International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 17 (2018) pp. 133340-13343

A New Approach to Fuzzy Soft Connectedness

J.Ruth[1] and S.Selvam[2]

[1]*Department of Mathematics, R.D. Govt. Arts College, Sivagangai-630561, India.*
[2]*Department of Mathematics, Govt. Arts and Science College, Tiruvadanai-623407, India.*

Abstract

In this paper we introduce the idea of connectedness in fuzzy soft topological spaces. We define fuzzy soft connected set with examples and study some of its properties.

Keywords: fuzzy soft set, fuzzy soft topology, fuzzy soft connected set.

2000 AMS Classification: 06D72, 54A05, 54A40.

INTRODUCTION AND PRELIMINARIES

The concept of fuzzy sets are initiated by L.A.Zadeh[7] which is a gener-alization of classical sets. R.Lowen[4] defined the notion of fuzzytopology in a non-empty set. The idea of soft sets was first given by Molodtsov[5]. In[3] the concept of soft topological spaces were developed. Some of the structural properties of soft topology was seenin [2]. Some properties of fuzzy soft topological spaces was studied in[6]. In this paper we approach the notion of connected nessin fuzzy soft topological spaces via Lowen's fuzzy topology. We investigated some theorems related to fuzzy soft connectedness.

The following definitions and theorems are in[1], which are needed for our study.

Through this paper, X be an initial universe and E be the set of all pa rameters for X, I^X is the set of all fuzzy sets on X (where, I=[0,1] and for $\lambda\in[0,1]$, $\bar{\lambda}(x)=\lambda$, for all x∈X.)

Definition[1]. Let A⊆ E. f_A is called a fuzzy soft set on X, where f is a mapping from E into I^X. i.e., fe ≜ f(e) ≜ $f_A(e)$ is a fuzzy set on X for each e ∈ A and $f_e = \bar{0}$ if e ∉ A, where $\bar{0}$ is zero function on X. f_e, for each e ∈ E, is called an element of the fuzzy soft set f_A.

FS(X,E) denotes the collection of all fuzzy soft sets on X and is called a fuzzy soft universe.

In this paper to each parameter e∈ E, f_e is equivalent to $f_e(x)$ for all x ∈ X and if f_B and f_C ∈ FS(X,E) then B⊆ E, C⊆ E.

Definition[1]. For two fuzzy soft sets f_A and g_B on X, we say that f_A is a fuzzy soft subset of g_B and write $f_A \sqsubseteq g_B$ if $f_e \le g_e$, for each e ∈ E.

Definition[1]. Two fuzzy soft sets f_A and g_B on X are called equal if $f_A \sqsupseteq g_B$ and $g_B \sqsupseteq f_A$.

Definition[1]. Union of two fuzzy soft sets f_A and g_B on X is the fuzzy soft set $h_C = f_A \sqcup g_B$, where C = A ∪ B and $h_e = f_e \vee g_e$, for each e ∈ E. That is, $h_e = f_e \vee \bar{0} = f_e$ for each e∈A-B, $h_e = \bar{0} \vee g_e = g_e$ for each e∈B-A and $h_e = f_e \vee g_e$, for each e∈ A ∪ B.

Definition[1]. Intersection of two fuzzy soft sets f_A and g_B on X is the fuzzy soft set $h_C = f_A \sqcap g_B$, where C = A ∩ B and $h_e = f_e \wedge g_e$, foreach e∈E.

Definition[1]. The complement of a fuzzy soft set f_A is denoted by f_A^c, where $f^c : E \to I^X$ is a mapping given by $f_e^c = \bar{1} - f_e$, for each e ∈ E.Clearly $(f_A^c)^c = f_A$.

Definition[1]. (Null fuzzy soft set) A fuzzy soft set f_E on X is called a null fuzzy soft set and denoted by ∅ if $f_e = \bar{0}$, for each e ∈ E.

Definition.[1] (Absolute fuzzy soft set) A fuzzy soft set f_E on X is called an absolute fuzzy soft set and denoted by $\tilde{E}$, if $f_e = \bar{1}$, for each e ∈ E.Clearly $(\tilde{E})^c = \emptyset$

Definition.[1] (λ -absolute fuzzy soft set) A fuzzy soft set f_E on X is called a λ-absolute fuzzy soft set and denoted by $\tilde{E}^\lambda$, if $f_e = \bar{\lambda}$, for each e ∈E. Clearly,

$(\tilde{E}^\lambda)^c = \tilde{E}^{1-\lambda}$.

