

MHD Flow through Vertical Channel with Porous Medium

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Abstract

MHD flow of fluids through vertical porous channel having porous medium placed in magnetic field is of industrial importance, therefore in this paper study has been made on the flow of a viscous incompressible, electrically conducting fluid through a vertical channel filled with porous channel, the field is applied perpendicular to the direction of flow. After forming the governing equation under suitable boundary conditions, a solution for velocity has been derived and taking different values of parameters, a graph has been plotted between velocity and applied magnetic field. Interference has been drawn to check the obtained results with physical nature of the problem. The study is useful in applying the result in industrial field.

Keyword: MHD, Porous medium, Magnetic field, Vertical Channel, Hall Parameter.

INTRODUCTION

Earlier so many studies have been done on MHD topics. In this paper, I have been taken a vertical porous channel having porous medium, an inclined magnetic field of uniform strength is applied with the parameter H_0 . Many researchers did work in the field of radiative convective flows. Radiation has large uses in physics, industrial processing for example heating and cooling of chambers and space vehicle re-entry. Manglesh and Gorla discussed here MHD free convective flow through porous medium in the presence of Hall current, radiation and thermal diffusion; I am extended the work of both above. We cannot neglect the effect of Hall current when uniform inclined magnetic field is strong. The growing interest in the study of hydro dynamical problems gets modified by Hall current. Mangles discussed here about the MHD free convective flow through porous medium in the presence of Hall current, radiation and thermal diffusion in this they studied here the effects of various parameters on velocity, temperature, concentration profiles with graphical study taking uniform magnetic field, here I extended the work of Mangles by taking inclined magnetic field by adding parameter $H_0^2 \sin^2 \alpha$ at the place of H_0^2 . Ohm's law here

applied. This study has been lots of industrial uses as above described.

FORMULATION OF THE PROBLEM

Consider the MHD flow through vertical porous channel filled with porous medium having inclined magnetic field. The vertical porous plates placed with distance d apart. X-axis and Y-axis taken as vertically upward and perpendicular to the wall of the channel respectively. There are two dimensional flow analysis occur. Injection and suction velocity assumed here constant v_0 and magnetic field inclination is taken with y-axis with angle $\sin^2 \alpha$. The permeability K_p^* of porous medium took constant, the intensity of magnetic field H_0 and all the quantities depends upon y and t only where y is layer distance and t is time.

The equation of continuity $\nabla \cdot \vec{v}$ on integration gives $v^* = v_0$ where u^* , v^* and w^* are component of velocities in x, y, z-directions respectively. j_x , j_y and j_z are component of \vec{j} . $j_y = \text{constant}$ by equation of conservation of electric charge so plates are electrically non-conducting $j_y = 0$ and it is zero throughout the flow. When the magnetic field is large, then generalized Ohm's law are as follows

$$j_x^* - \omega_e \tau_e j_z^* = -\sigma \mu_e H_0 \sin^2 \alpha \omega^* \quad (2.1)$$

$$j_z^* + \omega_e \tau_e j_x^* = \sigma \mu_e H_0 \sin^2 \alpha \omega^* \quad (2.2)$$

Using (2.1) and (2.2) we find

$$j_x^* = \frac{\sigma \mu_e H_0 \sin^2 \alpha (m u^* - \omega^*)}{1+m^2} \quad (2.3)$$

$$j_z^* = \frac{\sigma \mu_e H_0 \sin^2 \alpha (u^* + m \omega^*)}{1+m^2} \quad (2.4)$$

Where $m = \omega_e \tau_e$ is a hall parameter, ω_e : frequency, τ_e : electron collision time, μ_e : magnetic permeability, σ : electrical conductivity of the fluid.

Now Boussinesq's approximation taken here and the governing equations for momentum, energy and concentration are as follows

$$\frac{\partial u^*}{\partial t^*} + v_0 \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho} \frac{\partial P^*}{\partial x^*} + \nu \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta T^* + g\beta^* C^* - \frac{\sigma \mu_0^2 H_0^2 \sin^2 \alpha (u^* + m\omega^*)}{\rho(1+m^2)} - \frac{v \cdot u^*}{k_p^*} \quad (2.5)$$

