International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 17 (2018) pp. 13022-13030

Stability of Thermosolutal Free Convection on Electrically Conducting Superposed Fluid and Porous Layer

K.Sumathi[1*], T.Arunachalam[2] and S.Aiswarya[3]

Department of Mathematics,
[1,3]PSGR Krishnammal College for Women, Coimbatore – 641 004, TamilNadu, India.
[2]Kumaraguru College of Technology, Coimbatore – 641 049, TamilNadu, India.
*(*Corresponding Author)*

Abstract

The effect of thermosolutal convection on linear stability of an incompressible fluid in a horizontal fluid/porous interface in the presence of a vertical magnetic field has been analyzed. Asymptotic solution is obtained for rigid boundaries using normal mode analysis and the analysis is restricted to long wave approximations. The influence of various non-dimensional parameters such as Chandrasekhar number, magnetic Prandtl number, Schmidt number, Prandtl number, porosity, wave number and depth ratio on stability characteristics of flow field are represented numerically.

Keywords: Thermosolutal convection, magnetic field, fluid-porous interface, stability analysis.

INTRODUCTION

The combined effect of thermosolutal convection occurs in many practical situations. Nield [8] analyzed the linear stability of thermohaline convection induced by thermal and concentration gradients in a horizontal saturated porous medium by applying Fourier series. The effect of chemical reaction on stability of a binary mixture in a porous medium heated from below or above has been studied by Steinberg [9].

Gobin *et al.* [3], Zhao and Chen [12], Hirata *et al.*[5] analyzed the combined effect of thermal and solutal gradients on stability of fluid layer overlying a porous layer which is of great importance in thermal insulation, drying processes, ground water analysis etc.

Murthy and Lee [7] investigated the onset of convection due to temperature and solute concentration differences at the walls using linear stability analysis. The effect of magnetic field on onset of thermosolutal convection in a viscous incompressible fluid which is heated and soluted at both the walls has been examined by Abdullah [1] and he found that variable magnetic Prandtl number has no effect on stationary convection but induces overstability of the system by applying Chebyshev polynomials.

Pritchard and Richardson [10] carried out a numerical study to analyze the effect of temperature solvability on stability of thermosolutal convection in a saturated horizontal porous layer. Employing Brinkman model, Wang and Tan [11] investigated the onset of convection in a horizontal sparsely packed porous medium with imposed thermal and concentration gradient at both the boundaries and found that Darcy number destabilizes the system in case of stationary and oscillatory convection.

Jena *et al.* [6] by employing Darcy-Brinkman-Forchheimer model studied about the linear stability of onset of thermosolutal convection in a rectangular annulus which is highly heated and soluted from within and exposed to low thermal and solutal gradients from outside in a non-Darcian region. Abdullah *et al.* [2] studied the effect of thermosolutal convection of a viscous incompressible fluid in the presence of rotation on the stability of the system and observed that solute concentration and rotation stabilizes the system.

Hirata *et al.* [4] using linear stability analysis studied the thermal convection of a viscous incompressible fluid applying normal mode analysis. These earlier works concentrated on the marginal stability of thermosolutal convection under different physical conditions and very less emphasis has been given to analyze the linear stability of thermosolutal convection. Hence in this paper, the work of Hirata *et al.* [4] has been extended to study the linear stability of thermosolutal convection on an incompressible viscous fluid in the presence of vertical magnetic field using method of small oscillations and the analysis is restricted to long wave approximations.

MATHEMATICAL FORMULATION

Consider a viscous incompressible electrically conducting fluid which is bounded by a horizontal infinite parallel plate. The porous medium is assumed to be saturated, isotropic and homogeneous governed by Darcy – Brinkman model. The impermeable horizontal walls are maintained at different temperatures and concentrations such that T_u, S_u at the upper walls and T_l, S_l at the lower walls. It is assumed that the fluid obeys Boussinesq approximation and variations of density due to temperature and solute concentration is assumed to be $\rho(T,C) = [1 - \beta_T(T - T_0) - \beta_C(C - C_0)]$ where thermal expansion coefficient $\beta_T \geq 0$ and solute expansion coefficient $\beta_C \leq 0$.

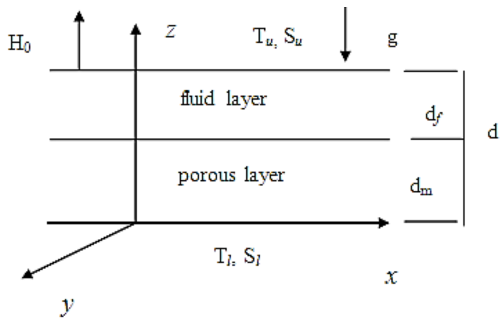

Figure 1. Physical Description of the Flow

Under the flow assumptions the governing equations takes the form

$$\nabla.\vec{u}=0 \quad (1)$$

$$\rho_0\left[\frac{\partial}{\partial t}\left(\frac{\vec{u}}{\phi}\right)+\frac{1}{\phi}\left(\vec{u}.\frac{\nabla\vec{u}}{\phi}\right)\right]=-\nabla p-\frac{\mu}{k}\vec{u}+g\rho+\frac{\mu_{eff}}{\mu_f}\nabla^2\vec{u}+\frac{\mu_e}{4\pi}(\nabla\times\vec{H})\times\vec{H} \quad (2)$$

$$\frac{\partial\vec{T}}{\partial t}+(\vec{u}.\nabla\vec{T})=\nabla.(\alpha\nabla\vec{T}) \quad (3)$$

$$\phi\frac{\partial\vec{C}}{\partial t}+(\vec{u}.\nabla\vec{C})=D_f\,\nabla.(\phi\nabla\vec{C}) \quad (4)$$

$$\nabla.\vec{H}=0 \quad (5)$$

$$\phi\frac{\partial\vec{H}}{\partial t}=\nabla\times(\vec{u}\times\vec{H})+\phi\eta\nabla^2\vec{H} \quad (6)$$

with boundary conditions

$$\vec{u}=\frac{\partial\vec{u}}{\partial z}=\vec{T}=\vec{C}=\vec{H}=0 \text{ at}$$

$$z=0 \text{ and } z=d \quad (7)$$

where $\vec{u}$, ρ, p, g , T, C, $\vec{H}(0,0,H_0)$, ϕ, μ_e, η, ρ_0, μ_{eff}, μ_f, α respectively denote the velocity vector, density, pressure, acceleration due to gravity, temperature, concentration, constant vertical magnetic field, porosity, magnetic permeability, electrical resistivity, density at reference level, effective viscosity of the porous medium, dynamic viscosity of the fluid and thermal diffusivity.