In this paper we write $\tilde{E}^\lambda$ as $\bar{\lambda}_E$.

Proposition[1]. Let Δ be an index set and f_A, g_B, h_C, $(f_A)_i \triangleq (f_i)_{Ai}$, $(g_B)_i \triangleq (g_i)_{Bi} \in$ FS(X,E), $\forall_i \in \Delta$, then we have the following properties:

(1) $f_A \sqcap f_A = f_A$, $f_A \sqcup f_A = f_A$.

(2) $f_A \sqcap g_B = g_B \sqcap f_A$, $f_A \sqcup g_B = g_B \sqcup f_A$.

(3) $f_A \sqcup (g_B \sqcup h_C) = (f_A \sqcup g_B) \sqcup h_C$.

(4) $f_A = f_A \sqcap (f_A \sqcup g_B)$, $f_A = f_A \sqcup (f_A \sqcap g_B)$

(5) $f_A \sqcap (\sqcup_{i\in\Delta} (g_B)i) = (\sqcup_{i\in\Delta} (f_A \sqcap (g_B)i))$

(6) $f_A \sqcup (\sqcap_{i\in\Delta} (g_B)i) = (\sqcap_{i\in\Delta} (f_A \sqcup (g_B)i))$

(7) $\emptyset \sqsubseteq f_A \sqsubseteq \tilde{E}$.

(8) $(f_A^c)^c = f_A$.

(9) $(\sqcap_{i\in\Delta} (f_A)_i)^c = \sqcup_{i\in\Delta} ((f_A)_i)^c$

(10) $(\sqcup_{i\in\Delta} (f_A)_i)^c = \sqcap_{i\in\Delta} ((f_A)_i)^c$

(11) If $f_A \sqsubseteq g_B$, then $g_B^c \sqsubseteq f_A^c$

Definition.[1]Let f:FS(X,E)→FS(Y,E') and p:E→E' be two functions. consider a fuzzy soft set h_A ∈FS(X,E) where A⊆E.Then the fuzzy soft set f(h_A)∈FS(Y,E'),(the image of h_A)is defined as

$$f(h_A)(\beta) = \begin{cases} \sqcup h_A(\alpha) & if\ \alpha \in p^{-1}(\beta) \cap A \\ \emptyset & if\ p^{-1}(\beta) = \emptyset \end{cases}$$

Where $\beta \in$ B = p(A)⊆ E' and $\emptyset$ denotes the crisp empty set.

Let $h_C \in$ FS(Y,E') where C⊆ E'.then the preimage of h_C,$f^{-1}(h_c)$∈ FS(X,E),defined by $f^{-1}(h_c)(\alpha) = h_c(p(\alpha))$ for $\alpha \in D = p^{-1}(C) \subseteq E$.

FUZZY SOFT TOPOLOGICAL SPACES

Definition. The support of a fuzzy soft set g_E on X is defined as $S(g_E) = \{ e \in E \,/\, g_e > \bar{0} \}$. A fuzzy soft set g_B is said to be finite fuzzy soft set of X iff $S(g_B)$ is a finite parameter set. A fuzzy soft set g_B is said to be countable fuzzy soft set of X iff $S(g_B)$ is a countable parameter set.

Definition. A non empty family $\mathcal{G} \subseteq$ FS(Y,E) of fuzzy soft sets is called fuzzy soft ideal on Y if

i) $i_E \in \mathcal{G}$, $j_E \sqsubseteq i_E$ implies that $j_E \in \mathcal{G}$.

ii) $i_E \in \mathcal{G}$, $j_E \in \mathcal{G}$ implies that $i_E \sqcup j_E \in \mathcal{G}$. (As $\mathcal{G}$is not empty, $\emptyset \in \mathcal{G}$)

Examples.

1. $\mathcal{G}_f$ - is the fuzzy soft ideal of fuzzy soft sets of Y with finite support.

2. $\mathcal{G}_c$ - is the fuzzy soft ideal of fuzzy soft sets of Y with countable support.

