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Fuzzy Criticalpath Method with Hexagonal and Generalised Hexagonal Fuzzy Numbers Using Ranking Method

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Abstract

In this paper we introduce fuzzy project network with hexagonal fuzzy numbers and Generalized Hexagonal fuzzy numbers. The arithmetic operations on hexagonal fuzzy numbers are performed. Based on these hexagonal fuzzy numbers we obtain fuzzy critical path by ranking method. These procedures are illustrated with numerical example.

Keywords: Fuzzy set, Fuzzy numbers, hexagonal fuzzy numbers, generalized hexagonal fuzzy numbers, ranking function.

INTRODUCTION

The critical path method worked out at the beginning of the 1960's with the help of the critical path, the decision maker can adopt a better strategy of optimizing the time and the available resource to ensure the earliest completion and the quality of the project. In a practical situation of a production process a manufactures has to work out for a minimum investment with maximum realization. In this situation the fuzzy set theory is applied using the membership function since the values are not clear and defined due to uncertainties.

In fuzzy environment ranking fuzzy numbers is a very important in decision making procedure. Ranking fuzzy number is used mainly in data analysis, artificial intelligence and various other fields of operations research. Ranking fuzzy numbers were first proposed by Jain [1] for decision making in fuzzy situations by representing the ill-defined quality as a fuzzy set. Some of these ranking method have been compared and reviewed by Bortolan and Degani[2] and recently by Chen and Hwang[3],Cheng[4] used in centroid based distance methods to rank fuzzy numbers in 1998. Abbabandy and Asady[5] suggested a sign distance method for ranking fuzzy numbers in 2006.

Chen et al[6] has studies that there is not compulsion for membership fuction for normal fuzzy numbers. Phanibhusan et al[7] investicate distance method by using ranking of fuzzy numbers with centroid. Sahaya Sudha et al[8] proposed a new ranking on hexagonal fuzzy numbers. Rajarajeswari et al [9] proposed ordering of generalized fuzzy numbers area, mode and divagence, spread and solved with generalized hexagonal fuzzy numbers.

In this paper, we presents another approach which has not been proposed in the literature so far. We introduce fuzzy project network in which all parameters are hexagonal fuzzy numbers and apply to expected duration for each activity. To solve this procedure a suitable numerical example is illustrated.

PRELIMINARIES

Definition Fuzzy Set [FS]

A fuzzy set $\tilde{A}$ defined on the universal set of real numbers R is said to be a fuzzy numbers of its membership function has the following characteristics.

(a) $\mu_{\tilde{A}}$: R⟶ [0,1] is continuous

(b) $\mu_{\tilde{A}}(x) = 0$ for all $x \in [-\infty, a_1] \cup [a_4, \infty]$

(c) $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a_1, a_2]$ and strictly decreasing on $[a_3, a_4]$

(d) $\mu_{\tilde{A}}(x) = 1$ for all $x \in [a_2, a_3]$ where $a_1 \le a_2 \le a_3 \le a_4$.

Fuzzy Number.

A Fuzzy set A of the real line R with membership function $\mu_{\tilde{A}}(x)$: R ⟶ [0,1] is called fuzzy number.

If

a) $\tilde{A}$ must be normal and convex fuzzy set.

b) The support of $\tilde{A}$ must be bounded.

c) α .$\tilde{A}$ must be closed interval for every α in [0,1]

Generalized fuzzy number

A fuzzy set $\tilde{A} = (a,b,c,d,\omega)$ is defined on universal set of real number if its membership function has the following attributes

a) $\mu_{\tilde{A}}(x)$: R⟶ [0,1] is continues

(e) $\mu_{\tilde{A}}(x) = 0$ for all $x \in [-\infty, a] \cup [d, \infty]$

(f) $\mu_{\tilde{A}}(x)$ is strictly increasing on [a,b] and strictly decreasing on [c,d]

(g) $\mu_{\tilde{A}}(x) = \omega$ for all $x \in [b,c]$ where $0 < \omega \le 1$

Trapezoidal Fuzzy Number.