In quiescent state, basic state flow field are given by $\vec{u}^*=(0,0,0)$, $P^*=P(z)$, $\vec{H}^*=(0,0,H_0)$, $T^*=T(z)$ and $C^*=C(z)$and so the temperature field and concentration field using the boundary conditions become

$$T=z+\frac{T_l-T_0}{T_u-T_l} \text{ and } C=z+\frac{C_l-C_0}{C_u-C_l} \quad (8)$$

Let the small disturbance in the initial states of velocity, temperature, concentration, pressure and magnetic field respectively be denoted by $\vec{u}'(x,z,t)$, $T'(x,z,t)$, $C'(x,z,t), P'(x,z,t)$ and $\vec{h}(h_x,0,h_z)$, then the linearized perturbed equation (1) – (6) becomes

$$\nabla.\vec{u}'=0 \quad (9)$$

$$\frac{\partial}{\partial t}\left(\frac{\vec{u}'}{\phi}\right)=-\frac{1}{\rho_0}\nabla p'-\frac{\nu}{k}\vec{u}'-g\beta_T T'\hat{k}+\frac{1}{\phi}\nabla^2\vec{u}'+\frac{\mu_e}{4\pi\rho_0}(\nabla\times\vec{h})\times\vec{H}^* \quad (10)$$

$$\frac{\partial T'}{\partial t}+(\vec{u}'.\nabla T^*)=\nabla.(\alpha\nabla T') \quad (11)$$

$$\phi\frac{\partial C'}{\partial t}+(\vec{u}'.\nabla C^*)=D_f\nabla.(\phi\nabla C') \quad (12)$$

$$\nabla.\vec{h}=0 \quad (13)$$

$$\phi\frac{\partial\vec{h}}{\partial t}=\nabla\times(\vec{u}'\times\vec{H}^*)+\phi\eta\nabla^2\vec{h} \quad (14)$$

By eliminating pressure term and introducing the non-dimensional variables for length, velocity, time, temperature, concentration and magnetic field respectively as follows

$$z=z^*d,\quad w=w^*\frac{\nu}{d},\quad t=t^*\frac{d^2}{\nu}, T'=\Delta T\,T^*,$$

$$C'=\Delta C\,C^*, h_z=\frac{H_0\nu h_z^*}{\eta}$$

The system of linearized perturbed equations becomes

$$\frac{\partial}{\partial t}\left(\frac{1}{\phi}\nabla^2 w\right)=-\frac{1}{Da}\nabla^2 w+Gr_T\frac{\partial^2 T}{\partial x^2}+Gr_C\frac{\partial^2 C}{\partial x^2}+\frac{1}{\phi}\nabla^4 w+Q\frac{\partial}{\partial z}\left(\frac{\partial^2 h_z}{\partial x^2}\right) \quad (15)$$

$$\frac{\partial T}{\partial t}+w=\frac{\hat{d}}{P_r}\nabla^2 T \quad (16)$$

$$\phi\frac{\partial C}{\partial t}+w=\frac{\phi}{Sc}\nabla^2 C \quad (17)$$

$$p_m\frac{\partial h_z}{\partial t}=\frac{1}{\phi}\frac{\partial w}{\partial z}+\nabla^2 h_z \quad (18)$$

where $Da=k/d^2$ (Darcy number),

$Gr_T=g\beta_T\rho_0\Delta Td^3/\nu^2$ (Thermal Grashof number) $Gr_C=g\beta_C\rho_0\Delta Cd^3/\nu^2$ (Mass Grashof number), $P_r=\nu/\alpha_f$ (Prandtl number), $Sc=\frac{\nu}{D_f}$ (Schmidt Number), $Q=\frac{\mu_e H_0^2 d^2}{4\pi\rho_0\eta\nu}$ (Chandrasekhar number), $p_m=\nu/\eta$ (Magnetic Prandtl number)

Also the special variations of α, ϕ and k are taken as null.

Applying normal mode analysis to the dependent variables

$$(w,T,C,h_z)=(W(z),\theta(z),S(z),H(z))e^{(ikx+\sigma t)}$$

where k is the nondimensional wave number and σ represents the growth rate. Substituting the above expression into equations (15) – (18) we get

$$\left(\frac{\partial^2}{\partial z^2}-k^2\right)\left(\frac{\partial^2}{\partial z^2}-k^2-\frac{\phi}{Da}-\sigma\right)W=k^2\phi Gr_T\theta+k^2\phi Gr_C S+k^2Q\phi\frac{\partial}{\partial z}H \quad (19)$$

$$\left(\frac{\partial^2}{\partial z^2}-k^2-\sigma\frac{pr}{\hat{d}}\right)\theta=\frac{pr}{\hat{d}}W \quad (20)$$

$$\left(\frac{\partial^2}{\partial z^2}-k^2-\sigma Sc\right)S=\frac{Sc}{\phi}W \quad (21)$$

$$\left(\frac{\partial^2}{\partial z^2}-k^2-\sigma\,p_m\right)H=-\frac{1}{\phi}\frac{\partial}{\partial z}W \quad (22)$$

with the corresponding boundary condition

$$W=\frac{\partial W}{\partial z}=\theta=S=H=0 \text{ at } z=0 \text{ and } z=1 \quad (23)$$

EIGEN VALUES AND EIGEN FUNCTIONS

Now we expand W, σ, θ , S and H in powers of k

$$\left.\begin{aligned} W &= W_0 + k^2 W_1 + k^4 W_2 + \cdots \\ \sigma &= \sigma_0 + k^2\sigma_1 + k^4\sigma_2 + \cdots \\ \theta &= \theta_0 + k^2\theta_1 + k^4\theta_2 + \cdots \\ S &= S_0 + k^2 S_1 + k^4 S_2 + \cdots \\ H &= H_0 + k^2 H_1 + k^4 H_2 + \cdots \end{aligned}\right\} \quad (24)$$

Substituting (24) in equations (19) to (23) and collecting the like powers of k we get

$$\frac{\partial^2}{\partial z^2}\left(\frac{\partial^2}{\partial z^2} - \frac{\phi}{Da} - \sigma_0\right) W_0 = 0 \quad (25)$$

$$\left(\frac{\partial^2}{\partial z^2} - \sigma_0 \frac{Pr}{d}\right)\theta_0 = \frac{Pr}{d} W_0 \quad (26)$$

$$\left(\frac{\partial^2}{\partial z^2} - \sigma_0 Sc\right) S_0 = \frac{Sc}{\phi} W_0 \quad (27)$$

$$\left(\frac{\partial^2}{\partial z^2} - \sigma_0\, p_m\right) H_0 = -\frac{1}{\phi} DW_0 \quad (28)$$

$$\frac{\partial^2}{\partial z^2}\left(\frac{\partial^2}{\partial z^2} - \frac{\phi}{Da} - \sigma_0\right) W_1 = (1+\sigma_1)\frac{\partial^2}{\partial z^2} W_0 + \left(\frac{\partial^2}{\partial z^2} - \frac{\phi}{Da} - \sigma_0\right) W_0 + \phi Gr_T \theta_0 + \phi Gr_C S_0 + Q\phi DH_0 \quad (29)$$

$$\left(\frac{\partial^2}{\partial z^2} - \sigma_0 \frac{Pr}{d}\right)\theta_1 = \frac{Pr}{d} W_1 + \left(1 + \sigma_1 \frac{Pr}{d}\right)\theta_0 \quad (30)$$

$$\left(\frac{\partial^2}{\partial z^2} - \sigma_0 Sc\right) S_1 = \frac{Sc}{\phi} W_1 + (1 + \sigma_1 Sc) S_0 \quad (31)$$

$$\left(\frac{\partial^2}{\partial z^2} - \sigma_0\, p_m\right) H_1 = -\frac{1}{\phi} DW_1 + (1 + \sigma_1\, p_m) H_0 \quad (32)$$

