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A Functional Time System Connectivity Analysis in Multi-Stage Processes

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Abstract

The research urgency of the problem under consideration is caused by the need to optimize discrete dynamical systems in industry, agriculture, technology, transport, economics, etc. in terms of systems theory for their sustainable development.

Dynamic programming and the bang-bang principle used in solving multi-step optimization tasks in the external (physical) time do not assume a systematic approach.

The goal of this article is to construct a mathematical model for optimizing multi-stage processes and to determine the harmony principle in terms of systems theory; the processes which basic representation is a symmetry group formulated by an objective function.

The research method, which becomes the leading approach to solving this problem, is the revealing of symmetry group in discrete technological, energy, kinematic, economic flows accompanying multi-stage processes in the form of a multiplicative group for multiplication by a set of real numbers defined by a corresponding objective function.

A set of objective functions formulate functional time periods on the basis of which a multi-stage process becomes a system in the form of a single deeply integrated structure. The bang-bang principle of multi-stage processes in the context of the functional time is reduced to synchronization of the functional time at individual stages. Therefore, the system harmony principle should be realized by superposing the bang-bang principle of multi-step processes in the external (physical) and the internal (functional) time.

As a result of this research, a mathematical model of harmonious representation of technical and economic efficiency of dynamic multi-step processes in their functional time periods is given; the most informative efficiency criteria, dimensionality of functional space of accompanying flows, the bang-bang principle, etc., are formulated.

The bang-bang principle of multi-stage processes in their functional time is reduced to synchronization of functional time periods of their individual stages. Therefore, the system harmony principle should be realized by superposing integrated bang-bang principles of multi-step processes in their external (physical) and internal (functional) time.

The content of the article can be useful for mathematicians engaged in optimization tasks, for engineers and designers who can apply this research in their practical activities.

Keywords: objective function, correlation, operator, group, harmony

INTRODUCTION

Many production, energy, gas transport, logistics, information, financial, etc. systems are discrete, deeply integrable structures, which optimal functioning is relevant [1, 2, 3, 4, 5].

A sufficiently large number of studies have considered issues of optimal control of processes in their external (physical) time [6, 7, 8, 9, 10]. Dynamic programming and the bang-bang principle, used in solving multi-step optimization tasks in external (physical) time, do not assume a systematic approach [11, 12,13, 14, 15].

At the same time, it should be kept mind that in a system, as a functionally linked set of subsystems, it is necessary to solve optimization problems in its internal (functional) time, integrated with its external time period by the group representation [16, 17].

RESEARCH GOAL AND OBJECTIVES

Increasing technical and economic efficiency of multi-stage processes is associated with the need to represent them as single deeply integrated systems. A symmetry group, which is formulated by an objective function, is the basic representation of a system [18, 19]. The group representation defines two linked semi-groups of state parameters in the external (physical) and the internal (functional) time, dynamic connectivity of which provides not only optimization, but also observance the harmony principle necessary for sustainable development. In the external (physical) time, subsystems are optimized and functional time links them into a single integrated structure.

In general, system optimization is a multi-criterial issue, therefore its objective function is formulated as a superposition of objective functions using a set of determining state parameters: technological, energy, transport, financial, information, etc., which in turn define a set of functional time, which determine operation of a system.

The time factor is the key attribute in operation of systems. For discrete processes, the principle of maximum performance (the bang-bang principle), formulated using the external time, should be integrated with the bang-bang principle formulated using functional (internal) time.

RESEARCH METHODOLOGY

Mathematical modelling of control in multi-stage processes is done using their linking internal functional time, are integrated with the relevant external time.

Dynamic effectiveness of processes should be considered in superposition of time : the external (physical) and the internal (functional), the latter is integrated (dually) with the former [16, 17].

The external time serves for parametrization of state of individual stages, and the functional time, as their linker, transforms a multi-stage process into a single deeply integrated and functionally related structure which effectiveness is determined by the process total functional time.

The external and the internal time complement each other, carrying a different information load in relation to a system: the first is necessary to characterize functional parameters of processes per unit time (physical), and the second characterizes the functional time necessary to obtain the process unit parameter.

