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A Study on Fuzzy (i,j)-β-compact spaces

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Abstract

The aim of this paper is introducing fuzzy (i,j)-β-compact spaces with respect to fuzzy ideals. First we investigate some properties of (i,j)-β-open fuzzy sets and finally we prove some theorems related to fuzzy (i,j)-$\mathcal{J}\beta$-compact spaces.

Keywords: fuzzy bitopological spaces, fuzzy ideals, (i,j)-β-open fuzzy sets , fuzzy (i,j)-$\mathcal{J}\beta$-compact sets.

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INTRODUCTION

Zadeh[11] introduced the definition of fuzzy sets and their properties. Lowen[5] defined the notion of fuzzy topology. Mahomoud[6] studied and investigated about the concept of fuzzy ideal and fuzzy local functions. Nouh[8] initiated the notion of fuzzy bitopological spaces. A fuzzy bitopological space[2] (shortly fbts) is a trible (X, τ_1, τ_2) where X is a non-empty set, τ_1 and τ_2 are any two fuzzy topologies on X. If A is fuzzy set on a fbts X, we denote the closure of A, $cl_i(A)$ and the interior of A, $int_j(A)$ with respect to the two topologies , τ_i and τ_j where $i \neq j \in \{1,2\}$. In this paper we follow the definition of fuzzy topology in Lowen's[5] sence. In Selvam[10] various concepts and properties of fuzzy topological spaces in Lowen's sence were studied, particularly the notion of fuzzy β-compact spaces with respect to fuzzy ideals were investigated. The idea of (i,j)-β-compact spaces was found in [3]. In this paper we study the properties of (i,j)-β-compact spaces with respect to fuzzy ideals.

PRELIMINARIES

Definition 2.1.[3] *Let (X, τ_1, τ_2) be a fbts and $A \in I^X$. Then A is called an (i,j)-β-open fuzzy set if $A \leq cl_j\left(int_i\left(cl_j(A)\right)\right)$*

Definition 2.2.[10] *Let (X, τ_1, τ_2) be a fbts with fuzzy ideal $\mathcal{J}$. A fuzzy set u is said to be fuzzy $\mathcal{J}\beta$-compact relative to X, iff for every family μ of β-open fuzzy sets with $\bigvee_{A\in\mu} A \geq u$ there is a finite subfamily $\eta \subseteq \mu$ such that $\left(u - \bigvee_{A\in\mu} A\right) \in \mathcal{J}$.*

On (i,j)- β-open fuzzy sets

If a fuzzy set $B \in (\tau_1 \cap \tau_2)$ then B is a (i,j)-β-open fuzzy set is both τ_1- open and τ_2- open.

Example 3.1. Let (X, τ_1, τ_2) be a fbts where $\tau_1 = \{$f : X → [0,1] / |f(x) – f(y)|≤ a for x,y ∈ X and a is a fixed real number with 0 < a < 1} and $\tau_2 = \{$f∈ $I^X/$ f : X → [0,1] }. Note that if f∈ τ_2 then f is both τ_2- open and τ_2- closed fuzzy set. B is a τ_1- closed fuzzy set iff (1-B) is τ_1-fuzzy open iff |(1-B)(x) – (1-B)(y)| ≤ a ∀x,y ∈ X iff |B(x) – B(y)| ≤ a ∀x,y ∈ X iff B is τ_1- open . Let h∈ I^X and g(x) = min{h(x), inf(h)+a} ∀x ∈ X. Then g ≤ h. As|g(x) – g(y)| ≤ a, g is τ_1- open fuzzy set. If g' is τ_1- open and $g' \leq$ h then $|g'$(x) - g'(y)| ≤ a ∀x,y ∈ X and g'(x) ≤ h(x) ∀ x ∈ X. So inf(g') ≤ inf(h) and g'(x) ≤ inf(h) + a. Thus g'(x) ≤ h(x)∧(inf(h)+a) = g(x). Therefore g =int_1(h). Therefore $cl_2(int_1$(h)) $= cl_2$(g) = g ≤ h as g is τ_1- closed and τ_1- open. Thus if h $\notin (\tau_1 \cap \tau_2)$ then $cl_2(int_1$(h)) < h. Therefore h is not a (i,j)-β-open fuzzy set [if h∈ $(\tau_1 \cap \tau_2)$, then g = h and h= $cl_1(int_2$(h))].