The corresponding boundary conditions are

$$\left.\begin{aligned} &W_0 = DW_0 = \theta_0 = S_0 = H_0 = 0 \\ &\text{at } z = 0 \text{ and } z = 1 \\ &W_1 = DW_1 = \theta_1 = S_1 = H_1 = 0 \\ &\text{at } z = 0 \text{ and } z = 1 \end{aligned}\right\} \quad (33)$$

On solving equations using boundary conditions we get

$$W_0 = A_1 + A_2 z + A_3 \cosh(rz) + A_4 \sinh(rz)$$

$$\theta_0 = A_5 \cosh(r_1 z) + A_6 \sinh(r_1 z) + A_7 + A_8 z + A_9 \cosh(rz) + A_{10} \sinh(rz)$$

$$S_0 = A_{11} \cosh(r_2 z) + A_{12} \sinh(r_2 z) + C_2 + C_3 z + C_4 \cosh(rz) + C_5 \sinh(rz)$$

$$H_0 = A_{13} \cosh(r_3 z) + A_{14} \sinh(r_3 z) + C_7 + C_8 \sinh(rz) + C_9 \cosh(rz)$$

$$\begin{aligned} W_1 = {} & A_{15} + A_{16} z + A_{17} \cosh(rz) + A_{18} \sinh(rz) + A_{19} z \sinh(rz) + A_{20} z \cosh(rz) + C_{24} \cosh(r_1 z) \\ & + C_{25} \sinh(r_1 z) + C_{26} + C_{27} z^3 + C_{28} \cosh(r_2 z) + C_{29} \cosh(r_2 z) + C_{30} \sinh(r_3 z) + C_{31} \cosh(r_3 z) \end{aligned}$$

$$\begin{aligned} \theta_1 = {} & A_{21} \cosh(r_1 z) + A_{22} \sinh(r_1 z) + C_{53} + C_{54} z + C_{55} \cosh(rz) + +C_{56} \sinh(rz) + C_{57} z \sinh(rz) + C_{58} z \cosh(rz) \\ & + C_{59} z \sinh(r_1 z) + C_{60} z \cosh(r_1 z) + C_{61} z^3 + C_{62} \cosh(r_2 z) + C_{63} \sinh(r_2 z) + C_{64} \sinh(r_3 z) + C_{65} \cosh(r_3 z) \end{aligned}$$

$$\begin{aligned} S_1 = {} & A_{23} \cosh(r_2 z) + A_{24} \sinh(r_2 z) + C_{81} + C_{82} z + C_{83} \cosh(rz) + +C_{84} \sinh(rz) + C_{85} z \sinh(rz) + C_{86} z \cosh(rz) \\ & + C_{87} \cosh(r_1 z) + C_{88} \sinh(r_1 z) + C_{89} z^3 + C_{90} z \sinh(r_2 z) + C_{91} z \cosh(r_2 z + C_{92} \sinh(r_3 z) + C_{93} \cosh(r_3 z) \end{aligned}$$

$$\begin{aligned} H_1 = {} & A_{25} \cosh(r_3 z) + A_{26} \sinh(r_3 z) + C_{107} + C_{108} \sinh(rz) + C_{109} \cosh(rz) + C_{110} z \cosh(rz) + C_{111} z \sinh(rz) \\ & + C_{112} \sinh(r_1 z) + C_{113} \cosh(r_1 z) + C_{114} z^2 + C_{115} \sinh(r_2 z) + C_{116} \cosh(r_2 z) + C_{117} z \sinh(r_3 z) + C_{118} z \cosh(r_3 z) \end{aligned}$$

The zeroth order eigen values are given by the following transcendental equation

$$A_3[\cosh(r) - 1] + A_4[\sinh(r) - r] = 0$$

$$A_3 \sinh(r) + A_4[\cosh(r) - 1] = 0$$

The solution of above expression will not give explicit values of σ_0. Hence the values of σ_0 are obtained using Mathematica 8.0.

The first order approximations of the growth rate is given by

$$\sigma_1 = -\frac{C_{37}}{C_{36}}$$

For brevity constants are given in appendix.

RESULTS AND DISCUSSION

To get physical insight into linear stability of thermosolutal convection of an electrically conducting viscous fluid bounded by saturated porous medium in presence of vertical magnetic field, the effects of various non-dimensional parameters such as Chandrasekhar number Q, magnetic Prandtl number p_m, Schmidt number Sc, Prandtl number Pr, Darcy number Da, depth ratio d and porosity ϕ on temporal growth rate, velocity, temperature, concentration and magnetic field has been discussed numerically and plotted in figures (2) – (21). We have fixed the values of parameters as $Da = 0.2,\ \phi = 0.6,\ Pr = 0.71,\ d = 0.08,\ Sc = 0.66,$ $Gr_T = 5.0, Gr_C = -5.0, q = 50.0,\ p_m = 5.0,$ $k = 0.9$ throughout the entire study of the problem.

Figures (2) – (3) illustrate the effect of Chandrasekhar number Q on growth rate and it is found that increase in Chandrasekhar number stabilizes the system for small Darcy numbers and induces instability for large Darcy numbers.

The influence of porosity ϕ, magnetic Prandtl number p_m and Schmidt number Sc on growth rate is depicted in figures (4) – (6). It is observed that increase in porosity creates stability/instability in the system, while magnetic Prandtl number increases the growth rate thereby inducing instability. Effect of Schmidt number destabilizes the system but it has less significant on temporal growth.

Figures (7) – (10) show the influence of wave number k, magnetic Prandtl number p_m, Chandrasekhar number Q and Darcy number Da with respect to porosity ϕ on frequency. It depicts that with increase in wave number system becomes unstable near the lower plate and stabilizes the system as we moves towards the upper plate, whereas increase in magnetic Prandtl number and Darcy number increases instability. Increase in Chandrasekhar number destabilizes the system for small values for Darcy number.

Increase in Chandrasekhar number Q and magnetic Prandtl number p_m destabilizes the system with increase in Darcy number as shown in figures (11) and (12).

Figure (13) represent the effect of wave number k on frequency parameter and it is found that increase in wave number decreases the growth rate thereby creating stability in the system.

The influence of Chandrasekhar number Q and magnetic Prandtl number p_m on velocity field is depicted in figures (14) – (15). It is shown that increase in Chandrasekhar number reduces the velocity flow field and that increase in magnetic Prandtl number enhances the flow field.

