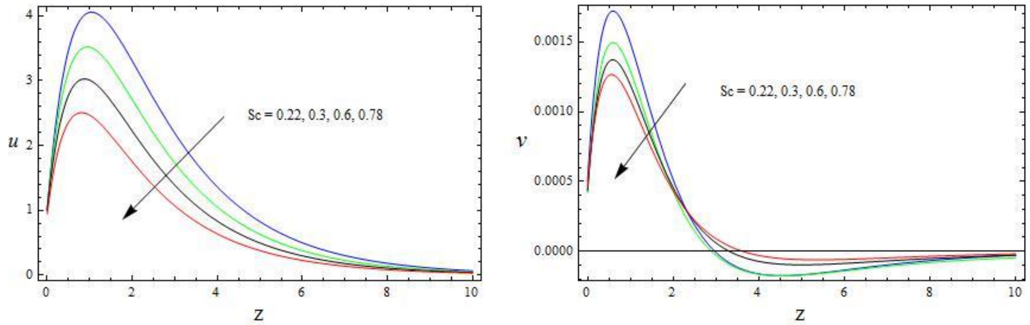
Figures 8. The velocity Profiles for *u* and *v* with Pr

$$M = 0.5, E = 0.1, K = 0.5, \mathrm{Gr} = 3, \mathrm{Gm} = 1, N = 0.3, \mathrm{Sr} = 0.22, \mathrm{Sc} = 0.22, t = 0.1$$



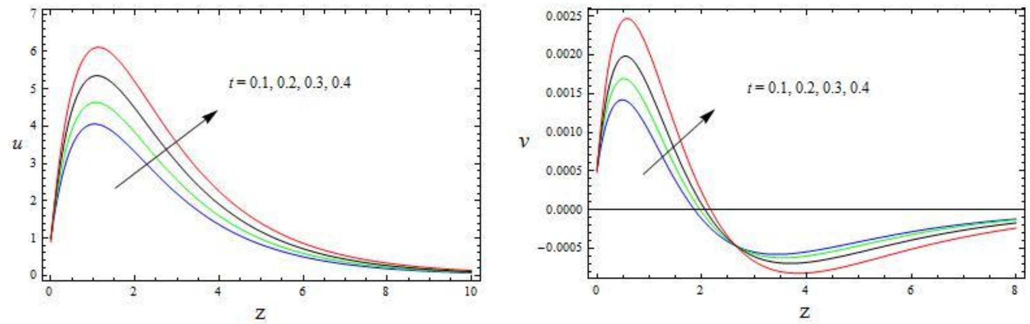
Figures 9. The velocity Profiles for *u* and *v* with Sr

$$M = 0.5, E = 0.1, K = 0.5, \mathrm{Gr} = 3, \mathrm{Gm} = 1, \mathrm{Pr} = 0.71, N = 0.3, \mathrm{Sc} = 0.22, t = 0.1$$



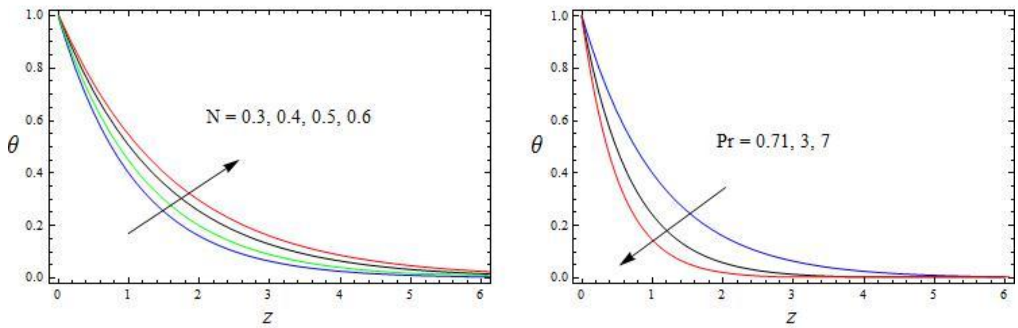
Figures 10. The velocity Profiles for *u* and *v* with Sc

$$M = 0.5, E = 0.1, K = 0.5, \mathrm{Gr} = 3, \mathrm{Gm} = 1, \mathrm{Pr} = 0.71, N = 0.3, \mathrm{Sr} = 0.5, t = 0.1$$