A fuzzy number $\tilde{A}$ is trapezoidal fuzzy number denoted by (a_1,a_2,a_3,a_4) and its membership function is given below

Where $a_1 \le a_2 \le a_3 \le a_4$.

$$\mu_{\tilde{A}}(x)=\begin{cases} \frac{x-a_1}{a_2-a_1}, & for\ a_1 \le x \le a_2 \\ 1, & for\ a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3}, & for\ a_3 \le x \le a_4 \\ 0, & othwise \end{cases} \quad (1)$$

Generalized Trapezoidal Fuzzy Number.

A generalized fuzzy number Ã= (a_1,a_2,a_3,a_4: w) is said to be generalized trapezoidal fuzzy number if its membership fuction is given by

$$\mu_A(x)=\begin{cases} w\left(\frac{x-a}{b-a}\right) a_1 \le x \le a_2 \\ w, \quad a_2 \le x \le a_3 \\ w\left(\frac{a_4-x}{a_4-a_3}\right), \ a_3 \le x \le a_4 \end{cases} \quad (2)$$

HEXAGONAL FUZZY NUMBERS

Definition

A fuzzy number F_H is a hexagonal fuzzy number denoted by $\tilde{F}_H$=(h_1,h_2,h_3,h_4,h_5, h_6) are real numbers and its membership function is given below

$$\mu\tilde{F}_H(x) = \begin{cases} \frac{1}{2}\left(\frac{x-h_1}{h_2-h_1}\right), & for\ h_1 \le x \le h_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-h_2}{h_3-h_2}\right), & for\ h_2 \le x \le h_3 \\ 1, & \text{for } h_3 \le x\, h_4 \\ 1-\frac{1}{2}\left(\frac{x-h_4}{h_5-h_4}\right., & for\ h_4 \le x \le h_5 \\ \frac{1}{2}\left(\frac{h_6-x}{h_6-h_5}\right), & for\ h_5 \le x \le h_6 \\ 0, & othwise \end{cases} \quad (3)$$

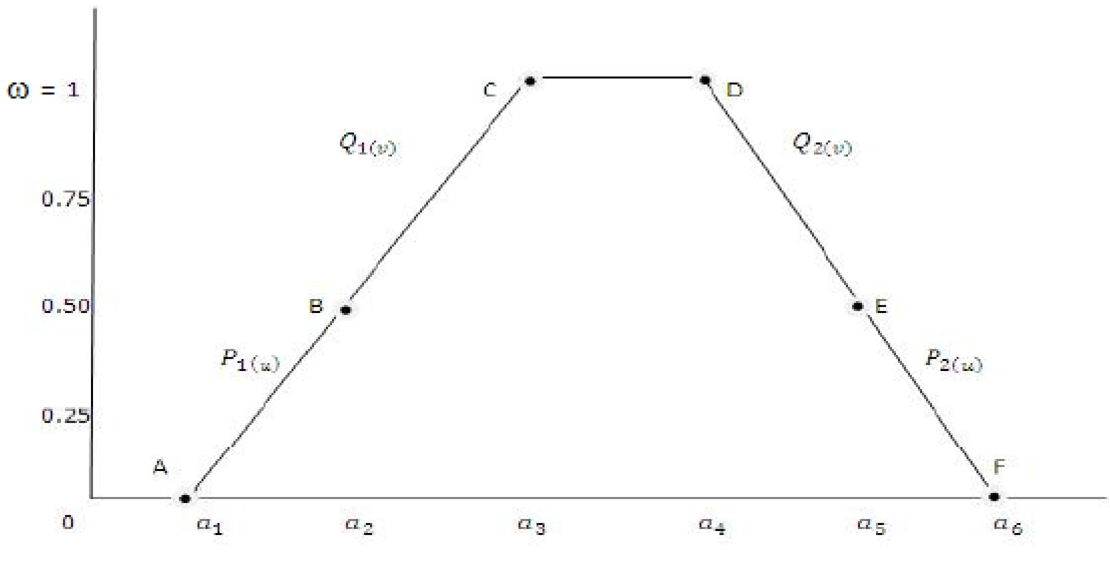


Figure 1. Graphical representation of a HFN

Definition

An HFN denoted by $\tilde{F}w$ is defined as

$\tilde{F}w$= ($P_1(v)$,$Q_1(v)$, $Q_2(v)$,$P_2(v)$) for v€ [0,0.5] and v€[0.5,w] having following characteristics

a) $P_1(v)$ is a bounded left continuous non decreasing function over [0,0.5]

b) $Q_1(v)$ is a bounded left continuous non decresing function over $[0.5, w]$

c) $Q_2(v)$ is bounded continuous non increasing function over$[w,0.5]$

d) $P_2(v)$ is increasing function a bounded left continuous non increasing function over $[0.5,0]$

Remark

In $\tilde{F}w$, w represents the maximum membership value if w=1 then the Hexagonal fuzzy number is called a normal Hexagonal Fuzzy Number.