Figures (16) – (17) illustrate the variations of temperature field due to variation in Chandrasekhar number Q and Prandtl number Pr. It is found that increase in Chandrasekhar number and Prandtl number decreases the temperature profile near the lower plate and increases as we move towards the upper wall.

The effect of Chandrasekhar number Q and Schmidt number Sc on concentration profile is plotted in figures (18) – (19). It may be inferred that increase in Chandrasekhar number decreases concentration profile whereas Schmidt number increases the concentration profile.

In figures (20) – (21), the variations of Chandrasekhar number Q and magnetic Prandtl number p_m on magnetic field is depicted and it is observed that increase in Chandrasekhar number decreases the magnetic field intensity while increase in magnetic Prandtl number increases the magnetic field effect.

CONCLUSION

We have investigated stability of thermosolutal convection in an electrically conducting viscous fluid under influence of vertical magnetic field confined between infinite horizontal plates using method of small oscillation and the effects of various non-dimensional parameters on characteristics of the flow has been analysed. The following interpretations were made from the findings.

- Increase in Chandrasekhar number stabilizes the system for small Darcy numbers and induces instability for large Darcy numbers.
- Increase in Darcy number and Chandrasekhar number destabilizes the system as porosity increases.
- It is shown that increase in Chandrasekhar number reduces the velocity flow field and that increase in magnetic Prandtl number enhances the flow field.
- It is found that increase in Chandrasekhar number and Prandtl number decreases the temperature profile near the lower plate and increases as we move towards the upper wall.
- It may be inferred that concentration decreases with increase in Chandrasekhar number and Schmidt number increases the concentration profile.
- Increase in magnetic Prandtl number increases the magnetic field effect.