3. Let $h_E$be a fixed fuzzy soft set in Y. Then $\mathcal{G}(h_E) = \{I_E \in FS(Y,E) / g_E \sqsubseteq h_E\}$ is a fuzzy soft ideal.

Definition. Let f_C and g_D are two fuzzy soft sets of X. Then f_C "intersection" g_D is redefined as follows:

$f_C \tilde{\sqcap} g_D = \max(\bar{0}, fe + ge - \bar{1})$ for each e ∈ E.

Now we prove a proposition which will be needed in the following part of this paper

Proposition.

If A, B, and C are the subsets of the parameter set E, then $g_C \tilde{\sqcap} (g_A \sqcup g_B) = (g_C \tilde{\sqcap} g_A) \sqcup (g_C \tilde{\sqcap} g_B)$.

Proof. For all e ∈ E,

$g_C \tilde{\sqcap} (g_A \sqcup g_B)(e) = \max\{\bar{0}, g_C(e) + (g_A \sqcup g_B)(e) - \bar{1}\}$

$= \max\{\bar{0}, g_C(e) + g_A(e) - \bar{1}, g_C(e) + g_B(e) - \bar{1}\}$

$= \max\{\max\{\bar{0}, g_C(e) + g_A(e) - \bar{1}\}, \max\{\bar{0}, g_C(e) + g_B(e) - \bar{1}\}\}$

$= \max\{(g_C \tilde{\sqcap} g_A)(e), (g_C \tilde{\sqcap} g_B)(e)\}$

$= ((g_C \tilde{\sqcap} g_A) \sqcup (g_C \tilde{\sqcap} g_B))(e)$.

Proposition. If f : FS(X,E)→ FS(Y,E') and p: E→ E', let $h_A, h_B \in$ FS (Y,E') where

A,B ⊆ E' then $f^{-1}(h_A \tilde{\sqcap} h_B) = f^{-1}(h_A) \tilde{\sqcap} f^{-1}(h_B)$.

Proof: For all e ∈ E,we have

$f^{-1}(h_A \tilde{\sqcap} h_B)(e) = (h_A \tilde{\sqcap} h_B)(p(e))$

$= \max\{h_A(p(e)) + h_B(p(e)) - \bar{1}, \bar{0}\}$

$= \max\{f^{-1}(h_A)(e) + f^{-1}(h_B)(e) - \bar{1}, \bar{0}\}$

$= (f^{-1}(h_A) \tilde{\sqcap} f^{-1}(h_B))(e)$

Definition. A family $\tau \subseteq$ FS(X,E) is called a fuzzy soft topology for X, if it satisfies the following axioms.

i) For all λ∈ [0,1], $\tilde{\lambda}_E \in \tau$.

ii) $f_E, g_E \in \tau$ implies that $f_E \sqcap g_E \in \tau$.

iii) If $\{f_{iE}\}_{i \in \Delta}$ is an indexed subfamily of τ , then $\underset{i \in \Delta}{\sqcup} f_{iE} \in \tau$.

The pair (X, τ) is called a fuzzy soft topological space. The elements of τ are called fuzzy soft open sets.The complement of fuzzy soft open sets are called fuzzy soft closed sets.

Examples for fuzzy soft topological spaces

1. τ = { λ - absolute fuzzy soft sets/ 0 ≤ λ ≤ 1} is a fuzzy soft topology and is called indiscrete fuzzy soft topology on X.

2. If τ equals FS(X,E) then τ is called discrete fuzzy soft topology on X.

3. Let A ⊆ E with |A| > 1. Then $\tau = \{f_E \in FS(X,E) / f_e = \bar{0}$ for all e ∈ A (or) $f_e \neq \bar{0}$ for all e ∈ A} is a fuzzy soft topology on X.

If $\tilde{\lambda}_E = \emptyset$ then $\tilde{\lambda}e = \bar{0}$ for all e ∈ A. Therefore $\emptyset \in \tau$. For all λ ∈ (0,1], $\tilde{\lambda}_e = \bar{\lambda} \neq \bar{0}$for all e ∈ A. Hence $\tilde{\lambda}_E \in \tau$.