Arithmetic operation on Hexagonal fuzzy numbers

If $\tilde{F}_H = (h_1,h_2,h_3,h_4,h_5,h_6)$, $\tilde{G}_H = (g_1,g_2,g_3,g_4,g_5,g_6)$ are two HFN's [11][12][13], then the following three operations can be performed as follows

Addition:

$\tilde{F}_H+\tilde{G}_H = (h_1+g_1,h_2+g_2,h_3+g_3,h_4+g_4,h_5+g_5,h_6+g_6)$

Subtraction:

$\tilde{F}_H-\tilde{G}_H = (h_1-g_6,h_2-g_5,h_3-g_4,h_4-g_3,h_5-g_2,h_6-g_1)$

Multiplication:

$\tilde{F}_H*\tilde{G}_H = (h_1*g_1,h_2*g_2,h_3*g_3,h_4*g_4,h_5*g_5,h_6*g_6)$

Magnitude of HFN

The Magnitude of a HFN [13] $\tilde{F}_H = (h_1,h_2,h_3,h_4,h_5,h_6)$ is defined as

$$\text{Mag}(\tilde{F}_H) = \left(\frac{2h1+3h2+4h3+4h4+3h5+2h6}{18}\right) \quad (4)$$

RANKING OF GENERALIZED HEXAGONAL FUZZY NYMBERS [14]

For Ranking of fuzzy numbers, we have to rank a more numbers define the ranking function of generalized hexagonal fuzzy numbers and solved by numerical examples.