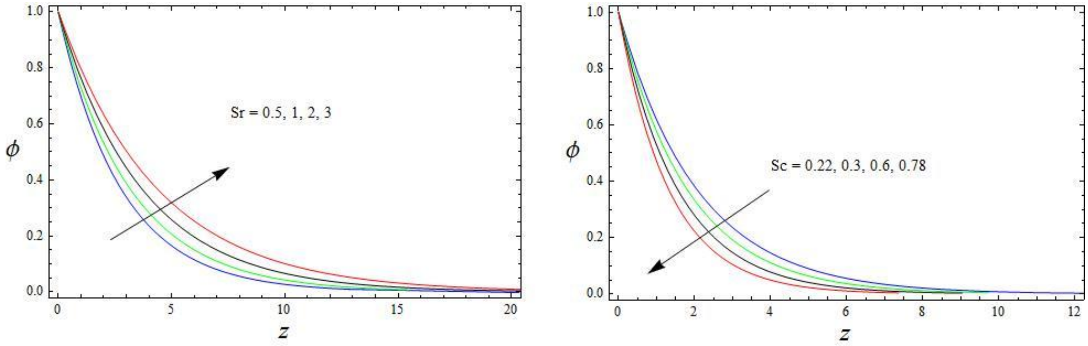


Figures 11. The velocity Profiles for *u* and *v* with *t*

$$M = 0.5, E = 0.1, K = 0.5, \mathrm{Gr} = 3, \mathrm{Gm} = 1, \mathrm{Pr} = 0.71, N = 0.3, \mathrm{Sr} = 0.5, \mathrm{Sc} = 0.22$$



Figures 12. Temperature Profiles with *N* (Pr=0.71) and Pr (*N*=0.3)



Figures 13. Concentration Profiles with Sr (Sc=0.22) and Sc (Sr=0.5)

Table 1. Skin friction ($\varepsilon = 0.001, Ec = 0.01$)

M	***E***	***K***	**Gr**	**Gm**	**N**	**Pr**	**Sr**	**Sc**	τ_{xy}	τ_{xz}
0.5	**0.1**	**0.5**	**3**	**1**	**3**	**0.71**	**0.5**	**0.22**	0.885547	-0.047851
1									0.952248	-0.036658
1.5									1.255658	-0.015478
	0.2								0.966985	-0.065589
	0.3								1.022462	-0.085848
		1							0.633253	-0.032256
		1.5							0.477850	-0.025047
			4						1.122521	-0.065585
			5						1.522462	-0.085985
				2					0.755258	-0.024117
				3					0.544185	-0.018547
					4				0.788554	-0.068958
					5				0.688958	-0.085047
						3			0.522485	-0.147749
						7			0.142552	-0.251047
							1		1.225524	-0.250214
							2		1.689589	-0.366966
								0.3	1.428740	-0.025001
								0.6	1.988554	-0.017989

Table 2. Nusselt number

($\varepsilon = 0.001, \mathrm{Gr} = 5, \mathrm{Gm} = 3, \mathrm{Sr} = 0.5, \ \mathrm{Sc} = 0.22, Ec = 0.01, M = 0.5, E = 2, K = 0.5$)

N	**Pr**	*n*	Nu
0.3	**0.71**	**1**	0.198858
0.4			0.165589
0.5			0.147853
	3		0.258954
	7		0.366258
		2	1.255246
		3	1.487056

Table 3. Sherwood number

($\varepsilon = 0.001, \mathrm{Gr} = 5, \mathrm{Gm} = 3, \mathrm{Pr} = 0.71, \mathrm{N} = 3, Ec = 0.01, M = 0.5, E = 2, K = 0.5$)

Sr	**Sc**	***n***	Sh
0.5	**0.22**	**1**	1.025985
1			0.855732
2			0.778543
	0.3		1.566982
	0.6		2.663547
		2	1.522415
		3	1.898566

CONCLUSIONS:

We have discussed unsteady MHD natural convective flow of a viscous, rotating fluid, electrically conducting and incompressible fluid over an impulsively moving vertical plate embedded in porous medium with thermal radiation and thermal diffusion. The conclusions are made as the following.