Let $\{f_{iE}\}_{i \in \Delta}$ be a collection of elements of τ . If $\underset{j \in \Delta}{\vee} f_{ie_0} \neq \bar{0}$ for some $e_0 \in A$, then $f_{k_{e_0}} \neq \bar{0}$ for atleast one k (say) ∈ Δ. Therefore $f_{k_{e_0}} \neq \bar{0}$ for all e ∈ A. Therefore $\vee f_{k_{e_0}} \neq \bar{0}$ for all e ∈ A. Hence $\underset{i \in \Delta}{\sqcup} f_{iE} \in \tau$.

Let $f_E, g_E \in \tau$, then $f_e = \bar{0}$ for all e ∈ E (or) $f_e \neq \bar{0}$ for all e ∈ E. Similarly we have $g_e = \bar{0}$ for all e ∈ E (or) $g_e \neq \bar{0}$ for all e ∈ E. If either $f_e = \bar{0}$ (or) $g_e = \bar{0}$ for all e ∈ E then $(f \wedge g)_e = \bar{0} \forall$ e∈ E. If $f_e \neq \bar{0}$ (or) $g_e \neq \bar{0} \forall$ e∈ E then $(f \wedge g)_e \neq \bar{0} \forall$ e∈ E. Therefore $f_E, g_E \in \tau \Rightarrow f_e \wedge g_e \in \tau$. Thus τ is a fuzzy soft topology on X

4. Let us take the parameter set E by the set of all natural numbers, for all n ∈ E, consider the subsetA_n = { 2n-1,2n } of E. Clearly by above example 2.8.3 to each n ∈ E,

$\tau_n = \{ f_E \in FS(X,E) / f_e = \bar{0} \forall e \in A_n$ (or) $f_e \neq \bar{0} \forall e \in A_n \}$ is a fuzzy soft topology on X. Hence $\bigcap_{n=1}^{\infty} \tau_n$ is a fuzzy soft topology on X.

So $\tau = \{f_E \in FS(X,E) /$ for all n ∈ E, $f_{2n} = \bar{0}$ if and only if $f_{2n-1} = \bar{0}$ }

Definition. . Let (Y, ζ) be a fuzzy soft topological space.Then the closure of a fuzzy soft set g_E, denoted as $cl(g_E)$, and defined by

$cl(g_E) = \sqcup\{\tilde{\beta}_E / g_B \in \zeta, g_B \sqsupset \tilde{\beta}_A^c \Rightarrow g_E \tilde{\sqcap} g_B \neq \emptyset\}$ Where B⊆ E.

Definition. Let(X,τ) and (Y,σ) be two fuzzy soft topological spaces and p:E → E'. The function h:FS(X,E) → FS(Y,E') is fuzzy soft continuous iff $h^{-1}(g_B) \in \tau$ for all $g_B \in \sigma$ where B ⊆E'.

Proposition let (X,τ) and (Y,σ) be two fuzzy soft topological spaces let g:FS(X,E) →FS(Y,E′)be a fuzzy soft continuous function and p:E→E′.consider a fuzzy soft set $f_A \in FS(Y,E')$ where $A \subseteq E'$. Then $cl(g^{-1}(f_A))(e) \leq g^{-1}(cl(f_A))$ (e) for all e∈ E.

Proof: $cl(g^{-1}(f_A)) = \sqcup\{\tilde{\lambda}_E / f_B \in \tau,\ f_B \sqsupset \tilde{\lambda}_E^c \Rightarrow g^{-1}(f_A)\ \tilde{\sqcap}\ f_B \neq \emptyset\}$.where B⊆ E.

$\sqsubseteq \sqcup\{\tilde{\lambda}_E / f_{B'} \in \sigma, g^{-1}(f_{B'}) \sqsupset \tilde{\lambda}_E^c \Rightarrow g^{-1}(f_A)\ \tilde{\sqcap}\ g^{-1}(f_{B'}) \neq \emptyset\}$ where B'⊆ E′

$= \sqcup\{\tilde{\lambda}_E / f_{B'} \in \sigma,\ f_{B'} \sqsupset \tilde{\lambda}_E^c \Rightarrow g^{-1}(f_A \tilde{\sqcap} f_{B'}) \neq \emptyset\}$ where $f_{B'}(p(\alpha)) = g^{-1}(f_{B'})(\alpha)$ such that $\alpha \in D = p^{-1}(B') \subseteq E$

$\sqsubseteq \sqcup\{\tilde{\lambda}_E / f_{B'} \in \sigma, f_{B'} \sqsupset \tilde{\lambda}_E^c \Rightarrow (f_A \tilde{\sqcap} f_{B'}) \neq \emptyset\}$

$= cl(f_A)$.