RESEARCH RESULTS

Analytical studies on the control effectiveness in multi-stage deterministic processes with the use of the functional time are feasible for the following basic process types: simple, splitting, merging and complex, which can be combined into multi-variant compositions [20].

A change in a stage state parameter is completely determined by control, which is regarded as small with respect to the stationary state.

Simple controllable multi-stage process. An example of a simple linked controllable multi-step process is shown in Figure 1.

Each stage in a system is characterized by controllable parameters: productivity of production means *P*, capacity of equipment *N*, flow rate *U*, cost *C*, etc., determined in units of the physical time.

To compare the technical and economic efficiency of systems, it is necessary to introduce the efficiency coefficient as the ratio of the efficiency in the functional time to the efficiency in the external time, according to given parameters.

Productivity. The formula of productivity at an individual stage is written in the following form:

$$P_{is} = P + O\,(P) = P + P_+ = P(1+ P_+ /P) = P \exp(P_+ /P) = P_* \quad (1)$$

where O is the control operator, P is the stationary production capacity, P_+ is the control variation.

The formula of functional time per unit of production at a controllable stage is written in the following form:

$$\tau_p = 1 / [P \exp(P_+ /P)] = 1 / P_*, \quad (2)$$

this follows from the representation of production process integration at an individual stage in the external and the functional time, as a multiplicative group [4].

$$P_* P^{-1}_* = 1 ,$$

Further, this representation corresponds to the subsequent state parameters (capacity, flow rate and cost).

The formula of functional cycle time per unit of production in a system is written in the following form:

$$T_{pc} = \sum 1/ [P_i \exp(P_{+i} /P_i)] = \sum 1/ P_{i*} \qquad i = 1, 2, 3, .., n , \quad (3)$$

where n is the number of stages;

With a continuous process, the functional cycle time per unit of production is calculated using the following formula:

$$T_{p1} = n^{-1} \sum 1/ [P_i \exp(P_{+i} /P_i)] = n^{-1} \sum 1/ P_{i*} , \quad i = 1, 2, 3, .., n . \quad (4)$$

The formulas (3) and (4) allow to define the productivity of a system in the functional time of a cycle, using the following formula:

$$P_c = 1 /\sum 1/ [P_i \exp(P_{+\,i} /P_i)] = 1/ \sum 1/ P_{i*} \qquad i = 1, 2, 3, .., n , \quad (5)$$

and with a continuous process:

$$P_{cc} = n /\sum 1/ [P_i \exp(P_i /P_i)] = n/ \sum 1/ P_{i*} \quad i = 1, 2, 3, .., n . \quad (6)$$

The process efficiency coefficient is defined as the ratio of the effective productivity in the functional time of a system to the total stationary productivity at all stages in the external time:

$$K_p = P_{cc} / \sum \Pi_i , \quad i = 1, 2, 3, .., n . \quad (7)$$

Capacity. The controllable capacity at an individual stage is written in the following formula:

$$N \rightarrow O_n(N) = N + N_+ = N(1+ N_+ /N) = N \exp(N_+ /N) = N_* ,$$

where O_n is the capacity control operator; *N* is the stationary capacity, represented as the ratio amount of energy per unit time, N_+ is the control variation.

The functional time per unit of energy at an individual stage is defined by the following formula:

$$\tau_e. = 1 / [N \exp(N_+ /N)] = 1/N_* ,$$

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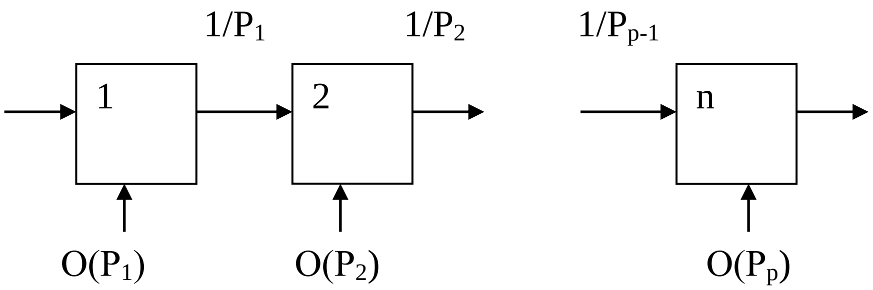