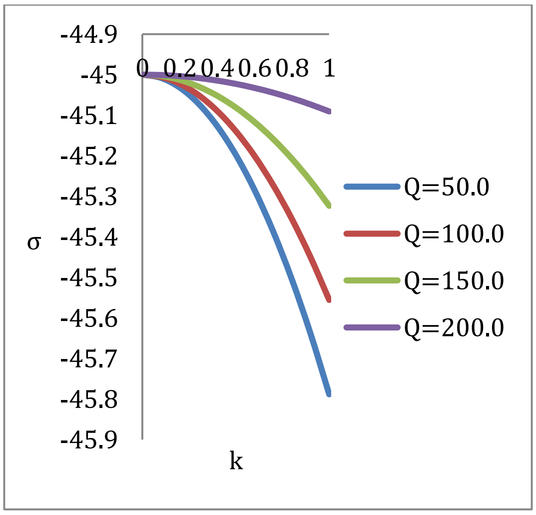

Figure 2. Effect of Chandrasekhar number on growth rate for small Darcy number

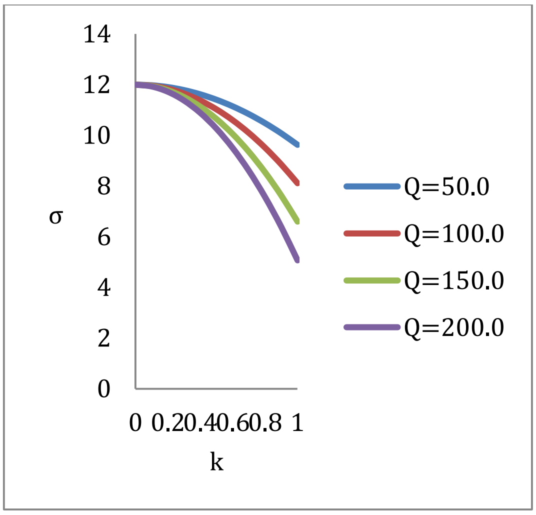

Figure 3. Effect of Chandrasekhar number on growth rate for large Darcy number

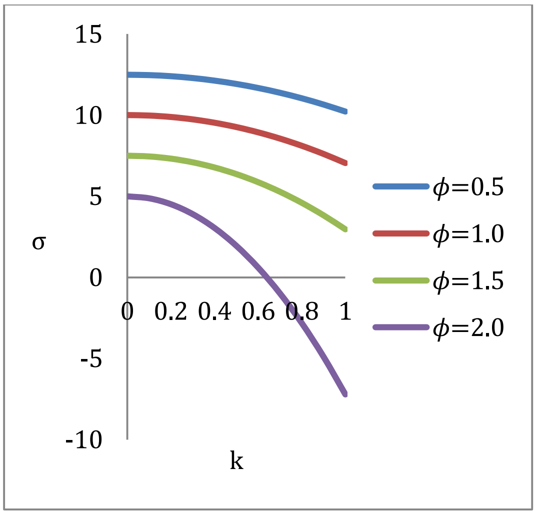

Figure 4. Effect of porosity on growth rate

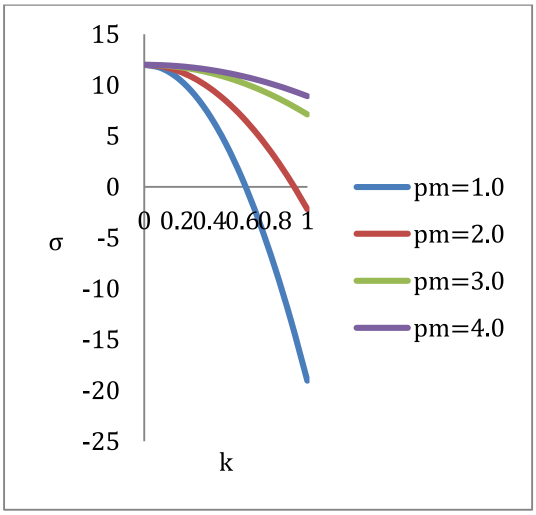

Figure5. Effect of magnetic Prandtl number on growth rate

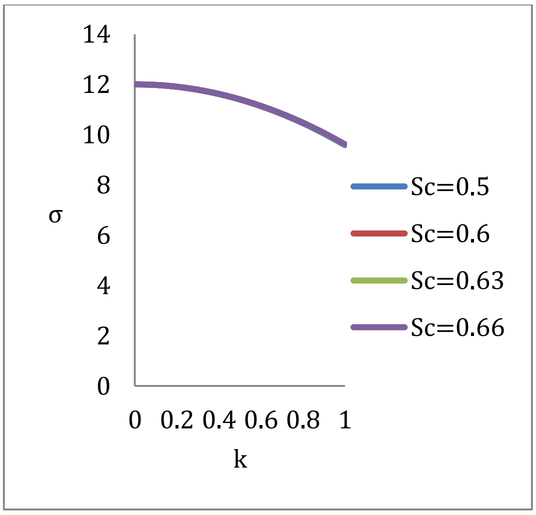

Figure 6. Effect of Schmidt number on growth rate

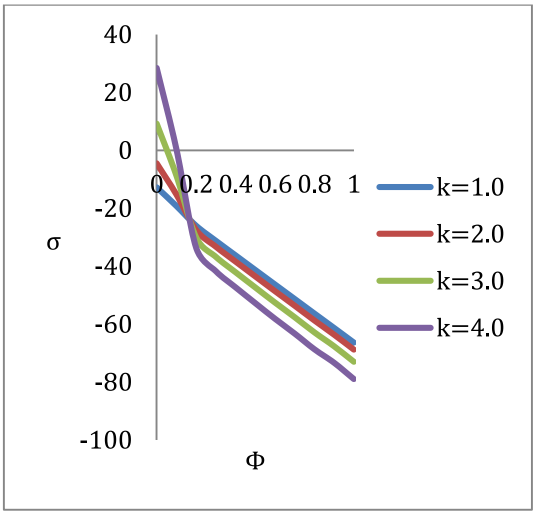

Figure 7. Effect of wave number on growth rate

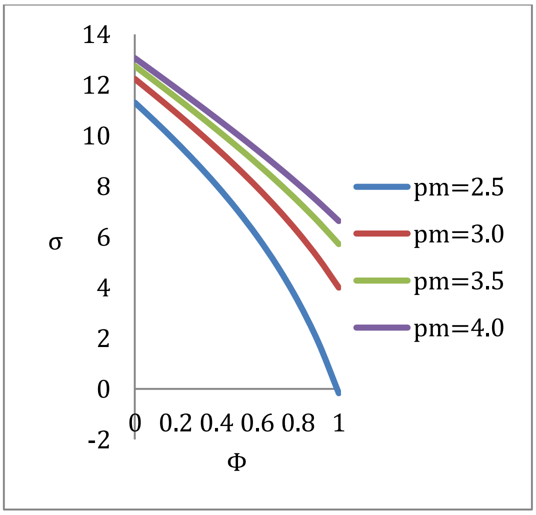

Figure 8. Effect of magneticPrandtl number on growth rate

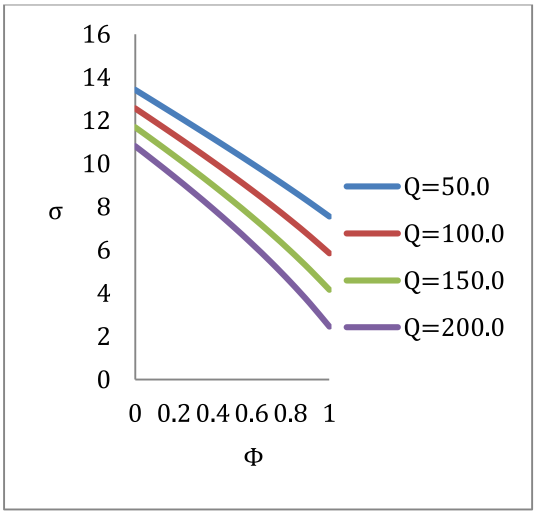

Figure 9. Effect of Chandrasekhar number on growth rate

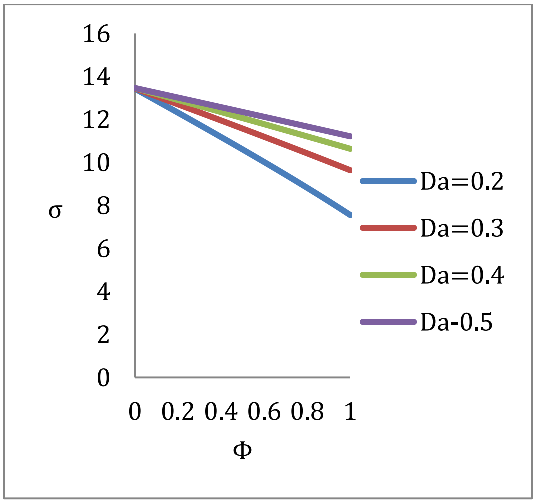

Figure 10. Effect of Darcy number on growth rate

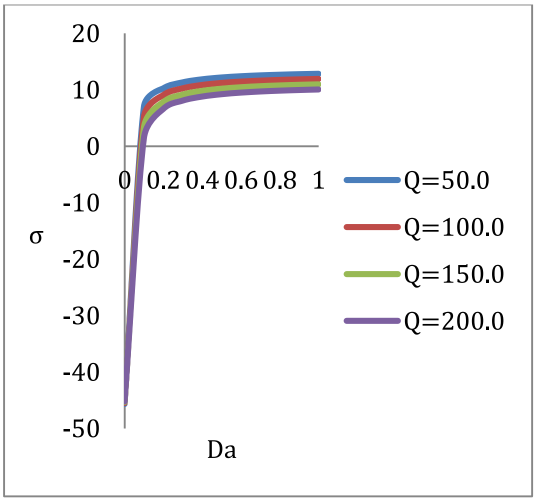

Figure 11. Effect of Chandrasekhar number on growth rate

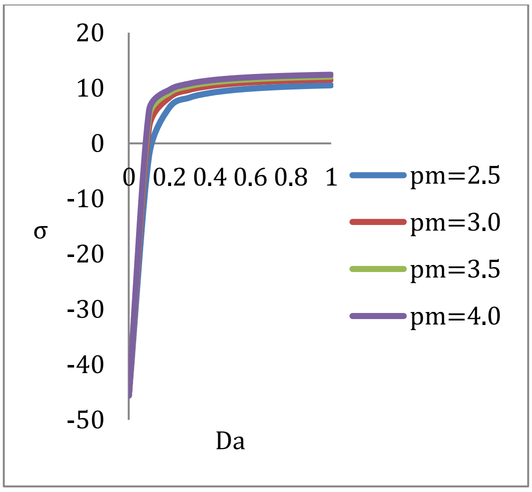

Figure 12. Effect of magnetic Prandtl number on growth rate

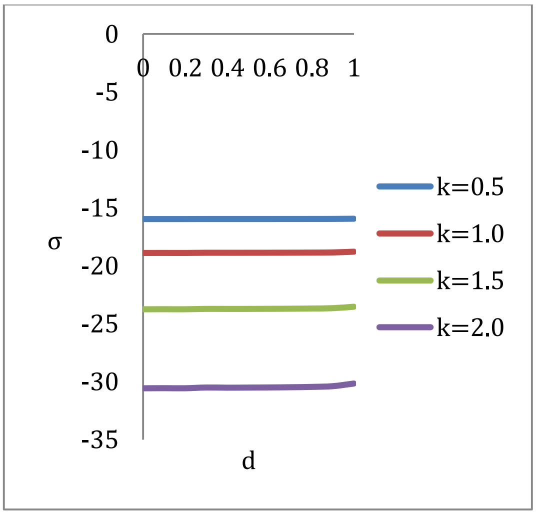

Figure 13. Effect of wave number on growth rate

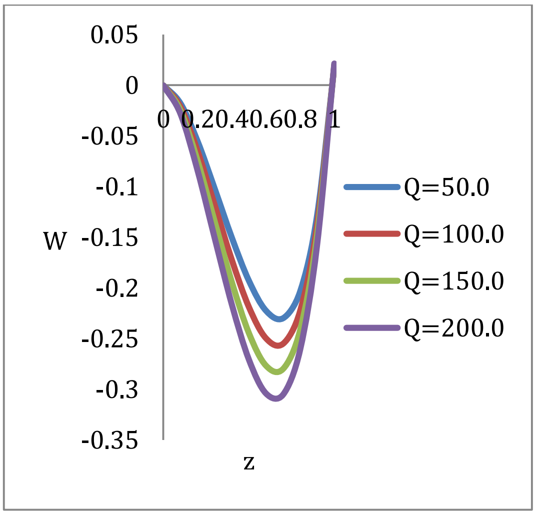

Figure 14. Effect of Chandrasekhar number on velocity field

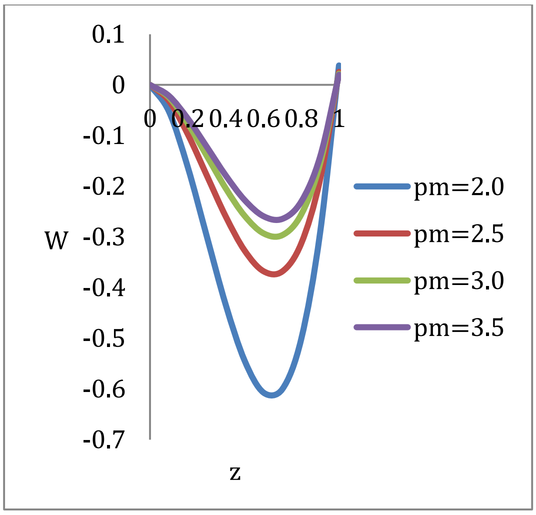

Figure 15. Effect of magnetic Prandtl number on velocity field

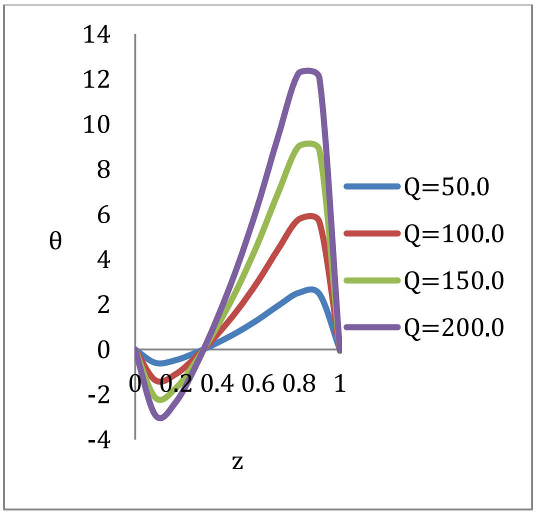

Figure 16. Effect of Chandrasekhar number on temperature field

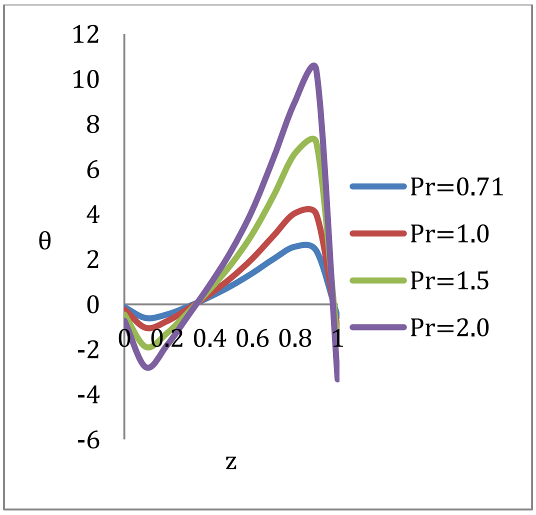

Figure 17. Effect of Prandtl number on temperature field

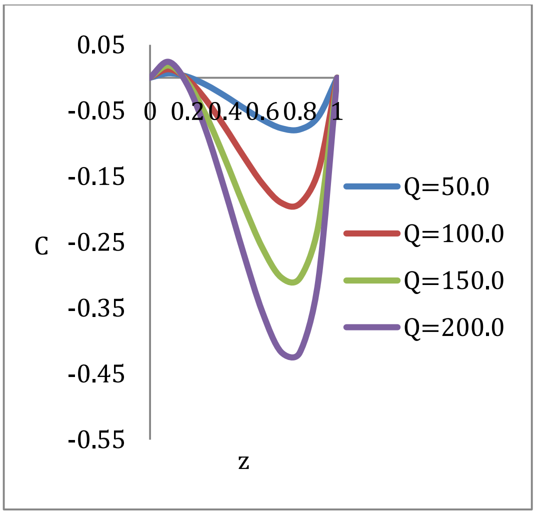

Figure 18. Effect of Chandrasekhar number on concentration field

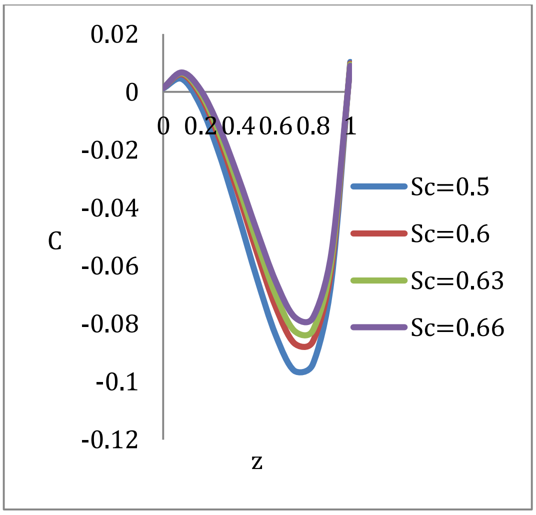

Figure 19. Effect of Schmidt number on concentration field

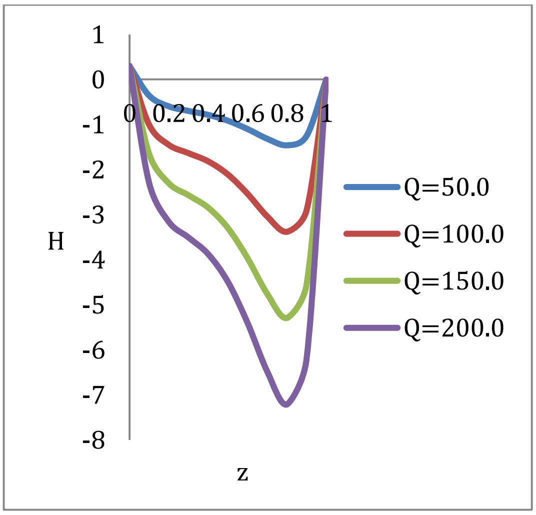

Figure 20. Effect of Chandrasekhar number on magnetic field

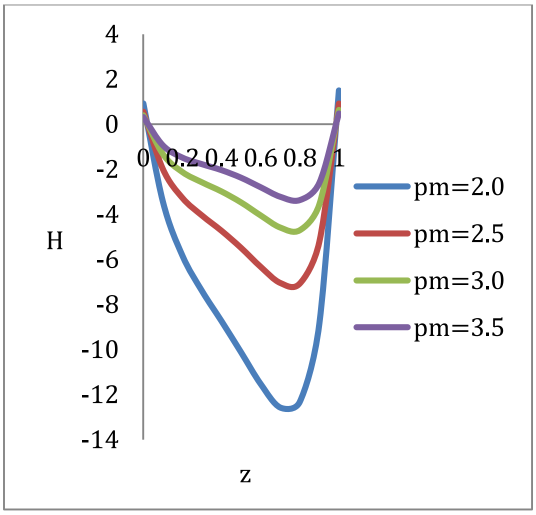