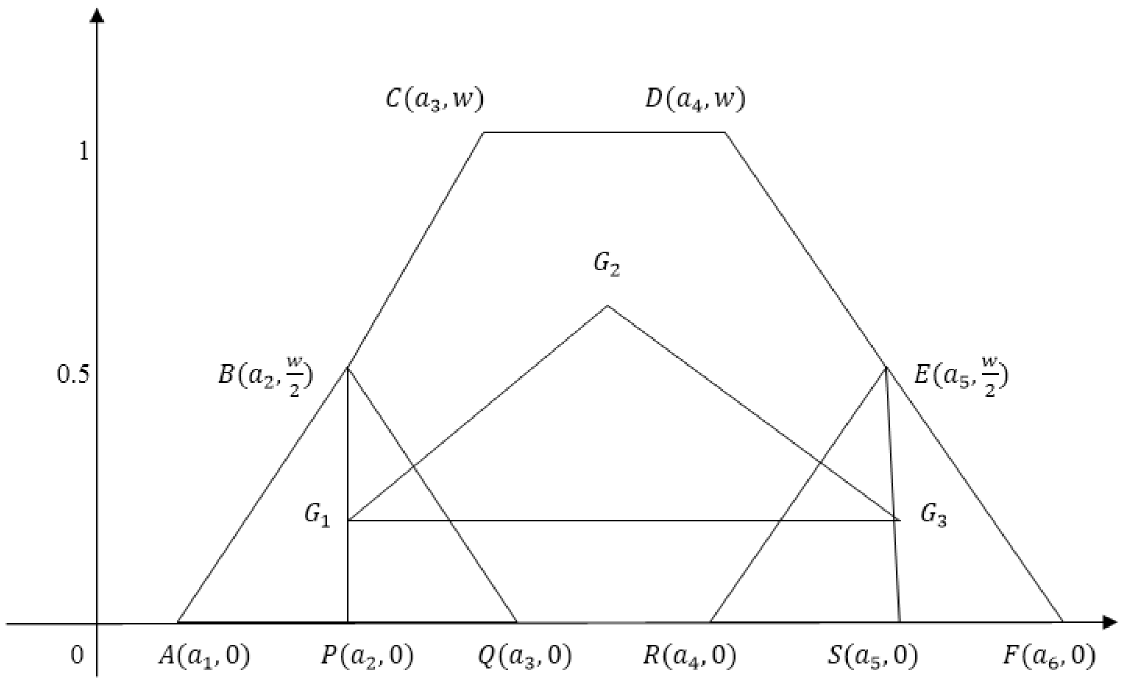


Figure 2. Generalized hexagonal fuzzy numbers

The centroid of a fuzzy number is considered to be the balancing point of the hexagon. Divide the hexagonal into three plane figures. These three plane figures are a triangle ABQ, Hexagon CDERQB and again a triangle REF respectively. The circumcenter of the centroids of these three plane figures is taken as the point of reference to define the ranking of generalized hexagonal fuzzy numbers. Let the centroid of three plane figures be G_1,G_2,G_3 resply.

The centroid of the three plane figures is

$G_1=(\frac{a_1+a_2+a_3}{3},\frac{W}{6})$, $G_2=(\frac{a_2+2a_3+2a_4+a_5}{6},\frac{W}{2})$,

$G_3=(\frac{a_4+a_5+a_6}{3},\frac{W}{6})$

Equation of the line G_1,G_3 is $y=\frac{w}{6}$ and G_2 does not lie on the line G_1,G_3. Thus G_1,G_2 and G_3 are non collinear and they form a triangle.

The ranking function of the generalized hexagonal fuzzy number $A_H=(a_1,a_2,a_3,a_4,a_5,a_6;\ w)$ which maps the set of all fuzzy numbers to a set of real numbers.

$$R(\tilde{A}_H)=(x_0)(y_0)=(\frac{2a_1+3a_2+4a_3+4a_4+3a_5+2a_6}{18}\ X\ \frac{5w}{18}) \qquad (5)$$

PROCEDURE FOR FUZZY CRITICAL PATH ALGORITHM

Let $\tilde{E}S_i$ and $\tilde{L}S_i$ be the earliest fuzzy event time, and the latestfuzzy event time for event i, respectively. Functions that define the earliest starting times, latest starting times and floats in terms of fuzzy activity durations are convex, normal whose membership functions are piecewise continuous, hence the quantities such as earliest fuzzy event time $\tilde{E}S_i$, the latest fuzzy event time $\tilde{L}S_i$ and the floats $\tilde{T}$ are also trapezoidal fuzzy numbers for an event i respectively.

Step 1: Identify Fuzzy activities in a fuzzy project.

Step 2: Establish precedence relationships of all fuzzy activities by applying a fuzzy ranking function.

Step 3: Construct the fuzzy project network with trapezoidal fuzzy numbers as fuzzy activity times

Step 4: Let $\tilde{E}S_i$ be the earliest fuzzy event time and $\tilde{L}S_i$ be the latest fuzzy event time for the initial event $\tilde{v}_1$ of the project network and assume that $\tilde{E}S_i=\tilde{L}S_i=\tilde{0}$. Compute the earliest fuzzy event time $\tilde{E}S_j$ of the event $\tilde{v}_j$ by using the formula

$$\tilde{E}S_j=\max_{i\in N:i\to j}\{\tilde{E}S_j+\tilde{t}_{ij}\} \qquad (i)$$

Step 5: Let $\tilde{E}S_n$ be the earliest fuzzy event time and $\tilde{L}S_n$ be the latest fuzzy event time for the terminal event $\tilde{v}_n$ of the fuzzy project network and assume that $\tilde{E}S_n=\tilde{L}S_n$. Compute the latest fuzzy event time $\tilde{L}S_i$ by using the following equation

$$\tilde{L}S_i=\min_{i\in N}\{\tilde{L}S_j-\tilde{t}_{ij}\} \qquad (ii)$$

Step 6: Compute the total float $\tilde{T}_{ij}$ of each fuzzy activity $\tilde{a}_{ij}$ by using the following equation.

$$\tilde{T}F_{ij}=\{\tilde{L}S_j-\tilde{E}S_i\} \qquad (iii)$$

Hence we can obtain the earliest fuzzy event time, latest fuzzy event time, and the total float of every fuzzy activity by using equations (i), (ii) and (iii).