$$\frac{\partial u^*}{\partial t^*} + v_0 \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho} \frac{\partial P^*}{\partial x^*} + \nu \frac{\partial^2 \omega^*}{\partial y^{*2}} + \frac{\sigma \mu_0^2 H_0^2 \sin^2 \alpha (m\omega^* - \omega^*)}{\rho(1+m^2)} - \frac{v \cdot \omega^*}{k_p^*} \quad (2.6)$$

$$\frac{\partial T^*}{\partial t^*} + v_0 \frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p} \frac{\partial q}{\partial y^*} \quad (2.7)$$

$$\frac{\partial C^*}{\partial t^*} + v_0 \frac{\partial C^*}{\partial y^*} = D_m \frac{\partial^2 C^*}{\partial y^{*2}} + D_T \frac{\partial^2 T^*}{\partial y^{*2}} \quad (2.8)$$

With boundary equations are

$$y^* = \frac{-d}{2}, u^* = \omega^* = 0, T^* = 0, C^* = 0$$

$$y^* = \frac{d}{2}, u^* = \omega^* = 0, T^* = T_0 \cos \omega^* t^*, C^* = C_0 \cos \omega^* t^* \quad (2.9)$$

Where

Where

*: Represents the dimensional quantity.

ν : Kinematics Viscosity of fluid.

t^* : Time.

ρ : Density of fluid.

p^* : Pressure of fluid.

T^* : Temperature of fluid.

H_0 : Intensity of magnetic field applied over channel.

C_p : Specific heat at constant pressure.

κ : Thermal conductivity of fluid.

g : Acceleration due to gravity.

β : Volumetric coefficient of thermal expansion.

C^* : Concentration of fluid.

β^* : Volumetric coefficient of thermal expansion with concentration.

D_m : Chemical molecular diffusivity.

D_T : Thermal diffusivity

$\frac{\partial q}{\partial y^*} = 4a^2 T^*$ where 'a' is mean radiation absorption coefficient.

$$u = \frac{u^*}{v_0}, \theta = \frac{T^*}{T_0}, \omega = \frac{\omega^*}{v_0}, x = \frac{x^*}{d}, y = \frac{y^*}{d},$$

$$t = \frac{t^* v}{d^2}, P = \frac{P^* d}{v_0 \mu}, \omega = \frac{\omega^* d^2}{v}, \lambda = \frac{v_0 d}{\nu}, Re = \frac{U d}{\nu}, C = \frac{C^*}{C_0}, Gr =$$

$$\frac{g\beta T_0 d^2}{v v_0}, Gm = \frac{g\beta^* C_0 d^2}{v v_0}, M = \frac{\sigma H_0^2 \sin^2 \alpha \mu_0^2 d^2}{\mu}, N = \frac{2ad}{\sqrt{\kappa}}, Pr = \frac{\mu C_p}{\kappa},$$

$$Sc = \frac{v}{D_m}, K_0 = \frac{K_0 U}{dv}, S_0 = \frac{D_T T_0}{v C_0}$$

Where

ω : Frequency parameter.

λ : suction parameter.

G_r : Grashoff number.

G_m : Modified Grashoff number.

M : Hartman number.

P_r : Prandlt number.

N : Radiation Parameter.

S_o : Soret number.

S_c : Schmidt number.

Using above non-dimensional parameters we get-

Velocity equations are

$$\frac{\partial u}{\partial t} + \lambda \frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \frac{M(u+m\omega)}{(1+m^2)} + G_r \theta + G_m C - \frac{u}{k_p} \quad (2.8)$$

and

$$\frac{\partial \omega}{\partial t} + \lambda \frac{\partial \omega}{\partial y} = -\frac{\partial P}{\partial y} + \frac{\partial^2 \omega}{\partial y^2} + \frac{M(m\omega - \omega)}{(1+m^2)} - \frac{\omega}{k_p} \quad (2.9)$$

Temperature equation

$$\frac{\partial \theta}{\partial t} + \lambda \frac{\partial \theta}{\partial y} = -\frac{N^2}{Pr} \theta + Pr \frac{\partial^2 \theta}{\partial y^2} \quad (2.10)$$

Concentration equation

$$\frac{\partial C}{\partial t} + \frac{\partial C}{\partial y} = S_o \frac{\partial^2 \theta}{\partial y^2} + \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} \quad (2.11)$$