Figure 21. Effect of magnetic Prandtl number on magnetic field

REFERENCES

[1] A. Abdullah, "Thermosolutal Convection in a Non-Linear Magnetic Fluid", International Journal of Thermal Sciences, vol.39, no.2, pp. 273-284, 2000.

[2] A. Abdullah, S.D. Ahmari, and A.J. Chamkha, "Thermosolutal instability in a horizontal fluid layer affected by rotation", Int. J. Mathematics Trends and Technology, vol. 20, no.1, pp. 55, 2015.

[3] D. Gobin, B. Goyeau and J.P. Songbe, "Double diffusive natural convection in a composite fluid porous layer", Transactions of the ASME, vol.120, no.1, pp.234-242, 1998.

[4] S. C. Hirata, B. Goyeau, D. Gobin, and R.M. Cotta,"Stability of Natural Convection in Superposed Fluid and Porous Layers Using Integral Transforms", Numerical Heat transfer, Part B, vol.50, pp. 409 – 424,2006.

[5] S. C. Hirata, B. Goyeau and D. Gobin, "Stability of thermosolutal natural convection in superposed fluid and porous layer", Transport porous Med., vol.78, pp. 525-536, 2009.

[6] S. K. Jena, S. K. Mahapatra, and A. Sarkar, "Thermosolutal Convection in a Rectangular Concentric Annulus: A Comprehensive Study", Transport in Porous Media, vol.98, no. 1, pp. 103 – 124, 2013.

[7] J. Murthy, and P. Lee, "Thermosolutal Convection in a Floating Zone: The Case of an Unstable Solute Gradient", International Journal of Heat and Mass Transfer, vol.31, no.9, pp. 1923 – 1932, 1988.

[8] D. A. Nield, "Onset of thermohaline convection in a porous medium", Water Resource Research, vol.4, no.3, pp. 553 -560, 1968.

[9] V. Steinberg, "Convective instabilities of binary mixtures with fast chemical reaction in a porous medium", The Journal of Chemical Physics, vol.78, pp. 2655, 1983.

[10] D. Pritchard, and C. N. Richardson, "The effect of temperature - dependent solubility on the onset of thermosolutal convection in a horizontal porous layer", J. Fluid Mech., vol.57, no.1, pp.59–95, 2007.

[11] S. Wang and W. Tan, "The onset of Darcy-Brinkman thermosolutal convection in a horizontal porous media, Physics Letters A", vol. 373, no.7, pp. 776–780, 2009.

