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Numerical Modelling of Heat Transfer for The First Kind Boundary Condition in the Pulsating Laminar Flow at the Initial Hydrodynamic Section in the Flat Channel

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Abstract

The mathematical model and technique of numerical modeling of heat transfer with a first kind boundary condition at the pulsating flow developing laminar flow in the flat channel is created. Results of numerical modeling heat transfer according to the dimensionless frequency of pulsations S, amplitude of pulsations of the average velocity in section *A*, the Reynolds number Re and Prandtl number Pr are received and analyzed.

Keywords: heat transfer, laminar flow, pulsing flow, flat channel, initial hydrodynamic section, initial thermal section.

INTRODUCTION

The amount of the marked-out warmth of modern microelectronic devices demands the use of active cooling systems among which air and liquid are often met. The liquid cooling system allows achieving much compactness [1]. It consists of the microchannel cooler (the heat sink or the planar heat exchanger), the systems of the pipeline, pumps and the convector. The most important is the research of features of heat transfer in microchannel structure of the heat sink. The flow regime is laminar because of the small sizes of channels. The planar heat exchangers, representing the system of slot-hole microchannels, possess high heat transfer coefficient in spite of the laminar flow regime of the coolant and are used not only for cooling the microelectronics, but also in power system and chemical industry.

Some researches show a possibility of increase the heat transfer in the pulsating flow [2], [3], [4]. Exist studies of the effect of pulsating flow on heat exchange on the initial hydrodynamic section of the circular tube [5], [6], [7], nevertheless there are no detailed researches of hydrodynamics and heat transfer in the pulsating laminar flow in the flat channel on the initial hydrodynamic site. The results of researches, which are available currently, don't answer the question how the heat transfer changes according to the pulsating laminar flow regime parameters changes. Thus, conducting settlement researches in this science sphere is important from the practical and theoretical point of view.

PROBLEM STATEMENT

Fields components of velocity and temperature at pulsating laminar flow are described by the equations

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{1}{\rho}\frac{dp}{dx}, \qquad (1)$$

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0, \qquad (2)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=a\frac{\partial^2 T}{\partial y^2}, \qquad (3)$$

a – coefficient of thermal diffusivity, p – pressure, T – temperature, t – time, u and υ – velocity along axes x and y, ν – kinematic viscosity, ρ – density.

Such a form of equations (1) and (2) corresponds to the notion of the narrow channel approximation. As $\upsilon << u$ there is no need for solving the equation of preservation of impulse for a cross component of velocity. Such simplification has been applied for solving the task about the developing stationary flow analytically in [8], and later numerically in [9]. The cross component of velocity can exert impact for solving the energy equation (3). It is easy to find this component from the equation of continuity (2). When Re < 100 at small distances from the entrance it is impossible to neglect the solution of the movement for a cross component of velocity equation.

Axial heat conductivity that can exert impact when Pe < 100, according to estimates [10], Pe = RePr.

For velocity we have the equations (1) and (2) that need to be added with the equation for finding pressure. The task about the stationary developing heat is solved analytically in [11], and the full numerical decision is presented in [12] later.

The equations (1), (2) for pulsating flow in a dimensionless look are presented in a such form:

$$\mathrm{S}^2\frac{\partial U}{\partial t_\omega}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{\partial^2 U}{\partial Y^2}+P, \qquad (4)$$

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0, \qquad (5)$$

$P=-\dfrac{d_\mathrm{h}\mathrm{Re}}{\rho<\bar{u}>}\dfrac{dp}{dx}$ – dimensionless pressure gradient, $\mathrm{Re}=\dfrac{<\bar{u}>d_\mathrm{h}}{\nu}$ – Reynolds number, $d_\mathrm{h}=2h$ – hydraulic diameter, h – height of the channel, $\mathrm{S} = d_\mathrm{h}\sqrt{\frac{\omega}{\nu}}$ – Stokes number (dimensionless frequency of pulsations), t_ω – dimensionless time, $U = \frac{u}{<\bar{u}>}$ and $V = \frac{\upsilon \mathrm{Re}}{<\bar{u}>}$ – dimensionless longitudinal and cross velocity, $X = \frac{x}{d_\mathrm{h}\,\mathrm{Re}}$ и $Y = \frac{y}{d_\mathrm{h}}$ – dimensionless longitudinal and cross coordinates, ω – circular frequency, tokens < > and ¯ mean averaging on section and time.