Figure 1. A schematic diagram of a controllable simple multi-stage process (vertical arrows denote control parameters at the stages).

the functional time per unit of energy in a system can be obtained from:

$$T_e = n^{-1}\sum\tau_{ei} \;, \quad i = 1, 2, 3, .., n \;,$$

The effective power of an equipment unit at a stage in a multi-stage complex is defined using the following formula:

$$N_{p1} = 1/T_e \;, \tag{8}$$

and the whole complex in the functional time:

$$N_p = n\,N_{n1} \;. \tag{9}$$

The energy efficiency of a system should be determined by the coefficient:

$$K_e = N_p / \sum N_i \;, \; i = 1, 2, 3, .., n \;, \tag{10}$$

as the ratio of the capacity of a system in the functional time to the total stationary capacity at all stages in the physical time.

Flow rate. The controllable flow rate of a process is defined by the following formula:

$$U \rightarrow O_u U = U + U_+ = U(1 + U_+ /U) = U \exp(U_+ /U) = U_*$$

where O_u is the flow rate control operator at an individual stage, U is the stationary flow rate, U_+ is the flow rate control variation.

The flow rate U_* can be represented by its inverse value:

$$\tau_x = 1 /[U \exp(U_+ /U)] = 1/U_* \;,$$

containing information about the process functional time at an individual stage per unit duration.

The total functional cycle time per unit duration is calculated according to the following formula:

$$T_x = \sum \tau_{xj}, \quad i = 1, 2, 3, \ldots, n,$$

with a continuous process, the functional time per unit duration is formulated as follows:

$$T_* = T_x / n \;.$$

The cycle flow rate in the functional time is calculated in accordance with the following formula:

$$U_c = 1 / T_x, \tag{11}$$

with a continuous process, the flow rate is formulated as follows:

$$U_c = 1 / T_* \;. \tag{12}$$

The efficiency of the flow rate for a process in whole is estimated by the kinematic coefficient as follows:

$$K_к = n\,U_c / \sum U_i \;, \;, \; i = 1, 2, 3, .., n \;. \tag{13}$$

Cost. The controllable cost at an individual stage of a process is defined by the following formula:

$$U \rightarrow O_c (C) = C + C_+ = C(1+ C_+ /C) = C \exp(C_+ /C) = C_* \;,$$

where O_c is the cost control operator at an individual stage of a process, C is the stationary cost, C_+ is the cost variation.

The functional time of controllable cost of a technological operation at an individual stage is defined by the amount of time spent for execution of the technological operation per financial unit (rouble, etc.):

$$\tau_c = 1 / [\, C \exp(C_+ /C) \,] = 1/C_* \;,$$

The total functional time of cycle cost in a system is determined by n financial units:

$$\tau_{cp} = \sum \tau_{c\,i} \;, \;, \; i = 1, 2, 3, .., n \;,$$

the functional time for performing a system technological operation per financial unit can be obtained from:

$$\tau_{p1} = n^{-1} \sum \tau_{ci} \;, \;, \; i = 1, 2, 3, .., n \;,$$

hence, the cost of the whole process in the functional time is found using the following formula:

$$C_c = n / \tau_{cp} \;. \tag{14}$$

The system cost efficiency coefficient in the functional time is obtained from:

$$K_c = C_c / \sum C_i \;, \;, \; i = 1, 2, 3, .., n \;. \tag{15}$$

Controllable splitting multi-stage process.

Figure 2 shows a schematic diagram of a controllable splitting multi-step process characterized by parameters of splitting: α and β ($\alpha + \beta = 1$).