1. The rotation parameter increases the velocity close to the wall but there have been a decrease in velocity outlying from the wall.
2. The velocity increases with increase in *K*, *N*, Gr, Gm, Sr and *t*.
3. The resultant velocity diminishes with increasing *M* or Sc or Pr.
4. The temperature reduces with increase in the value of Prandtl number and increases with Radiation parameter.
5. In presence of Sr increase the concentration. It linearly decreases at all points of the flow field with increase in Sc.
6. The skin friction coefficients both τ_{xy} or τ_{xz} increase with the increasing in *E*, Gr and Sr and also reduce with increasing *K* and Gm. The component τ_{xy} enhances and τ_{xz} reduces with increasing *M* and Sc. The reversal behaviour is observed with increasing Radiation parameter or Pr.
7. Nusselt number (*Nu*) reduces with increase radiation parameter. Also enhance the rate of heat transfer with increase Prandtl number or the frequency parameter.
8. The rate of mass transfer increases with increasing of Schmidt number and in the presence of Soret number decreases Sherwood number.

REFERENCES

[1]. C. Israel-Cookey, A Ogulu and V B Omubo-Pepple, Influence of viscous dissipation on unsteady MHD free convection flow past an infinite heated vertical plate in porous medium with time-dependent suction, Int. J. Heat mass transfer, 46 (2003) 2305-2311.

[2]. Y. J. Kim, Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction, International Journal of Engineering Sciences, 38 (2000) 833-845.

[3]. M. C. Raju and S.V. K.Varma, Unsteady MHD free convective oscillatory couette flow through a porous medium with periodic wall temperature, imanager's Journal on Future Engineering and Technology, 6(4) (2011) 7-12.

[4]. U. S. Rajput and Surendra Kumar, MHD Flow Past an Impulsively Started Vertical Plate with Variable Temperature and Mass Diffusion, Applied Mathematical Sciences, 5(3) (2011) 149 – 157.

[5]. P. R. Sharma, N. Kumar and P. Sharma, Influence of chemical reaction and radiation on unsteady MHD free convection flow and mass transfer through viscous incompressible fluid past a heated vertical plate immersed in porous medium in the presence of heat source, Appl. Math. Sciences, 5(46) (2011) 2249-2260.

[6]. T. S. Reddy, M.C. Raju and S.V. K. Varma, Unsteady MHD radiative and chemically reactive free convection flow near a moving vertical plate in porous medium, JAFM, 6(3) (2013) 443-451.

[7]. A. Mohamed, M. SeddeekEmad and D. Aboel, Radiation effects on unsteady MHD free convection with hall current near an infinite vertical porous plate, Int. J. Mathematics and Mathematical Sciences, 26(4) (2001) 249-255.

[8]. K. D. Singh and R. Kuma, Fluctuating heat and mass transfer on unsteady MHD free convection flow of radiating and reacting fluid past a vertical porous plate in slip- flow regime, JAFM, 4(4) (2011) 101-106.

[9]. R. Muthucumaraswamy and V. Visalakshi, Radiative flow past an exponentially accelerated vertical plate with variable temperature and mass diffusion, Int. J. of Enng Annals of faculty Engineering Hunedoara, Tom IX, Fascicule, 2 (2011) 137-140.

[10]. Sumonsaha, Goutamsaha, Md. Quamrulislam and M C Raju, Convective heat transfer inside an enclosure with double discrete heaters exit configurations, International Journal of Applied Mathematics, 2(1) (2010) 85-90.

[11]. M. C. Raju, S. V. K. Varma, and N. Ananda Reddy, Radiation and mass transfer effects on a free convection flow through a porous medium bounded by a vertical surface, Journal of Future Engineering and Technology, 7(2) (2000) 7-12.

[12]. M. A. Hossain and H. S. Takhar, Radiation effect on mixed convection along a vertical plate with uniform surface temperature, Heat and Mass Transfer, 31 (1996) 243-248.

[13]. R. Muthucumaraswamy and Janakiraman, MHD and radiation effects on moving isothermal vertical plates with variables mass diffusion, Teoret. Appl. Mech, 33(1) (2006) 17-29.

[14]. M .A. Hossain, M .A. Alim, and D. A. S. Rees, The effect of radiation on free convection from a porous vertical plate, Int. J. Heat Mass Transfer, 42 (1999) 181- 191.

[15]. V. R. Prasad, N. B. Reddy and R. Muthucumaraswamy, Radiation and mass transfer effects on two-dimensional flow past an impulsively started infinite vertical plate, Int. J. Thermal Sci., 46(12) (2007) 1251-1258.