The equation (3) in a dimensionless form:

$$\mathrm{S_T}^2 \frac{\partial \vartheta}{\partial t_\omega} + \mathrm{Pr}\left(U\frac{\partial \vartheta}{\partial X} + V\frac{\partial \vartheta}{\partial Y}\right) = \frac{\partial^2 \vartheta}{\partial Y^2}, \qquad (6)$$

Pr – Prandtl number, $\mathrm{S_T} = \mathrm{S}\sqrt{\mathrm{Pr}}$ – thermal Stokes number (dimensionless thermal frequency of pulsations), $\vartheta = \frac{T - T_0}{T_\mathrm{w} - T_0}$ – dimensionless temperature, T_0 – liquid temperature on the entrance to the channel, T_w – temperature of the wall.

On the fig. 1 there is a scheme of settlement area with boundary conditions.

The velocity profile pulsating in time, uniform on section is set on the entrance to the channel $U = 1 + A\sin(t_\omega)$.

The solution of the task depends on S, amplitude of pulsations of velocity, average on section A, Re and Pr.

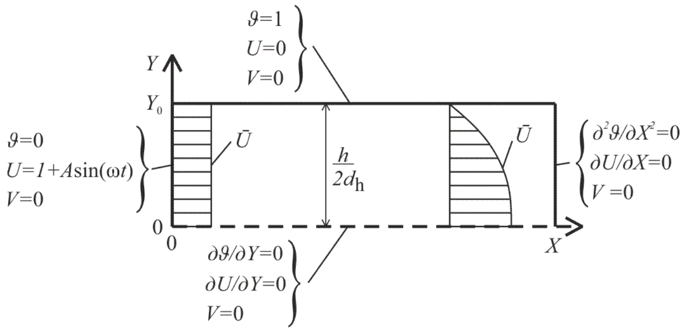


Figure 1: scheme of settlement area.

In practice, in front of the entrance to the channel there is some preincluded space. When $A > 1$, the stream of liquid moves to the opposite direction some part of time and comes into this space that is a problem for modeling pulsating flow connected with variety of its geometrical parameters. The geometry of the preincluded space is also required to be considered in considerable influence of axial heat conductivity. The explored area of parameters is limited by $0 < A \le 1$, $0{,}7 \le \mathrm{Pr} \le 70$, $100 \le \mathrm{Re} \le 1000$, $0 < \mathrm{S} \le 16$.

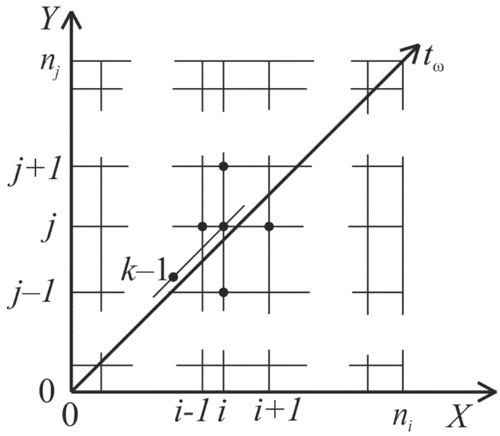


Figure 2: spatial grid.

The solution of the equations (4) and (6) was made by finite differences method by means of the implicit steady certainly differential two-layer by the time scheme. The scheme "against flow" is applied to approximation of the first derivative on longitudinal coordinate, as even when $A < 1$ and $\mathrm{S} > 1$ emergence of returnable flows near the wall is possible [13].

On the fig. 2 there is a spatial grid applied for solving the equations (4), (5), (6). i, j, k – number of a point on coordinates X, Y, t_ω. Uneven divide on longitudinal coordinate with condensation near an entrance to the channel is applied to reduce the machine time. The provision of points is defined by the formula: $X(i) = \frac{e^{ki} - 1}{e^{kn_i} - 1} X_0$, n_i – number of divide by X.

The length of settlement area X_0 is selected so it doesn't influence the results of calculations. The number of divide on coordinates X, Y, t_ω, equal 32×64×256, is consecutive doubling and control of difference of the received results.

L.M. Simuni method [14] is applied to find a dimensionless gradient of pressure of P entering the equation (4).

The Gauss-Seidel method is applied to solve the systems of linear equations. It is well suited for multi-core computing devices, since it does not require a sequential transition from point to point, which is necessary, for example, in the sweep method.