Step 7: If $\tilde{T}_{ij}=0$, then the activity $\tilde{a}_{ij}$ is said to be a Fuzzy critical activity. That is activities with zero total float are called Fuzzy critical activities, and are always found on one or more Fuzzy critical paths.

Step 8: The length of the longest Fuzzy critical path from the start of the fuzzy project to its finish is the minimum time required to complete the Fuzzy Project. This (or these) Fuzzy critical path(s) determine the minimum fuzzy project duration.

Numerical Example:

Suppose that there is a project network with the set of fuzzy events $\tilde{}$ = {1, 2, 3, 4, 5, 6, 7, } the fuzzy activity time for each activity is shown in table 1(all duration are in hours)

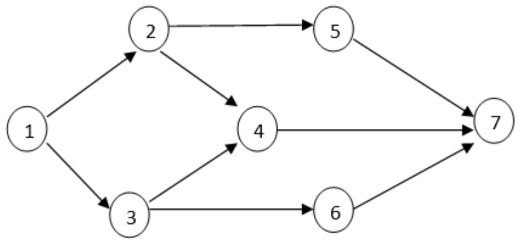


Figure 3. Fuzzy Project network

Table 1: Fuzzy Activity duration of each activity in ex1.

Activity	Fuzzy activity time
1-2	(3,7,11,15,19,24)
1-3	(3,5,7,9,10,12)
2-4	(11,14,17,21,25,30)
3-4	(3,5,7,9,10,12)
2-5	(5,7,10,13,17,21)
3-6	(7,9,11,14,18,22)
4-7	(7,9,11,14,18,22)
5-7	(2,3,4,6,7,9)
6-7	(5,7,8,11,14,17)

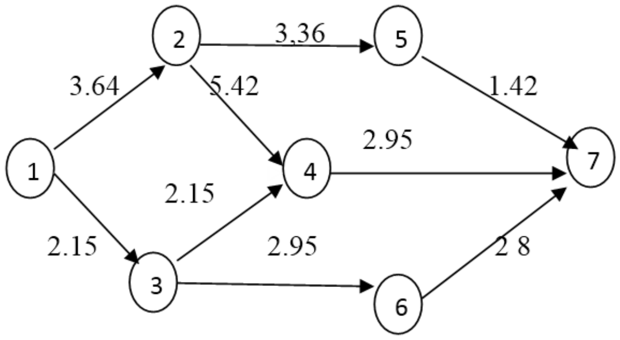
Description of the Model

Hexagonal fuzzy numbers are converted into expected time (normal time) by using magnitude of hexagonal fuzzy numbers. These expected time treated as the time between the nodes and fuzzy critical path is calculated by using conventional method.

Table 2: Calculation of Expected time in each activity of fuzzy project network and identify the critical path

Activity	Fuzzy activity time (hexagonal fuzzy no's)	Fuzzy activity time converted in to Expected time (by Gen $R(F_H)$
1-2	(3,7,11,15,19,24)	3.64
1-3	(3,5,7,9,10,12)	2.15
2-4	(11,14,17,21,25,30)	5.42
3-4	(3,5,7,9,10,12)	2.15
2-5	(5,7,10,13,17,21)	3.36
3-6	(7,9,11,14,18,22)	2.95
4-7	(7,9,11,14,18,22)	2.95
5-7	(2,3,4,6,7,9)	1.42
6-7	(5,7,8,11,14,17)	2.82

By applying expected time in the given numerical example. The revised form of the project network shown below



From the above network we can identify the critical path as follows

Table 2. Result of Critical path

Path	Project completion time
P_1:1-2-5-7	8.42
P_2:1-2-4-7	12.01
P_3:1-3-4-7	7.25
P_4: 1-3-6-7	7.92

Here path P_2 : 1-2-4-7 is identified as fuzzy critical path.