That is $cl(g^{-1}(f_A))(e) \leq cl(f_A)(p(e))$ for all e∈ E.

Therefore $cl(g^{-1}(f_A))(e) \leq cl(f_A)(p(e)) = g^{-1}(cl(f_A))(e)$.

FUZZY SOFT CONNECTED SET

Definition let (X,τ) be a fuzzy soft topological space. A fuzzy soft set f_C is said to be fuzzy soft connected set if there exists two fuzzy soft sets f_A and $f_B \in FS(X,E)$ where A and B are subsets of E and C = A ∪ B such that (i) $f_c = f_A \sqcup f_B$ (ii) there exists $e_0 \neq k_0 \in E$ such that $f_A(e_0) = f_c(e_0) \neq \bar{0}$and $f_B(k_0) = f_c(k_0) \neq \bar{0}$(iii) $cl((f_A)\ \tilde{\sqcap}\ f_B = \emptyset = f_A\ \tilde{\sqcap}\ cl(f_B)$.$f_c$ is called fuzzy soft connected if and only if $f_c$is not fuzzy soft disconnected.

Examples

(i) If (X,τ) be a fuzzy soft topological space with the indiscrete fuzzy soft topology, Then the fuzzy soft set f_X is fuzzy soft connected.

(ii) If (X,τ)) be a fuzzy soft topological space with the discrete fuzzy soft topology, Then the fuzzy soft set f_X is fuzzy soft disconnected.

Example

Let (X,τ) be a fuzzy soft topological space with $|E| \geq 2$ and f_A be fuzzy soft set on X such that $f_A \sqsubseteq \tilde{E}^{\frac{1}{2}}$,then f_A isfuzzy soft disconnected.

Let B be the proper subset of E. Take C=E-B. Let f_1 and f_2 be two fuzzy soft sets on E defined by

$$f_1(e) = \begin{cases} \bar{1} & \text{if } e \in B \\ \frac{\bar{1}}{2} & \text{if } e \in C. \end{cases} \quad \text{and} \quad f_2(e) = \begin{cases} \bar{1} & \text{if } e \in C \\ \frac{\bar{1}}{2} & \text{if } e \in B. \end{cases}$$

Let $g_1 = f_1 . f_A$ (that is $g_1(e) = f_1(e).f_A(e)$ for all e∈ E) and $g_2 = f_2 . f_A$.Then (i)

$f_A = g_1 \sqcup g_2$ (ii) $g_1 = f_A$ on B and $g_2 = f_A$ on C (iii) as g_1 , $g_2 \sqsubseteq \tilde{E}^{\frac{1}{2}}$, $cl(g_1) \sqsubseteq \tilde{E}^{\frac{1}{2}}$and $cl(g_2) \sqsubseteq \tilde{E}^{\frac{1}{2}}$ (iv) $g_1 \tilde{\sqcap}\ cl(g_2) = \emptyset = cl((g_1)\ \tilde{\sqcap}\ g_2$.Thus f_A is the fuzzy soft disconnected.

Proposition

Let (X,τ) be a fuzzy soft topological space.Let f_A be fuzzy soft connected set of X and f_B is not fuzzy soft connected set of X with $f_A \sqsubseteq f_B$.If there exists two fuzzy soft sets f_C and f_D such that $f_B = f_C \sqcup f_D$ with

$f_C(x_0) = f_B(x_0) \neq \bar{0}$ and $f_D(y_0) = f_B(y_0) \neq \bar{0}$for some $x_0 \neq y_0 \in E$ and $cl((f_C)\ \tilde{\sqcap}\ f_D = \emptyset = f_C \tilde{\sqcap}\ cl(f_D)$ then either $f_A \sqsubseteq f_C$ or$f_A \sqsubseteq f_D$