Next we give an example for a (i,j)-β-open fuzzy set which is not both τ_i- open and τ_j- open.

Example 3.2. Let $Y = \mathcal{R}$(real line) with standard topology σ. Let $\tau_1 = \{\bar{\alpha} \in I^Y/0 \leq \alpha \leq 1\}$ and $\tau_2 = \{$g : $Y \to$ [0,1]/ to each $\lambda \in$ [0,1),{y∈ Y/g(y) > λ} is a σ-open set in $\mathcal{R}$}. Note that f∈ I^Y is τ_1- closed iff (1-f) is τ_1- open. Also f is τ_2-closed iff (1-f) is τ_2- open iff to each λ, { y∈ Y/(1-f)(y) > λ} is an open set in Y } iff to each λ, {y/(1- λ) > f(y)} is an open set in Y}.

Let fϵI^Y defined by $f(y) = \begin{cases} 1 & if\ \ 0 < y \leq 1 \\ 0 & otherwise \end{cases}$

Let f'=$\Psi_{[0,1]}$. So to each $\lambda\epsilon[0,1)$,$\{y\epsilon Y/(1-\lambda)>$ $f'(y)\}$=$(-\infty, 0) \cup (1,\infty)$ and f' is τ_2-closed. Let k' be a τ_2-closed set and f≤ k'. Then to each λ in [0,1), $B = \{y/(1-\lambda) > k'$(y)} ∩ (0,1] = Φ(crisp empty set). Since B={y/(1-λ)> k'(y)} is an open set in $\mathcal{R}$, we get that 0 ∉ B. Thus k'(0)=1 and $\Psi_{[0,1]} \leq k'$. Now we have cl_2(f) = $f' = \Psi_{[0,1]}$. $int_1(cl_2$(f)) =$\Psi_{(0,1)}$ and $cl_2(int_1((cl_2$(f))) = $\Psi_{[0,1]}$)= f'. Therefore f ≤ $cl_2(int_1((cl_2$(f))) . Thus f is a (i,j) -β-open fuzzy set, but f is not both τ_1- open and τ_2- open.

FUZZY A (i,j)-β-COMPACT SPACES WITH FUZZY IDEALS

Definition 4.1. *A fuzzy set f in a fuzzy bitopological space X together with fuzzy ideal $\mathcal{J}$ is said to be fuzzy (i,j)- $\mathcal{J}\beta$-compact relative to X, iff for every family μ of (i,j)- β-open fuzzy sets with $\bigvee_{A\in\mu} A \geq f$ there is a finite subfamily $\eta \subseteq \mu$ such that $\left(f - \bigvee_{A\in\eta} A\right) \in \mathcal{J}$.*

Theorem 4.2. *A fuzzy bitopological space X is fuzzy (i,j))-$\mathcal{J}\beta$- compact iff for every collection $\{A_i : i \in I\}$ of (i,j)-β-closed fuzzy sets of X with $\wedge_{i\in F} A_i \notin \mathcal{J}$ for every finite subfamily $F \subseteq I$, then $\wedge_{i\in I} A_i \neq \bar{0}$.*