[12] P. Zhao and C. F. Chen, "Stability analysis of double diffusive convection in superposed fluid and porous layers using a one equation model", Int. J. heat and mass transfer, vol. 44, no.24, pp. 4625-4633,2001.

APPENDIX

$r = \sqrt{\phi/_{Da} + \sigma_0}$; $A_1 = 1$;

$A_3 = -1$; $A_2 = \frac{-r(\cosh(r)-1)}{(\sinh(r)-r)}$;

$A_4 = \frac{\cosh(r)-1}{(\sinh(r)-r)}$; $r_1 = \sqrt{\sigma_0 Pr/_d}$;

$A_5 = -(A_7 + A_9)$; $A_7 = -\left(\frac{A_1}{r_1^2}\frac{Pr}{d}\right)$;

$A_6 = \left(\frac{-1}{\sinh(r_1)}\right)(A_5\cosh(r_1) + A_7 + A_8 + A_9\cosh(r) + A_{10}\sinh(r))$;

$r_2 = \sqrt{\sigma_0 sc}$; $A_8 = -\left(\frac{A_2}{r_1^2}\frac{Pr}{d}\right)$;

$A_9 = \frac{A_3}{(r^2-r_1^2)}\left(\frac{Pr}{d}\right)$; $A_{10} = \frac{A_4}{(r^2-r_1^2)}\left(\frac{Pr}{d}\right)$;

$C_1 = \frac{sc}{\emptyset}$; $C_2 = \frac{-C_1A_1}{r_2^2}$; $C_3 = -\frac{C_1A_2}{r_2^2} C_4 = \frac{C_1A_3}{r^2-r_2^2}$; $C_5 = \frac{C_1A_4}{r^2-r_2^2}$; $A_{11} = -(C_2 + C_4)$;

$A_{12} = \left(\frac{-1}{\sinh(r_2)}\right)(A_{11}\cosh(r_2) + C_2 + C_3 + C_5\sinh(r) + C_4\cosh(r))$;

$r_3 = \sqrt{\sigma_0 P_m}$; $C_6 = -\frac{1}{\phi}$;

$C_7 = -\left(\frac{C_6A_2}{r_3^2}\right)$; $C_8 = \left(\frac{C_6rA_3}{(r^2-r_3^2)}\right)$;

$C_9 = \left(\frac{C_6rA_4}{(r^2-r_3^2)}\right)$; $A_{13} = -(C_7 + C_9)$;

$$A_{14} = \frac{-1}{\sinh(r_3)}\{A_{13}\cosh(r_3) + C_7 + C_8\sinh(r) + C_9\cosh(r)\}$$

$C_{10} = Gr_T\phi$; $C_{11} = Gr_e\phi$; $C_{12} = Q\phi$;

$C_{13} = r^2A_3 + C_{10}A_9 + C_{11}C_4 + C_{12}rC_8$

$C_{14} = r^2A_4 + C_{10}A_{10} + C_{11}C_5 + C_{12}rC_9$; $C_{15} = C_{10}A_5$; $C_{16} = C_{10}A_5$;

$C_{17} = C_{10}A_7 + C_{11}C_2$; $C_{18} = C_{10}A_8 + C_{11}C_3$; $C_{19} = C_{11}A_{11}$; $C_{20} = C_{11}A_{12}$;

$C_{21} = C_{12}r_3A_{13}$; $C_{22} = C_{12}r_3A_{14}$; $C_{23} = \frac{1}{2r^3}$; $C_{24} = \frac{C_{15}}{r_1^2(r_1^2-r^2)}$; $C_{25} = \frac{C_{16}}{r_1^2(r_1^2-r^2)}$