With Boundary condition

$$u = \omega = \theta = C = 0 \text{ at } y = -\frac{1}{2}$$

$$u = \omega = 0, \theta = C = \cos \omega t \text{ at } y = \frac{1}{2} \quad (2.12)$$

SOLUTION

For oscillatory internal flow in the channel we shall assume the fluid also flows under the influence of non-dimensional pressure gradient oscillating in the direction of x-axis only which is of the form -

$$-\frac{\partial p}{\partial x} = A \cos \omega t, \frac{\partial p}{\partial y} = 0, \text{ where } A \text{ is constant.}$$

Where the complex velocity $F = u + i\omega$, we find that equation (2.9) and (2.10) can be ombined in to a single form

$$\frac{\partial F}{\partial t} + \lambda \frac{\partial F}{\partial y} = A \cos \omega t + \frac{\partial^2 F}{\partial y^2} - \frac{M(1-i\omega)F}{(1+m^2)} + G_r \theta + G_m C - \frac{F}{k_p} \quad (2.13)$$

With transformed boundary conditions

$$u = \omega = \theta = C = 0 \text{ at } y = -\frac{1}{2}$$

$$u = \omega = 0, \theta = C = 1, \text{ at } y = \frac{1}{2} \quad (2.14)$$

To solve the equations (2.13), (2.11), (2.10) under the boundary condition (2.14) assume the solution is of the form

$$U(y, t) = U_o(y)e^{i\omega t}$$

$$\theta(y, t) = \theta_o(y)e^{i\omega t}$$

$$C(y, t) = C_o(y)e^{i\omega t} \quad (2.15)$$

Substituting equations (2.15) in equation (2.13), (2.10), (2.11) we get

$$U_o'' - \lambda U_o' - I^2 U_o = -A - G_r \theta_o - G_m C_o \quad (2.16)$$

$$C_o'' - \lambda C_o' - i\omega S_c C_o = -S_c S_o \theta_o'' \quad (2.17)$$

$$\theta_o'' - \lambda P_r \theta_o' - (i\omega P_r + N^2) \theta_o = 0 \quad (2.18)$$

Now the solutions of above equations are

$$\theta_o(y) = (A_o e^{a_0 y} + B_o e^{b_0 y}) \cdot e^{i\omega t} \quad (2.19)$$

$$C_o(y) = (A_3 e^{a_1 y} + B_3 e^{b_1 y} + A_1 e^{a_0 y} + B_1 e^{b_0 y}) \cdot e^{i\omega t} \quad (2.20)$$

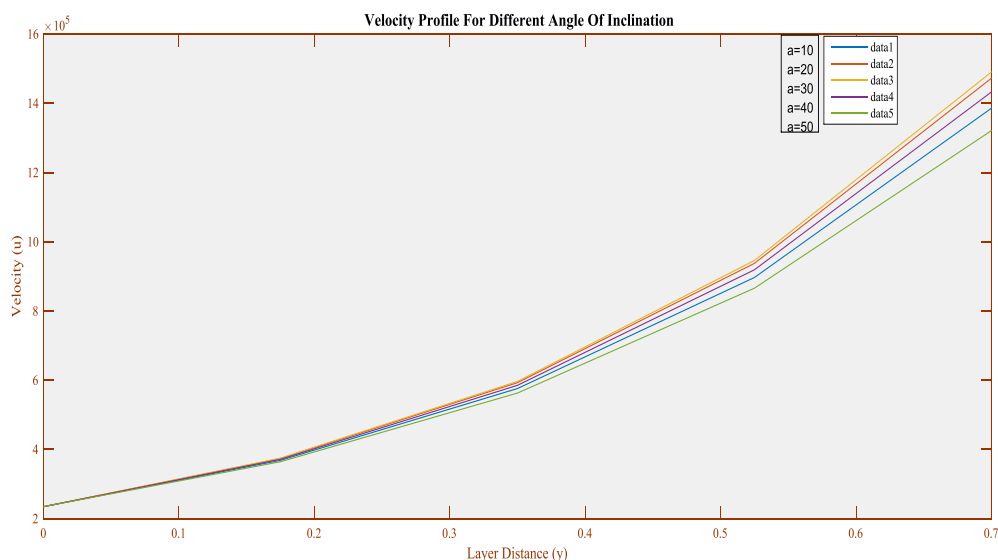
$$F_o(y) = (A_5 e^{a_2 y} + B_5 e^{b_2 y} + \frac{A}{I^2} - A_3 e^{a_0 y} - B_3 e^{b_0 y} - A_4 e^{a_1 y} - B_4 e^{b_1 y}) \cdot e^{i\omega t} \quad (2.21)$$

Arbitrary parameters values are chosen here to draw graph.