Productivity. After the nth stage there is a splitting of flows. Taking into account the above mentioned, the controllable performance is calculated using the following formula(hereinafter the subscript "*" denotes the value of the controllable parameters, the analytical representation of which is given above):

$$P_{p*} = P_{p\alpha*} + P_{p\beta*} ,$$

$$P_{p\alpha*} = P_{p*}\ \alpha , \quad P_{p\beta*} = P_{p*}\ \beta ,$$

the controllable productivity after the nth stage is:

$$P_{p*} = n / (\sum P^{-1}_{i*}) , \quad i = 1, 2, 3, .., , n ,$$

the controllable productivity for stages in a group

$$j = 1, 2, .., m ,$$

is

$$P_{m*} = (m+1) / [1/ P_{p\alpha*} + \sum P^{-1}_{j*}] , \quad j = 1, 2,. \ldots, m,$$

and, for a stage in the group

$$l = 1, 2, \ldots. , k ,$$

hence, the controllable productivity is:

$$P_{k*} = (k + 1) / [1/ P_{p\beta*} + \sum P^{-1}_{l*}] , \quad l = 1, 2, \ldots, k . \quad (16)$$

where the coefficient of effective productivity is found from the formula:

$$K_{ep} = (P_{m*} + P_{k*}) / (\sum P_i + \sum P_j + \sum P_l) , \quad (17)$$

$$i = 1, 2, .., n; \ j = 1, 2, \ldots. , m ; \ l =1, 2,.., k .$$

Controllable capacity. The controllable capacity for stages before splitting is obtained from:

$$N_{n*} = n^2 / \sum N^{-1}_{i*} , \ i = 1, 2, 3, .. ., n ,$$

after splitting:

$$N_{m*} = (m + 1)^2 / [1 / N_{n*} \alpha + \sum N^{-1}_{j*}] , \quad j = 1, 2, \ldots, m, \quad (18)$$

and

$$N_{k*} = (k + 1)^2 / [1 / N_{n*} \beta + \sum N^{-1}_{l*}] , \quad l = n+1, \ldots, k . \quad (19)$$

Where the energy efficiency coefficient is defined using the following formula:

$$K_e = (N_{m*} + N_{k*}) / (\sum N_i + \sum N_j + \sum N_l) , \quad (20)$$

$$i = 1, 2, .., n, \ j = 1, 2, .., m; \ l = n + 1, \ldots. , k .$$

Controllable flow rate. The controllable flow rates for split flows in the functional kinematic time are calculated using the relevant formulas:

$$U_{m*} = (n+ m) / [\sum(U^{-1}_{i*} + U^{-}_{1j*})] , \quad i = 1, 2, .., n, \ j =1, 2, \ldots, m ,$$

and

$$U_{k*} = (n+ k) /[\sum (U^{-1}_{\backslash i*} + U^{-1}_{l*})], \quad i = 1, 2, .., n, l = 1, 2, \ldots, k.$$

The coefficients of kinematic productivity is found from the formula:

$$K_{km} = (n+m)\, U_{m*} /[\sum (U_{i*} + U^{-1}_{j*})], \quad i = 1, 2, .., n, \ j= 1, 2, \ldots, m ,$$

and

$$K_{kk} = (n+k)\, U_{k*} / [\sum(U_{i*} + U^{-1}_{l*})] , \quad i = 1, 2, .., n, l= 1, 2, \ldots, k .$$