$C_{26} = \frac{-C_{17}}{r^2}$; $C_{27} = \frac{-C_{18}}{6r^2}$; $C_{28} = \frac{C_{16}}{r_2^2(r_2^2-r^2)}$; $C_{29} = \frac{C_{20}}{r_2^2(r_2^2-r^2)}$; $C_{30} = \frac{C_{21}}{r_3^2(r_3^2-r^2)}$;

$C_{31} = \frac{C_{22}}{r_3^2(r_3^2-r^2)}$; $C_{32} = r^2A_4C_{23} - r^2A_3C_{23}\sinh(r) - r^2A_4C_{23}\cosh(r)$;

$C_{33} = \{-C_{13}C_{23}\sinh(r) + C_{14}C_{23}(1-\cosh(r)) + C_{24}[1-\cosh(r_1)] + C_{25}[r_1 - \sinh(r_1)] - C_{27} + C_{28}[1-\cosh(r_2)] + C_{29}[r_2 - \sinh(r_2)] + C_{30}[r_3 - \sinh(r_3)] + C_{31}[1-\cosh(r_3)]\}$;

$C_{34} = r^2A_4C_{23} - r^2A_3C_{23}[r\cosh(r) + \sinh(r)] - r^2A_4C_{23}[r\sinh(r) + \cosh(r)]$;

$C_{35} = \{C_{14}C_{23}[1 - r\sinh(r) - \cosh(r)] - r_1C_{24}\sinh(r_1) + r_1C_{25}[1-\cosh(r_1)] - C_{13}C_{23}[r\cosh(r) + \sinh(r)] - 3C_{27} - r_2C_{28}\sinh(r_2) + r_2C_{29}[1-\cosh(r_2)] + r_3C_{30}[1-\cosh(r_3)] - r_3C_{31}\sinh(r_3)\}$;

$C_{36} = C_{32}r\sinh(r) - C_{34}(\cosh(r) - 1)$;

$C_{37} = C_{33}r\sinh(r) - C_{35}(\cosh(r) - 1)$;

$A_{17} = 1$; $A_{18} = \frac{\sigma_1C_{32} + C_{33} - (\cosh(r)-1)}{\sinh(r) - r}$

$A_{19} = (\sigma_1r^2A_3 + C_{13})C_{23}$;

$A_{20} = (\sigma_1r^2A_4 + C_{14})C_{23}$;

$C_{38} = \left(1 + \frac{Pr}{d}\sigma_1\right)$; $C_{39} = \frac{Pr}{d}$;

$C_{40} = [C_{39}(A_{15} + C_{26}) + C_{38}A_7]$;

$C_{41} = [C_{39}A_{16} + C_{38}A_8]$;

$C_{42} = [C_{39}A_{17} + C_{38}A_9]$;

$C_{43} = [C_{39}A_{18} + C_{38}A_{10}]$;

$C_{44} = C_{39}A_{19}$; $C_{45} = C_{39}A_{20}$;

$C_{46} = C_{39}C_{24} + C_{38}A_5$;

$C_{47} = C_{39}C_{25} + C_{38}A_6$;

$C_{48} = C_{39}C_{27}$;

$C_{49} = C_{39}C_{28}$; $C_{50} = C_{39}C_{29}$;

$C_{51} = C_{39}C_{30}$; $C_{52} = C_{39}C_{31}$; $C_{53} = -\frac{C_{40}}{r_1^2}$;

$C_{54} = \frac{-C_{41}}{r_1^2} - \frac{6C_{48}}{r_1^4}$; $C_{55} = \frac{C_{12}}{r^2-r_1^2} - \frac{2rC_{44}}{(r^2-r_1^2)^2}$;

$C_{56} = \frac{C_{43}}{r^2-r_1^2} - \frac{2rC_{45}}{(r^2-r_1^2)^2}$; $C_{57} = \frac{C_{44}}{r^2-r_1^2}$;

$C_{58} = \frac{C_{45}}{r^2-r_1^2}$; $C_{59} = \frac{C_{46}}{2r_1}$; $C_{60} = \frac{C_{47}}{2r_1}$;

$C_{61} = \frac{-C_{48}}{r_1^2}$; $C_{62} = \frac{C_{49}}{r_2^2-r_1^2}$; $C_{63} = \frac{C_{50}}{r_2^2-r_1^2}$;

$C_{64} = \frac{C_{51}}{r_3^2-r_1^2}$; $C_{65} = \frac{C_{52}}{r_3^2-r_1^2}$;

$A_{21} = -(C_{53} + C_{55} + C_{62} + C_{65})$;

$A_{22} = \frac{-1}{\sinh(r_1)}\{A_{21}\cosh(r_1) + C_{53} + C_{54} + C_{55}\cosh(r) + C_{56}\sinh(r) + C_{57}\sinh(r) + C_{58}\cosh(r) + C_{59}\sinh(r_1) + C_{60}\cosh(r_1) + C_{61} + C_{62}\cosh(r_2) + C_{63}\sinh(r_2) + C_{64}\sinh(r_3) + C_{65}\cosh(r_3)\}$;

$C_1 = \frac{Sc}{\emptyset}$; $C_{67} = (1 + \sigma_1Sc)$;

$C_{68} = C_1(A_{15} + C_{26}) + C_{67}C_2$;

$C_{69} = C_1A_{16} + C_{67}C_3$; $C_{70} = C_1A_{17} + C_{67}C_4$; $C_{71} = C_1A_{18} + C_{67}C_5$; $C_{72} = C_1A_{19}$;

$C_{73} = C_1A_{20}$; $C_{74} = C_1C_{24}$; $C_{75} = C_1C_{25}$; $C_{76} = C_1C_{26}$; $C_{77} = C_1C_{28} + C_{67}A_{11}$;

$C_{78} = C_1C_{29} + C_{67}A_{12}$; $C_{79} = C_1C_{30}$; $C_{80} = C_1C_{31}$; $C_{81} = -\frac{C_{68}}{r_2^2}$;

$C_{82} = -\frac{C_{69}}{r_2^2}$; $C_{83} = \frac{C_{70}}{r^2-r_2^2} - \frac{2rC_{72}}{(r^2-r_2^2)^2}$;

$C_{84} = \frac{C_{71}}{r^2-r_2^2} - \frac{2rC_{73}}{(r^2-r_2^2)^2}$;

$C_{85} = \frac{C_{72}}{r^2-r_2^2}$; $C_{86} = \frac{C_{73}}{r^2-r_2^2}$; $C_{87} = \frac{C_{74}}{r_1^2-r_2^2}$; $C_{88} = \frac{C_{75}}{r_1^2-r_2^2}$; $C_{89} = -\frac{C_{76}}{r_2^2}$; $C_{90} = \frac{C_{77}}{2r_2^2}$; $C_{91} = \frac{C_{78}}{2r_2^2}$; $C_{92} = \frac{C_{79}}{r_3^2-r_2^2}$; $C_{93} = \frac{C_{80}}{r_3^2-r_2^2}$;

$A_{23} = -(C_{81} + C_{83} + C_{87} + C_{93})$;