Integration on the section of the channel is carried out by Simpson method, and in time – method of trapezes. The order of accuracy of integration doesn't concede to the order of accuracy of differential approximation of the differential equations that allows to use machine resources effectively.

Averages in time values of a heat flow on a wall were determined by completion of calculation $\bar{Q} = \int_0^{2\pi}\left(\frac{\partial \vartheta}{\partial Y}\right)_{Y=Y_0} dt_\omega \Big/ 2\pi$ average mass temperature of liquid $\bar{\vartheta}_\mathrm{li}$ and Nusselt number $\overline{\mathrm{Nu}} = \bar{Q}/\overline{\Delta\vartheta}$, $\overline{\Delta\vartheta} = 1 - \bar{\vartheta}_\mathrm{li}$ – difference between temperature of a wall and average in time temperature of liquid.

For determination of average mass temperature of liquid it is necessary to write down the balance of thermal energy in the section of the channel $<\overline{U}>\overline{\vartheta_{li}}2\pi Y_0=\int\limits_0^{2\pi}\int\limits_0^{Y_0}U\vartheta dYdt_\omega$,

whence it follows $\overline{\vartheta_{li}}=\int\limits_0^{2\pi}\int\limits_0^{Y_0}U\vartheta dYdt_\omega \Big/ 2\pi Y_0$.

RESULTS OF CALCULATIONS

In fig. 3 there are results and verification of calculations of stationary heat transfer with a first kind boundary condition in laminar flow in the flat channel. In fig. 3(a) there is an increase in thickness and a closure of interfaces. Results are presented for completely stabilized and developing flows Pr = 0,7, 7, 70. When Pr > 1 the length of the initial hydrodynamic site is less than initial thermal site. Outside the initial hydrodynamic site, the development of heat transfer at pulsating laminar flow investigated in details in work [15]. In fig. 3(b) there is a change of temperature on length of the channel on an axis and in two other points of section of the channel. At small distances from an entrance with increase of number Pr the temperature near the wall increases, and decreases a little in the axe of the channel. It is explained by considerable influence of a cross component of velocity *V* near the entrance to the channel. In fig. 3(c) we observe how temperature of liquid and a thermal stream approach values for completely developed flow with increase the number Pr, without influence of axial heat conductivity that is explained by reduction of length of the initial hydrodynamic site, rather thermal, and decrease in influence of axial heat conductivity. The difference is observed only for a curve *1* as, despite the stabilized flow, the influence of axial heat conductivity is considerable. At large numbers Pr, Nusselt number changes as in stabilized flow that is presented in fig. 3(d). The developing hydrodynamics on the initial thermal site increases a thermal stream, temperature of liquid and Nusselt number.

In the quasi-steadystate regime of pulsations (S→0) in increase in amplitude of pulsations there is a significant increase $\Delta\overline{\vartheta}$ and $\overline{Q}$, and concerning its stationary values (fig. 4(a)) the value $\overline{Q}/Q_S$ considerably decreases near the entrance to the channel. It leads to decrease of Nusselt number on the whole length of the warmed site (fig. 4(b)). When S increases, the distance with reduction decreases and several lengths of a thermal wave fit on it. The value $\Delta\overline{\vartheta}/\Delta\vartheta_S$ increases and stabilizes on some case. As well as in the quasi-steadystate regime of pulsations $\overline{Q}/Q_S$, the value decreases acutely and then approaches $\Delta\overline{\vartheta}/\Delta\vartheta_S$.