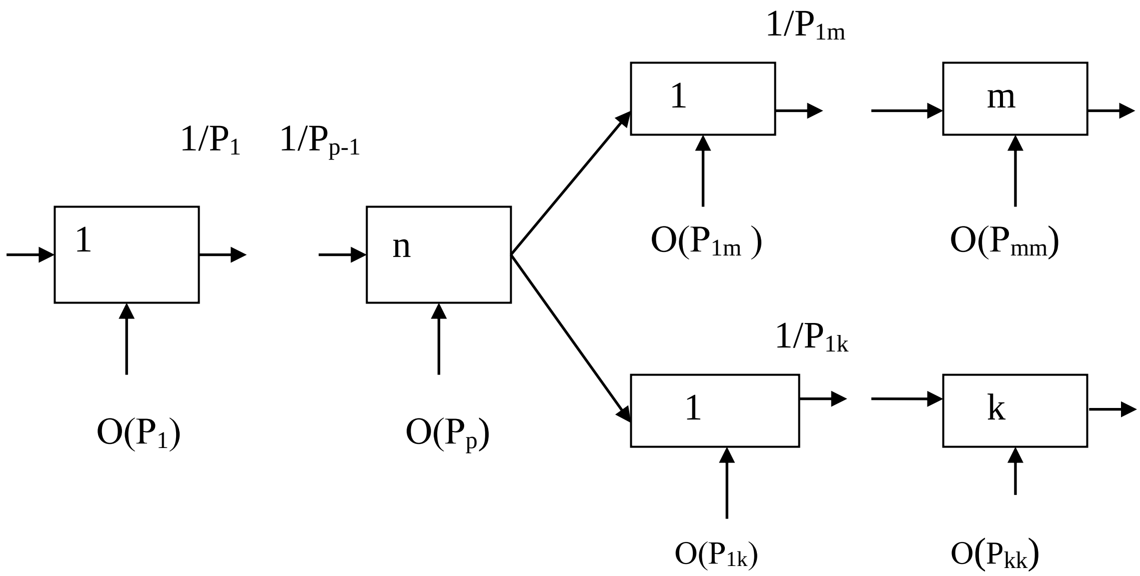

Figure 2. A schematic diagram of a controllable splitting multi-step process

Controllable cost. On the basis of previous representations, the controllable cost for split flows in the functional time is estimated using the following expressions: the cost of steps before spitting

$$C_n = n^2 / \sum C^{-1}_{i*} \,, \quad i = 1, 2, \ldots, n \,, \qquad (21)$$

and after splitting

$$C_m = (m + 1)^2 / [\, 1 / C_n \,\alpha + \sum C^{-1}_{j*} \,] \,, \quad j = 1, 2, \ldots, m, \qquad (22)$$

and

$$C_k = (k + 1)^2 / [\, 1 / C_n \,\beta + \sum C_{l*} \,] \,, \quad l = 1, 2, \ldots, k \,. \qquad (23)$$

where the cost efficiency coefficient can be found from:

$$K_e = (C_m + C_k) / \left(\sum C_i + \sum C_j + \sum C_l \right) , \qquad (24)$$

$$i = 1, 2, .., n, \; j = 1, 2, \ldots, m; \; l = 1, 2, \ldots, k \,.$$

Controllable merging multi-stage process.

Figure 3 shows a schematic diagram of a controllable merging multi-step process.

Controllable productivity. The productivity of flows before splitting is defined by the expression:

$$P_{mp} = n / \sum P^{-1}_{i*} \,, \quad i = 1, 2, 3, .., n \,,$$

and (25)

$$P_{mm} = m / \xi_m \sum P^{-1}_{j*} \,, \quad j = 1, 2, 3, .., m..$$

As a result of flow merging, the productivity of the system under consideration is obtained from:

$$P_k = (k+ 1) / \{ [\, ½ \,(1/P_{mp*} + 1/\Pi_{mm*}\,) + \sum P^{-1}_{l*} \,]\} \,, \quad l = 1, 2,.., k \,, \qquad (26)$$

and the productivity efficiency coefficient

$$K_p = P_k / \left(\sum P_{i*} + \sum P_{j*} + \sum P_{l*}\right), \qquad (27)$$