Proof. Suppose $f_B = f_C \sqcup f_D$ with $f_C(x_0) = f_B(x_0) \neq \bar{0}$ and $f_D(y_0) = f_B(y_0) \neq \bar{0}$ for some $x_0 \neq y_0 \in E$ and $cl((f_C)\ \tilde{\sqcap}\ f_D = \emptyset = f_C \tilde{\sqcap}\ cl(f_D)$ .Then $cl(f_C \sqcap f_A) \tilde{\sqcap}\ (f_D \sqcap f_A) = \emptyset = (f_C \sqcap f_A)\ \tilde{\sqcap}\ cl(f_D \sqcap f_A)$[since $f_C \sqcap f_A \sqsubseteq f_C$ and $f_D \sqcap f_A \sqsubseteq f_D$] $f_A = f_C \sqcup f_D$ implies that $f_A = (f_C \sqcap f_A) \sqcup (f_D \sqcap f_A)$.select $i_0 \in E$ such that $f_A(i_0) \neq \bar{0}$ .Therefore

$f_A(i_0) = (f_C \sqcap f_A)(i_0)$ or $f_A(i_0) = (f_D \sqcap f_A)(i_0)$.Suppose $f_A(i_0) = (f_C \sqcap f_A)(i_0)$.As f_A is fuzzy soft connected,there is no $j \in E$ such that $f_A(i) = (f_C \sqcap f_A)(i) \neq \bar{0}$,So if

$f_A(i) \neq \bar{0}$.Then $f_D(i) < f_A(i)$ .Therefore $f_A(i) = (f_C \sqcap f_A)(i)$ for all $i \in E$.Hence

$f_A \sqsubseteq f_C$ .Similarly $f_A(i_0) = (f_D \sqcap f_A)(i_0)$ then $f_A \sqsubseteq f_D$.Hence either $f_A \sqsubseteq f_C$ (or) $f_A \sqsubseteq f_D$

Proposition.

Let $\{f_{A\alpha}\}_{\alpha \in J}$ be a collection of fuzzy soft connected sets of X with $\tilde{\sqcap}_{\alpha \in J} f_{A_\alpha} \neq \emptyset$ then $\sqcup_{\alpha \in J} f_{A_\alpha}$ is fuzzy soft connected.

Proof. $\tilde{\sqcap}_{\alpha \in J} f_{A_\alpha} \neq \emptyset$ implies that there exists$e_0 \in E$ such that $(\tilde{\sqcap}_{\alpha \in J} f_{A_\alpha}(e_0) \neq \bar{0}$. That is $(\tilde{\sqcap}_{\alpha \in K} f_{A_\alpha})(e_0) \neq \bar{0}$ for every finite subset K of J............(1)

Obviouslyto each, $f_{A_\alpha}(e_0) \neq \bar{0}$.Therefore$(\sqcup_{\alpha \in J} f_{A_\alpha})(e_0) \neq \bar{0}$. Let $g_B = \sqcup_{\alpha \in J} f_{A_\alpha}$ .We will prove that g_B is the fuzzy soft connected. Suppose g_B is not fuzzy soft connected,then there exists two fuzzy soft sets f_C and f_D such that (i) $g_B = f_C \sqcup f_D$ (ii) there exists $i_0 \neq j_0 \in E$ such that $g_B(i_0) = f_C(i_0) \neq \bar{0}$ and $g_B(j_0) = f_D(j_0) \neq \bar{0}$.

(iii) $cl((f_C)\ \tilde{\sqcap}\ f_D = \emptyset = f_C \tilde{\sqcap}\ cl(f_D)$.Then by proposition 3.4.for each $\alpha \in J$,either

$f_{A_\alpha} \sqsubseteq f_C$ (or)$f_{A_\alpha} \sqsubseteq f_D$ .If there are$\alpha_1 \neq \alpha_2 \in J$ such that $f_{A_{\alpha 1}} \sqsubseteq f_C$ and $f_{A_{\alpha 2}} \sqsubseteq f_D$,

Then $\emptyset \neq f_{A_{\alpha 1}}\ \tilde{\sqcap}\ f_{A_{\alpha 2}} \sqsubseteq f_C \tilde{\sqcap}\ f_D \sqsubseteq cl((f_C)\ \tilde{\sqcap}\ f_D = \emptyset$,which is a contradiction. Therefore either $f_{A_\alpha} \sqsubseteq f_C$ for all α (or) $f_{A_\alpha} \sqsubseteq f_D$ for all α.