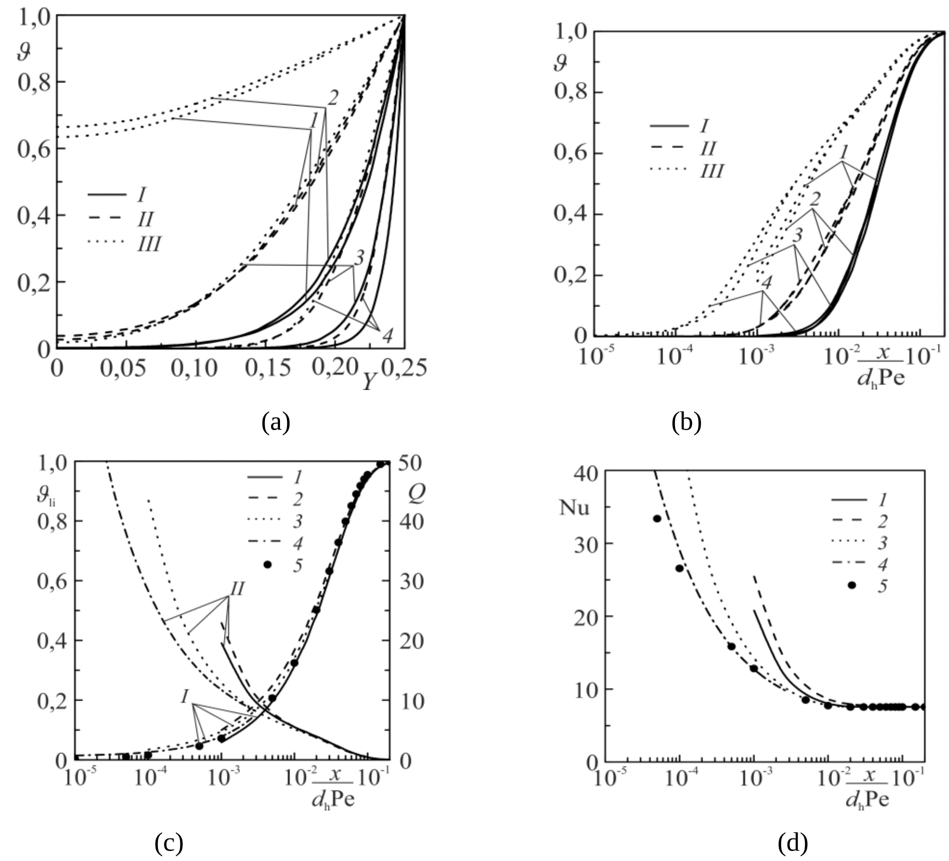
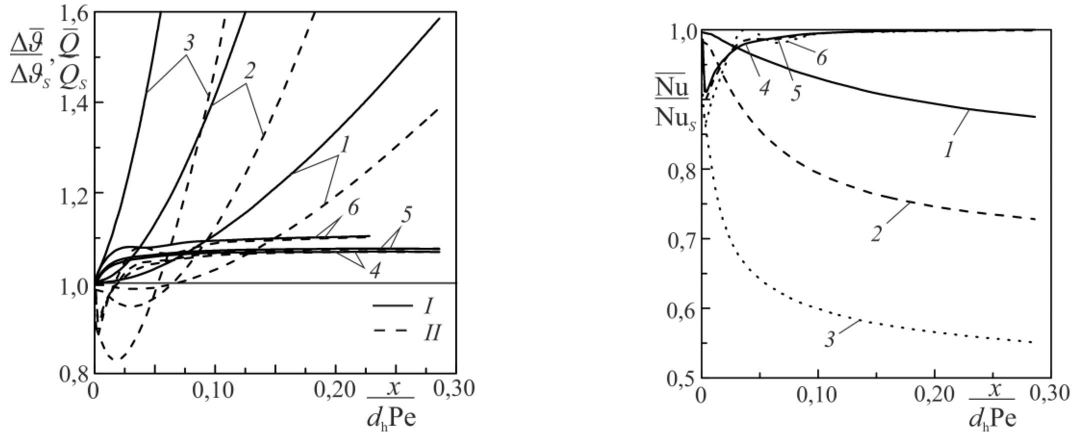


Figure 3: (a) – temperature profiles in sections of the channel (I – $x/(d_h\mathrm{Re})$ = 10^{-3}; II – $4{,}8\cdot10^{-3}$, III – 0,042), (b) – change of temperature on length of the channel (I – Y = 0; II – $0{,}5Y_0$, III – $0{,}75Y_0$), (c) – change of liquid temperature and thermal stream on length of the channel (I – $\vartheta_ж$; II – Q), (d) – change of Nusselt number on length of the channel. *1* – stabilized flow Pr = 0,7, *2* – developing flow Pr = 0,7, *3* – 7, *4* – 70, *5* – data [16].