$$i = 1, 2, 3, .., n; \quad j = 1, 2, 3, .., m \,; \quad l = 1, 2,.., k \,.$$

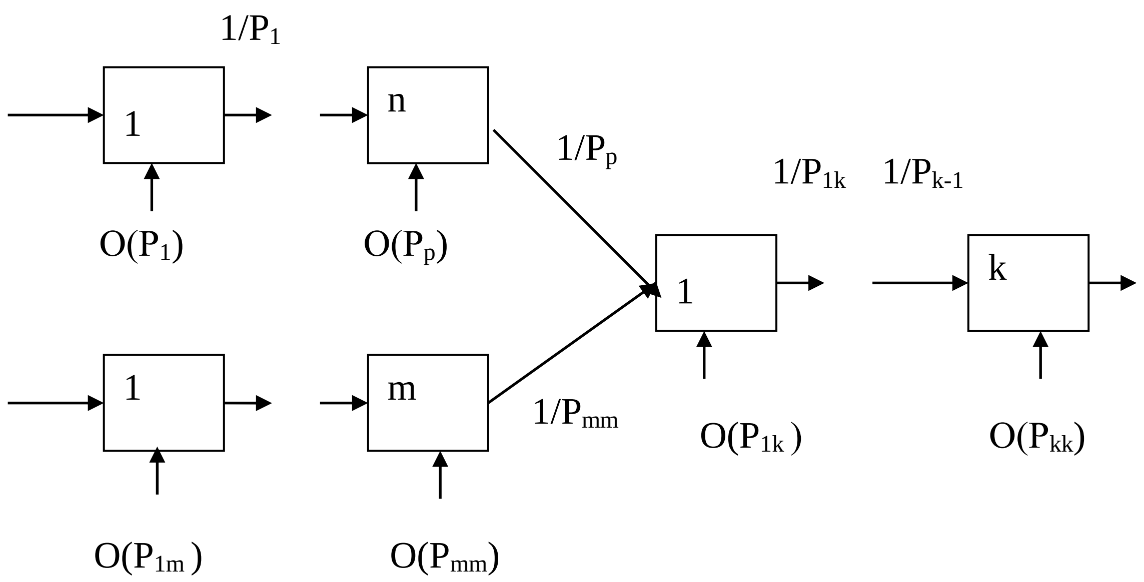

Figure 3. Schematic diagram of a controllable merging multi-step process (vertical arrows denote control).

Controllable capacity. The controllable capacity for stages before merging is obtained from:

$$N_p = n^2 / \sum N^{-1}_{i*} \,, \quad i = 1, 2, 3, .., n \,,$$

and

$$N_m = m^2 / \sum N^{-1}_{j*} \,, \quad j = 1, 2, 3, .., m.$$

As a result of flaw merging and the subsequent process flows, the capacity of the considered merging multi-stage system can be obtained from:

$$N_k = (k+ 1)^2 / \{[½ \,(1/N_p + 1/N_m) + \sum N^{-1}_{l*]} \,]\} \,, \quad l = 1, 2,.., k \,, \qquad (28)$$

and the energy efficiency coefficient

$$K_e = N_k / \left(\sum N_i + \sum N_j + \sum N_l\right),$$

$$i = 1, 2, 3, .., n; \quad j = 1, 2, 3, .., m \,; \quad l = 1, 2,.., k \,.$$

Controllable flow rate. The controllable flow rates before merging are calculated in the functional kinematic time using the relevant formulas:

$$U_p = n / \sum U^{-1}_{i*} \,, \quad i = 1, 2, 3, .., n \,,$$

and

$$U_m = m / \sum U^{-1}_{j*} \,, \quad j = 1, 2, 3, .., m..$$

As a result of flow merging, the flow rate of the system under consideration is obtained from:

$$U_k = (k+1) / \{ \tfrac{1}{2} \ (1/U_p + 1/U_m) + \sum U^{-1}_{l*}] \} , \quad l = 1, 2,.., k ,$$

and the flow rate efficiency coefficient

$$K_k = (n+m+k) \ U_к / (\sum U_i + \sum U_j + \sum U_l), \qquad (29)$$

$$i = 1, 2, 3, .., n; \quad j = 1, 2, 3, .., m ; \quad l = 1, 2,.., k .$$

Controllable cost. The flow costs in the functional time before merging are found from the following expressions:

$$C_p = n^2 / \sum C^{-1}_{i*} , \quad i = 1, 2, 3, ..,n ,$$

and

$$C_m = m^2 / \sum C^{-1}_{j*} , \ j = 1, 2, 3, .., m.$$

As a result of flaw merging and the subsequent process flows, the cost of the considered merging multi-stage system can be obtained from:

$$C_k = (k+1)^2 / \{ [\tfrac{1}{2} (1/C_p + 1/C_m) + \sum C^{-1}_{l\,]*}] \} , \quad l = 1, 2,.., k , \quad (30)$$

and the cost efficiency coefficient

$$K_c = C_k / (\sum C_{i*} + \sum C_{j*} + \sum C_{l*}),$$

$$i = 1, 2, 3, .., n; \quad j = 1, 2, 3, .., m ; \quad l = 1, 2,.., k .$$

Complex controllable multi-stage process. Figure 4 shows a schematic diagram of a complex controllable multi-step process. On the basis of previous representations, the formulas for calculating the considered system state parameters are given below.