Table 3: Calculation of Expected time in each activity of fuzzy project network in the numerical example and identify the critical path

Activity	Fuzzy activity time (hexagonal fuzzy no's)	Fuzzy activity time converted in to expected time by Magnitude of HFN
1-2	(3,7,11,15,19,24)	13.11
1-3	(3,5,7,9,10,12)	7.72
2-4	(11,14,17,21,25,30)	19.5
3-4	(3,5,7,9,10,12)	7.72
2-5	(5,7,10,13,17,21)	12
3-6	(7,9,11,14,18,22)	13.28
4-7	(7,9,11,14,18,22)	13.28
5-7	(2,3,4,6,7,9)	5.11
6-7	(5,7,8,11,14,17)	10.17

By applying expected time in the given numerical example. The revised form of the project network shown below

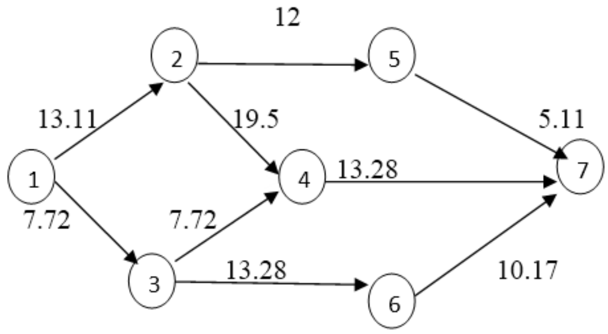


Figure 5. Project Network 3

From the above network we can calculate and identify the critical path as follows

Table 4. Result of Critical path

Path	Length of the Project
P_1:1-2-5-7	30.22
P_2:1-2-4-7	45.89
P_3:1-3-4-7	28.72
P_4: 1-3-6-7	31.17

Here path P_2 : 1-2-4-7 is identified as fuzzy critical path.

CONCLUSION

In this paper fuzzy critical path method were introduced by ranking of fuzzy numbers. We executed numerical example and found satisfactory result. Here ranking method(Magnitude) used to solve hexagonal fuzzy numbers with centroid ranking techniques. Both the method critical paths are same. However the result obtained in this paper is generalized ranking method has less completion time as compared to the Magnitude of hexagonal fuzzy numbers.

REFERENCES

[1] Jain, R. , 1976, "Decision Making in the Presence of Fuzzy Variables," IEEE Transactions on systems, Man and Cybernetics, 6, pp. 698-703.

[2] Bortolan. G and Degani. R," A review of some methods for ranking fuzzy subsets, Fuzzy set and system" 15(1985) 1-19

[3] Chang. C.H, " A new approach for ranking fuzzy numbers by distance method, Fuzzy set and System"Vol 95(1998),307-316.

[4] C.H.Cheng, A new approach for ranking fuzzy

numbers by distance method, *Fuzzy sets and system s,* 95 (1998) 307-317.

[5] S. Abbasbandy, B.Asady, Ranking of fuzzy numbers by sign distance, *Information Science,* 176 (2006) 2405-2416.

[6] Chen, S. , 1985, "Operations on Fuzzy Numbers with Function Principal," Tamkang, Journal of management sciences, 6, pp. 13-25.

[7] Phanibhushan, R. , and Ravishankar, N. , 2011, "Ranking Fuzzy Numbers with Distance Method Using Circumcentre of Centroids and an Index of Modeling, "Advances in fuzzy system, 10, pp. 1155-1161.

[8] Sahaya Sudha.A and Revathy.M ," A new ranking on Hexagonal fuzzy numbers, International Journal of fuzzy logic system volume 6, No:4 (Oct 2006)

[9] Rajarajeshwari, P. , and Sudha, S. , 2014, "Ordering Generalized Hexagonal Fuzzy Numbers Using Rank, Mode, Divergence and Spread," IOSR Journal Mathematics, 10, 3, pp. 15-22.

[10] Kaufmann A, Gupta .M.M, "Fuzzy Mathematical models in engineering and management science, Elsever Science Publishers, Netherlands,1988

[11] A.Bansal, "Some non linear arithmetic operations on triangular fuzzy numbers (m,B, a)", *Advances in fuzzy mathematics*, 5,pp.147-156, 2011.

[12] D. Dubois and H. Prade, Possibility Theory: An Approachto Computerized Processing of Uncertainty, New York: Plenum, 1988.

[13] Thamaraselvi. A and Shanthi. R "Optimal solution of fuzzy TP using Hexagonal fuzzy numbers", International Journal of scientific and Engineering Research, Vol 6,issue 3 , March 2015.

[14] Ghadle, K. , and Pathade, P. , 2017, "Optimal Solution of balanced and Unbalanced Fuzzy Transportation Problem by Using Octagonal Fuzzy Numbers" IJPAM (Bulgaria). (Accepted)