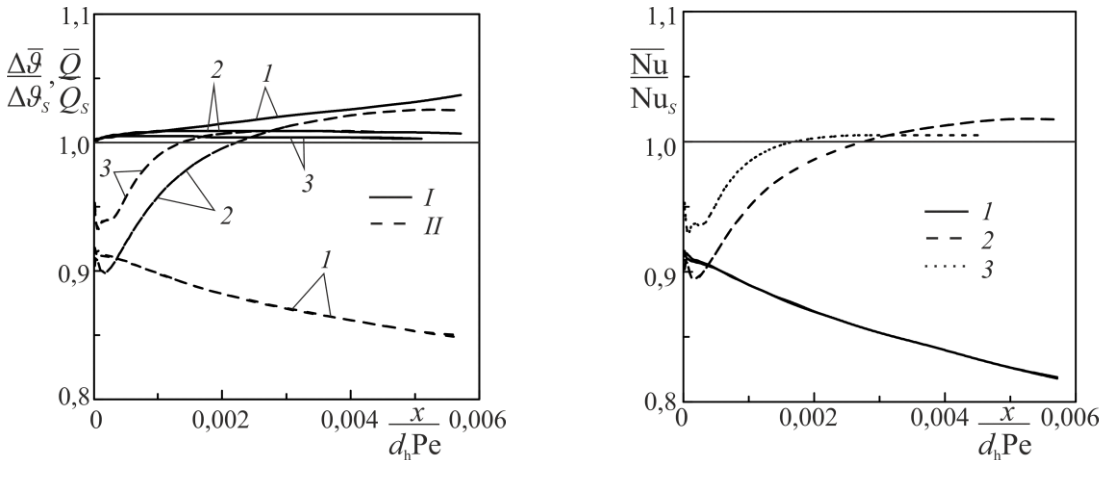


(a) (b)

Figure 4: change of tempereature average in time difference, a thermal stream (a) and Nusselt number (b) to their stationary values when Re = 1000 и Pr = 0,7. *1*, *2*, *3* – S → 0; *4*, *5*, *6* – S = 16; *1*, *4* – *A* = 0,25; *2*, *5* – *A* = 0,5; *3*, *6* – *A* = 1,0; *I* – $\Delta\overline{\vartheta}/\Delta\vartheta_S$, *II* – $\overline{Q}/Q_S$.

The length of the initial hydrodynamic site is much less than initial thermal site with large numbers Pr. The heat transfer isn't stabilized on the studied length (fig. 5).

In fig. 5 there are curves for Re numbers = 100 and 1000 which practically coincide. When Re > 100 and Pr > 1 number of Pe is very big and axial heat conductivity doesn't influence the decision. The increase of S, as well as when Pr = 0,7 leads to reduction of length on which it decreases, and at $S \to 0$ decreases on all length of the warmed site. There is a small increase $\overline{Q}/Q_S$, much more $\Delta\overline{\vartheta}/\Delta\vartheta_S$ on length of the channel. It happens because the temperature profile pulsating in time and changes on length of the channel. Nusselt number also strives for stationary value on big lengths $\overline{Q}/Q_S \approx \Delta\overline{\vartheta}/\Delta\vartheta_S$. The thermal Stokes number S_T increases with increase of Pr and length of a thermal wave decreases $\sim 1/S_T^2$.



(a) (b)

Figure 5: change of average temperatures in time of a difference, a thermal stream (a) and Nusselt number (b) to their stationary values when *A* = 1 and Pr = 35. *1* – S → 0; *2* – S = 4, *3* – 16.

CONCLUSION

1. The mathematical model and technique of numerical modeling of heat transfer for a first kind boundary condition at the pulsating developing laminar flow in the flat channel is created.
2. The heat transfer in developing pulsating laminar flow in the flat channel has its features. The value $\overline{Q}/Q_S$ near the entrance to the channel decreases, and then it increases for the same value as $\Delta\overline{\vartheta}/\Delta\vartheta_S$ at the distance from the entrance. It leads to reduction of Nusselt number, and then to approach to stationary value. Length

on which there is a reduction of Nussel number decreases with increase of dimensionless frequency of pulsations S.

When $S \to 0$ Nusselt number decreases on all length of the channel. Nusselt number decreases more when the amplitude of pulsations A increases.

Reynolds number doesn't influence the heat transfer at values more than 100 and Pr > 0,7.

The Increase of Prandtl number leads to the increase of Stokes thermal number, the reduction of length of a thermal wave and reduction of distance on which pulsations fade on length of the channel, and Nusselt number is close to stationary value.

3. Imposing of pulsations on an average flow in time leads to deterioration in a heat transfer near the entrance to the channel and to minor improvement on small removal when length of a thermal wave is less than length of the channel.

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