[16]. K. Manivannan, R. Muthucumaraswamy and V. Thangaraj, Radiation and chemical reaction effects on isothermal vertical oscillating plate with variable mass diffusion, Thermal Sciences, 13(2) (2009) 155-162.

[17]. A. J. Chamkha, H. S. Takhar, V. M .Soundalgekar, Radiation effects on free convection flow past a semi-infinite vertical plate with mass transfer, Chemical Engineering Journal, 84 (2001) 335-342.

[18]. Dual Pal and TBabulal, Combined effects of Joule heating and chemical reaction on unsteady magneto hydrodynamic mixed convection of a viscous dissipating fluid over a vertical plate in porous media with thermal radiation, Mathematical and computer modeling, 54 (2011) 3016-3036.

[19]. VeeraKrishna.M and B.V.Swarnalathamma (2016) Convective Heat and Mass Transfer on MHD Peristaltic Flow of Williamson Fluid with the Effect of Inclined Magnetic Field," AIP Conference Proceedings 1728:020461 doi: http://dx.doi.org/10.1063/1.4946512

[20]. Swarnalathamma. B. V. and M. Veera Krishna (2016) Peristaltic hemodynamic flow of couple stress fluid through a porous medium under the influence of magnetic field with slip effect AIP Conference Proceedings 1728:020603 doi: http://dx.doi.org/10.1063/1.4946654

[21]. VeeraKrishna.M and M.Gangadhar Reddy (2016) MHD free convective rotating flow of Visco-elastic fluid past an infinite vertical oscillating porous plate with chemical reaction IOP Conf. Series: Materials Science and Engineering 149:012217 doi: http://dx.doi.org/10.1088/1757-899X/149/1/012217.

[22]. VeeraKrishna/M and G.Subba Reddy (2016) Unsteady MHD convective flow of Second grade fluid through a porous medium in a Rotating parallel plate channel with temperature dependent source IOP Conf. Series: Materials Science and Engineering, 149:012216 doi: http://dx.doi.org/10.1088/1757-899X/149/1/012216.

[23]. M.Veera Krishna, M.Gangadhar Reddy, MHD Free Convective Boundary Layer Flow through Porous medium Past a Moving Vertical Plate with Heat Source and Chemical Reaction, Materials Today:

Proceedings, vol. 5, pp. 91–98, 2018. https://doi.org/10.1016/j.matpr.2017.11.058.

[24]. M.Veera Krishna, G.Subba Reddy, MHD Forced Convective flow of Non-Newtonian fluid through Stumpy Permeable Porous medium, Materials Today: Proceedings, vol. 5, pp. 175–183, 2018. https://doi.org/10.1016/j.matpr.2017.11.069.

[25]. M.Veera Krishna, Kamboji Jyotghi, Hall effects on MHD Rotating flow of a Visco-elastic Fluid through a Porous medium Over an Infinite Oscillating Porous Plate with Heat source and Chemical reaction, Materials Today: Proceedings, vol. 5, pp. 367–380, 2018. https://doi.org/10.1016/j.matpr.2017.11.094.

[26]. B. Siva Kumar Reddy, M. Veera Krishna , K.V.S.N. Rao, R. Bhuvana Vijaya, HAM Solutions on MHD flow of nano-fluid through saturated porous medium with Hall effects, Materials Today: Proceedings, vol. 5, pp. 120–131, 2018. https://doi.org/10.1016/j.matpr.2017.11.062.

[27]. Veera Krishna.M., B.V.Swarnalathamma and J. Prakash, "Heat and mass transfer on unsteady MHD Oscillatory flow of blood through porous arteriole, Applications of Fluid Dynamics, Lecture Notes in Mechanical Engineering, vol. XXII, pp. 207-224, 2018. Doi: 10.1007/978-981-10-5329-0_14.

[28]. Veera Krishna.M, G.Subba Reddy, A.J.Chamkha, "Hall effects on unsteady MHD oscillatory free convective flow of second grade fluid through porous medium between two vertical plates," Physics of Fluids, vol. **30**, 023106 (2018); doi: 10.1063/1.5010863

[29]. Veera Krishna.M, A.J.Chamkha, Hall effects on unsteady MHD flow of second grade fluid through porous medium with ramped wall temperature and ramped surface concentration, Physics of Fluids **30**, 053101 (2018), doi: https://doi.org/10.1063/1.5025542