Assume that$f_{A_\alpha} \sqsubseteq f_C$ for all $\alpha \in J$.Then $g_B = \sqcup_{\alpha \in J} f_{A_\alpha} \sqsubseteq f_C$ for all $\alpha \in J$ .Hence $g_B = f_c$ .As $g_B(j_0) = f_D(j_0) \neq \bar{0}$,select a positive integer m such that $\frac{\bar{1}}{m} < f_D(j_0)$.As $g_B(j_0) = (\sqcup_{\alpha \in J} f_{A_\alpha})(j_0)$,select α_2 such that $\frac{\bar{1}}{2m} < g_B(j_0) - \frac{\bar{1}}{2m} < f_{A_{\alpha 2}}(j_0) \leq g_B(j_0) = f_D(j_0)$.Hence $\bar{0} \neq f_{A_{\alpha 2}}(j_0) = (f_{A_{\alpha 2}} \sqcap g_B)(j_0)$

= =($f_{A_{\alpha 2}} \sqcap f_D$)($j_0$) (since$g_B(j_0)=f_D(j_0)$) and for all e≠ j_0,$f_{A_{\alpha 2}}$(e)=($f_{A_{\alpha 2}} \sqcap f_C$)(e)= $\bar{0}$ (otherwise$f_{A_{\alpha 2}}$=($f_{A_{\alpha 2}} \sqcap f_C$) ⊔ ($f_{A_{\alpha 2}} \sqcap f_D$)is fuzzy soft disconnection of $f_{A_{\alpha 2}}$,which is a contradiction).Therefore $f_{A_{\alpha 2}}$(e)= $\bar{0}$for all e≠ j_0 . As $f_{A_{\alpha 2}}(e_0) \neq \bar{0}$, We get e_0=j_0. *As* $f_{A_{\alpha 2}}(e_0) \leq f_C(e_0)$=$f_C(j_0) \leq \bar{1}$-$f_D(j_0)$ (as $f_C \tilde{\sqcap} f_D = \emptyset$)≤ $\bar{1}$-$f_{A_{\alpha 2}}(j_0)$ =$\bar{1}$-$f_{A_{\alpha 2}}(e_0)$.

Hence $f_{A_{\alpha 2}}(e_0) \leq \frac{\bar{1}}{2}$.Select any $\beta_1 \neq \beta_2 \in J$.If g_B)(j_0)-$\frac{\bar{1}}{2m} <$ $f_{A_{\beta 1}}(j_0)$,then $f_{A_{\beta 1}}(e_0) \leq \frac{\bar{1}}{2}$.

If $f_{A_{\beta 1}}(j_0) \leq g_B(j_0)$- $\frac{\bar{1}}{2m} < f_{A_{\beta 1}}(j_0)$,then also $f_{A_{\beta 1}}(e_0) \leq \frac{\bar{1}}{2m}$.That is($f_{A_{\beta 1}} \tilde{\sqcap} f_{A_{\beta 2}}$)($e_0$)= $\bar{0}$,Which implies a contradiction to equation (1).Thus g_B is fuzzy soft connected.

Proposition

The image of a fuzzy soft connected set under a fuzzy soft continuous map is fuzzy soft connected.

Proof. let (X,τ) and (Y,σ) be two fuzzy soft topological spaces,p:E→E′ and g:FS(X,E) →FS(Y,E′)be a fuzzy soft continuous map. Let f_A be fuzzy soft connected set in FS(X,E).We will prove that g(f_A) is a fuzzy soft connected set in FS(Y,E′).suppose g(f_A) is fuzzy soft disconnected. Then there exists f_C and f_D∈FS(Y,E′) such that (i) g(f_A) =$f_C \sqcup f_D$.(ii)there exists $e_1 \neq e_2$ ∈E′

such that f_C (e_1)=g(f_A)(e_1) ≠ $\bar{0}$ and f_D (e_2)=g(f_A)(e_2) ≠ $\bar{0}$ and (iii)cl((f_C) $\tilde{\sqcap}$ f_D= ∅ = $f_C \tilde{\sqcap}$ cl(f_D).As $f_C \tilde{\sqcap} f_D$= ∅,by proposition 2.6. and proposition2.11.