Controllable productivity. The productivity before splitting is can be found from:

$$P_p = п / \sum P^{-1}_{i*} , \ i = 1, 2,.., n ,$$

and after splitting:

$$P_{pα} = P_{p*} α , \ P_{pβ} = P_{p*} β ,$$

$$P_m = (m+1) / [\ (\sum P^{-1}_{j*} + P^{-1}_{nα})] , \ j = 1, 2,…, m ,$$

$$P_k = (k+1) / [\ (\sum P^{-1}_{l*} + P^{-1}_{nβ})] , \ l = 1, 2,…, k ,$$

and for merging:

$$P_{mk} = 2 / \ (P^{-1}_{m} + P^{-1}_{k}) ,$$

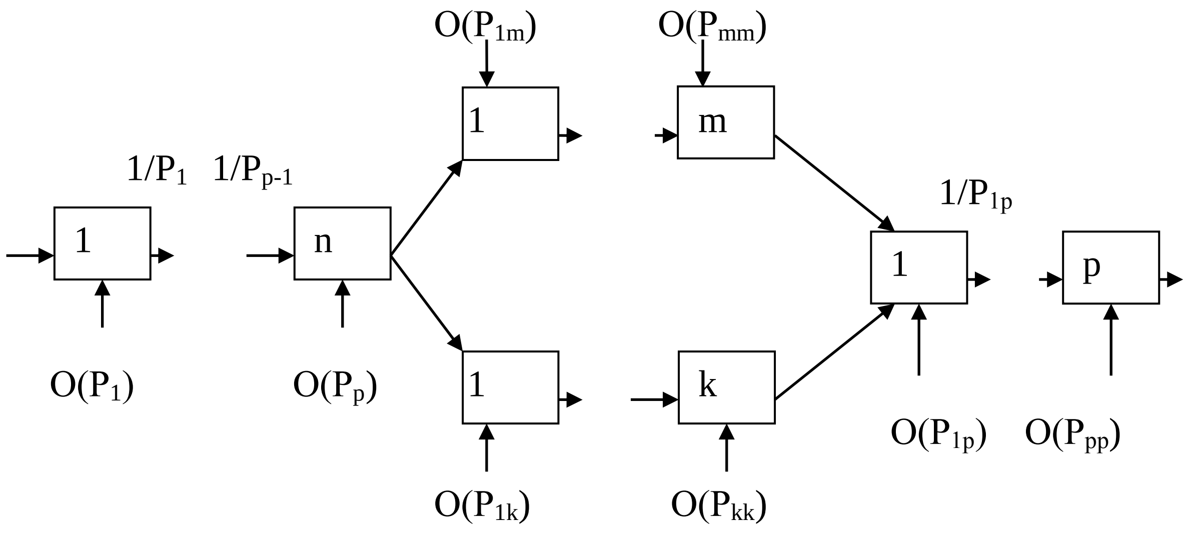

Figure 4. Schematic diagram of a complex controllable multi-step process (vertical arrows denote control)

and productivity at the output of the process:

$$P_p = (p+1) / \ (\sum P^{-1}_{s*} + P^{-1}_{mk}) , \ s = 1, 2, …, p. \qquad (31)$$

The efficiency coefficient of the prossess under consideration is found using the following expression:

$$K_p = P_p / (\sum P_i + \sum P_j + \sum P_l + \sum P_s) ,$$

$$i = 1, 2,.., n , \quad j = 1, 2,…, m , \quad l = 1, 2,…, k , \quad s = 1, 2, …, p.$$

Controllable capacity. The capacity before splitting is written as:

$$N_{p1} = p / \sum N^{-1}_{i*} , \ i = 1, 2,.., n ,$$

and after splitting:

$$N_{pα} = N_{p1} α , \ N_{pβ} = N_{p1} β ,$$

$$N_m = (m+1)^2 / [(\sum N^{-1}_{j*} + N^{-1}_{nα})] , \ j = 1, 2,…, m ,$$

$$N_k = (k+1)^2 / [(\sum N^{-1}_{l*} + N^{-1}_{nβ})] , \ l = 1, 2,…, k ,$$

the effective power after merging

$$N_{mk} = 2^2 / (N^{-1}{}_m + N^{-1}{}_k),$$

$$N_p = (p+1)^2 / (\sum N^{-1}{}_{s*} + N^{-1}{}_{mk}),\ s = 1, 2, \ldots, p. \quad (32)$$