Table 1. Velocity profile for different angle of inclination

(Pr=0.75, A=2.2, So=2, λ=0.5, Cp=1.5, d=1, Ho=12, μ=0.1, μ_e=0.5, Gm=3, Gr=2.2, K_p=1, ω=3)

Sr. No.	Layer distance (y)	Velocity of fluid (u)				
		Hartman no. (M=1.03)	Hartman no. (M=1.73)	Hartman no. (M=1.87)	Hartman no. (M=1.41)	Hartman no. (M=1.49)
		Angle (a=10)	Angle (a=20)	Angle (a=30)	Angle (a=40)	Angle (a=50)
1	0.000	0.2347	0.2346	0.2346	0.2346	0.2348
2	0.175	0.3685	0.3738	0.3748	0.3714	0.3645
3	0.350	0.5759	0.5929	0.5964	0.5852	0.5629
4	0.525	0.8967	0.9376	0.9460	0.9190	0.8658
5	0.700	1.3857	1.4723	1.4902	1.4327	1.3210



RESULT AND DISCUSSION

In order to get physical understanding of the problem we draw a graph here between layer distance (y) and velocity of fluid (u) for different angle of inclination with the plate associated with magnetic field. Here we find the result which shows in graph when the angles of inclination with plate are increases; velocity also increases with the increase of Layer distance (y). The curvatures of the graph are started with the same (approximately) initial values after that they scatter for more values of ' y '. Both the intensity of magnetic field and inclination of field control the fluid velocity. This is concluded that this study has been used for industrial applications and it is highly significant for the effects of different parameters on velocity profile. This study can be further extended for other parameters and various shaped channels.

REFERENCES

- [1] Ahmed N and Das KK, Unsteady MHD mass transfer flow past a suddenly moving vertical plate in a porous medium in rotating system, *Int. journal of engineering sciences and technology* 5, 1906-1923(2013).
- [2] Aiyesimi YM, Okedayo GT and Lawal OW, MHD flow of a dusty viscoelastic fluid through a horizontal circular channel, *Academic journal of scientific research* 1(3):056-062(2013).
- [3] Bejan A and Khair KR, Heat and mass transfer by natural convection in a porous medium, *Int. J. Heat Mass*, 28, pp.909-918 (1985).
- [4] Chamkha Ali J, The heat and mass transfer from MHD flow over a moving permeable cylinder, *International Scientific Publication and Consulting Services Article ID cna-00109*, 20 pages(2011).
- [5] Chaudhary, RC and Jain A, Combined heat and mass transfer effects on MHD free convection flow past an oscillating plate embedded in porous medium, *Rom.J.physics*.52,505-524(2007).
- [6] Chand Khem, Kumar Dinesh and Kumar Sanjeev, MHD flow of radiating and chemically reacting viscoelastic fluid through porous medium, *International Journal of engineering science invention* volume 2(2013).
- [7] Das KS, Jana M and Jana RN, Radiation effect on natural convection near a vertical plate embedded in porous medium with ramped wall temperature, *Open Journal Fluid Dynamics*1, 1-11(2011).
- [8] Gireesha BJ, Madhura KR and Bagewati CS, Flow of a unsteady dusty fluid through porous medium in a uniform pipe, *International Journal of pure and applied mathematics* volume 76,no.1,29-47(2011).
- [9] Khan S Kumar and Sanjyanand E, Viscoelastic boundary layer MHD fluid through porous medium over a stretching sheet, *International journal of engineering and science invention* volume 2, pp-97-108(2004).
- [10] Manglesh Arti and Gorla MG, MHD free convective flow through porous medium in the presence of hall current radiation and thermal diffusion, *Indian journal pure and applied mathematics*,44(6):743-756(2012).
- [11] Lai FC and Kulacki FA, Coupled heat and mass transfer from a sphere buried in an infinite porous medium, *Int. J. Heat Mass Transfer*, 33, pp.209-215(1990).
- [12] Singh AK, MHD free convective flow in Stokes problem for a porous vertical plate, *Astrophysics space science*, 89(1982), 455-466.