$g^{-1}(f_C) \tilde{\sqcap} g^{-1}(f_D)$=$g^{-1}(f_C \tilde{\sqcap} f_D)$=$g^{-1}(\emptyset)$=∅ and cl($g^{-1}(f_C)$)$\tilde{\sqcap}$ $g^{-1}(f_D) \leq g^{-1}$(cl(f_C))$\widetilde{\sqcap}$ $g^{-1}(f_D)$ = $g^{-1}(\emptyset)$=∅[as cl((f_C) $\tilde{\sqcap}$ f_D= ∅]. That is cl($g^{-1}(f_C)$)$\tilde{\sqcap}$ $g^{-1}(f_D)$=∅ .similarly we can show that g^{-1}((f_A)) $\widetilde{\sqcap\, \mathrm{cl}}$($g^{-1}(f_D)$)= ∅.As g($f_A$)($e_1$) ≠ $\bar{0}$,we get $p^{-1}(e_1) \neq \emptyset$ (empty crisp set) and $\bar{0}$=g(f_A)(e_1)=∨{f_A (α_1)/if $\alpha_1 \in p^{-1}(e_1)$ ∩A} where α_1 ∈B=P(A)⊆E′. find β_1 ∈E such that p(β_1)=e_1 and $f_A(\beta_1) > \bar{0}$.similarly find β_2 ∈E such that p(β_2)=e_2 and $f_A(\beta_2) > \bar{0}$.Therefore

$g^{-1}(f_C)(\beta_1)$=$f_C(p(\beta_1))$=$f_C(e_1) \neq \bar{0}$ and
$f_C(p(\beta_1))$=$f_C(e_1)$=g(f_A)(e_1)

$$=g(f_A(p(\beta_1)))= \vee_{p(e')=p(\beta_1)} f_A(e') \geq f_A(\beta_1) \neq \bar{0} \text{ and}$$

$f_C\big(p(\beta_1)\big)$=f_C(e_1 $\geq f_A(\beta_1) > \bar{0}$.

($g^{-1}(f_C) \sqcap$ f_A)(β_1)=$g^{-1}(f_C)(\beta_1)$∧$f_A(\beta_1)$=$f_C\big(p(\beta_1)\big)$ ∧$f_A(\beta_1)$=$f_C(e_1)$)∧f_A (β_1)

=$f_A(\beta_1) \neq \bar{0}$.similarly $g^{-1}(f_D) \sqcap$ f_A)(β_2)=$g^{-1}(f_D)(\beta_2)$∧$f_A(\beta_2)$=$f_D\big(p(\beta_2)\big)$ ∧$f_A(\beta_2)$=$f_C(e_2)$)∧f_A (β_2)

=$f_A(\beta_2) \neq \bar{0}$.[since $f_D\big(p(\beta_2)\big)$=g(f_A($p(\beta_2)$)≥ $f_A(\beta_2)$].Thus f_A=$g^{-1}(f_C) \sqcap f_A$) ⊔ $g^{-1}(f_D) \sqcap f_A$),which is a contradiction (as $f_A$is fuzzy soft connected). Hence fuzzy soft continuous image of a fuzzy soft connected set is fuzzy soft connected.

CONCLUSION

In this paper we have investigated connectedness in fuzzy soft topological spaces and some of its properties.

REFERENCES

[1] A.Aygunoglu.et.al, *An introduction to fuzzy soft topological spaces*, Hacettepe Journal of Mathematics and Statistics, Volume 43(2)(2014), 193-204.

[2] Bashir Ahmad.et.al, *On some structures of soft topology*, Ahmad and Hussain Mathematical Sciences, 2012, 6:64

[3] A.Kandil.et.al, *Soft ideal theory soft local function and generated soft topological spaces*, Appl.Math.Inf.Sci.8,No.4,(2014),1595-1603.

[4] R.Lowen, *Fuzzy topological spaces and fuzzy compactness*, J.Math.Appl., 56(1976), s621-633.

[5] D.Molodtsov, *Soft set theory - _first results*, Computers and Mathematics with applications 37(1999) 19-31.

[6] Pradip Kumar Gain.et.al., *On some structural properties of fuzzy soft topological spaces*, Intern.J.Fuzzy Mathematical Archive, Vol.I, 2013, 1-15

[7] L.A.Zadeh, *Fuzzy sets*, Information and Control,8(1965), 338-353.