The efficiency coefficient can be obtained from the following expression:

$$K_e = N_p / (\sum N_i + \sum N_j + \sum N_l + \sum N_s),$$

$$i = 1, 2,.., n,\quad j = 1, 2,\ldots, m,\quad l = 1, 2,\ldots, k,\quad s = 1, 2, \ldots, p.$$

Controllable flow rate. The flow rate for spit stages is defined as follows:

$$U_m = (p+m) / [\sum(U^{-1}{}_{i*} + U^{-1}{}_{j*})],$$

$$i = 1, 2,.., n,\ ,\ j = 1, 2,.., m,$$

$$U_k = (n+k) / [\sum(U^{-1}{}_{i*} + U^{-1}{}_{l*})],\ i = 1, 2,\ldots, n,\ l = 1, 2.., k,$$

$$U_{mk} = 2 / (U^{-1}{}_m + U^{-1}{}_k),$$

and at the output:

$$U_p = (p+1) / (\sum U^{-1}{}_{s*} + U^{-1}{}_{mk}),\ s = 1, 2, \ldots, p. \quad (33)$$

The flow rate efficiency coefficient is obtained from:

$$K_k = (n+m+k+p)\, U_p / (\sum U_i + \sum U_j + \sum U_l + \sum U_s),$$

$$i = 1, 2,.., n,\quad j = 1, 2,\ldots, m,\quad l = 1, 2,\ldots, k,\quad s = 1, 2, \ldots, p.$$

Controllable cost. The cost before splitting is defined as follows:

$$C_{p1} = p^2 / \sum C^{-1}{}_{i*},\ i = 1, 2,.., n,$$

and as a result of splitting:

$$C_{p\alpha} = C_{p1}\,\alpha,\ C_{p\beta} = C_{p1}\,\beta,$$

$$C_{m1} = (m+1) / [(\sum C^{-1}{}_{j*} + C^{-1}{}_{n\alpha})],\ j = 1, 2,\ldots, m,$$

$$C_{k1} = (k+1) / [(\sum C^{-1}{}_{l*} + C^{-1}{}_{n\beta})],\ l = 1, 2,\ldots, k,$$

and in case of merging:

$$C_{mk} = 2^2 / (C^{-1}{}_{m1} + C^{-1}{}_{k1}),$$

$$C_p = (p+1)^2 / (\sum C^{-1}{}_{s*} + C^{-1}{}_{mk}),\ s = 1, 2, \ldots, p. \quad (34)$$

The process cost efficiency coefficient can be found using the following expression:

$$K_c = C_p / (\sum C_i + \sum C_j + \sum C_l + \sum C_s), \quad (35)$$

$$i = 1, 2,.., n,\quad j = 1, 2,\ldots, m,\quad l = 1, 2,\ldots, k,\quad s = 1, 2, \ldots, p.$$

Conclusions. The research of multi-stage processes in their functional time, determined in the objective function by group connectivity of the external and the internal time, is necessary to obtain the most informative quantitative assessment of technical and economic efficiency, allowing to solve optimization problems.

The functional time, formulated by the objective function is just the core parameter that turns a multi-stage process into a deeply integrable structure of interlinked substructures.

The bang-bang principle of multi-stage processes in the context of the functional time is reduced to synchronization of the functional time at individual stages. Therefore, the system harmony principle should be realized by superposing the bang-bang principle of multi-step processes in the external (physical) and the internal (functional) time.

A set of integrated flows in a system: manufacturing, energy, kinematic, financial, etc., corresponds to a set of the relevant functional time periods (there are as many integrated flows, as functional time periods) that stipulate the need to analyze systems in a multidimensional phase space based on the coefficients of technical and economic effectiveness. An analysis of the dynamics of the modulus of a vector in this multidimensional phase space allows us to solve optimization problems in multi-stage processes with observance of the principle of harmony.

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