Proof Let $\{A_i : i \in I\}$ be a collection of (i,j)-β-closed fuzzy sets with $\wedge_{i\in F} A_i \notin \mathcal{J}$ for every finite subfamily $F \subseteq I$. Suppose $A_i = \bar{0}$. Then $\vee_{i\in I}(1 - A_i) = 1$. That is $\vee_{i\in I} A_i' = 1$ since $\{A_i : i \in I\}$ is a collection of (i,j) - β fuzzy sets of X with covers 1_X, as X is fuzzy (i,j)- $\mathcal{J}\beta$-compact, there is a finite subfamily $M \subseteq I$ such that $(1 - \vee_{i\in M} A_i') \in \mathcal{J}$. That is , this is a contradiction. Therefore $\wedge_{i\in I} A_i \neq \bar{0}$. Conversely, let $\{A_i' : i \in I\}$ be a collection of (i,j)- β-open fuzzy sets cover of X. That is $\vee_{i\in I} A_i' = 1$. Suppose that for every finite subset $F \subseteq I$, we have $(1 - \vee_{i\in F} A_i') \notin \mathcal{J}$. That is $\wedge_{i\in F} A_i \notin \mathcal{J}$. We have $\wedge_{i\in F} A_i \neq \bar{0}$, which implies that 1 - $\wedge_{i\in I} A_i \neq 1$. This is $\vee_{i\in I} A_i' \neq 1$. This is a contradiction to the fact that $\{A_i' : i \in I\}$ is a (i,j)-β open cover of X. Thus X is a fuzzy (i,j)-$\mathcal{J}\beta$-compact.

Definition 4.3. *A collection of fuzzy sets ζ of a fuzzy bitopological space is said to form a fuzzy $\mathcal{J}$- filter bases iff for every finite collection $\{A_i : i = 1,2,3,\ldots\ldots n\}$ of ζ, $\wedge_{i=1}^{n} A_i \notin \mathcal{J}$.*

Theorem 4.4. *A fuzzy bitopological space X is fuzzy (i,j)-$\mathcal{J}\beta$ -compact iff every fuzzy $\mathcal{J}$ -filter bases ζ in X, $\wedge_{G\in\zeta}(i,j) - \beta cl(G) \neq \bar{0}$.*

Proof Suppose for every fuzzy $\mathcal{J}$-filter bases ζ in X, $\wedge_{G\in\zeta}(i,j) - \beta cl(G) \neq \bar{0}$. Let μ be a (i,j)-β- open fuzzy sets covering of X. Then for every finite subcollection $\{B_1, B_2, \ldots\ldots B_n\}$ of μ, $(1 - \vee_{i=1}^{n} B_i) \notin \mathcal{J}$. That is $\wedge_{i=1}^{n} B_i' \notin \mathcal{J}$. Therefore $\{B_i' : B_i \in \mu\} = \zeta$ forms a fuzzy $\mathcal{J}$-filter bases in X. Since μ is a (i,j)- β open fuzzy set cover of X, $\vee_{B_i\in\mu} B_i = 1$. Therefore $\wedge_{B_i'\in\zeta} (i,j) - \beta cl(B_i') = \wedge_{B_i'\in\mu} (i,j) - \beta cl(B_i') = \wedge_{B_i\in\mu} B_i' = \bar{0}$. That is $\wedge_{B_i'\in\zeta} (i,j) - \beta cl(B_i') = 0$, this gives a contradiction. Thus if μ is a (i,j)-β- open fuzzy set covering of X, there exists a finite subcovering $\{B_1, B_2, \ldots\ldots B_n\}$ of μ such that $(1 - \vee_{i=1}^{n} B_i) \in \mathcal{J}$. Therefore X is fuzzy (i,j)-$\mathcal{J}\beta$- compact.

Conversely assume that X is fuzzy (i,j)-$\mathcal{J}\beta$- compact space. If there exists a fuzzy $\mathcal{J}$-filter bases ζ such that $\wedge_{G\in\zeta}(i,j) - \beta cl(G) = \bar{0}$. Then $\vee_{G\in\zeta}\big((i,j) - \beta cl(G)\big)' = 1$. Now $\mu = \left\{\big((i,j) - \beta cl(G)\big)' : G \in \zeta\right\}$ is a (i,j)- β- open fuzzy set cover of X. Since X is a fuzzy (i,j)-$\mathcal{J}\beta$-compact, there exists a finite subcollection $\left\{\big((i,j) - \beta cl(G_i)\big)' : \mathrm{i} = 1,2,\ldots \mathrm{n}\right\}$ such that $\left(1 - \vee_{i=1}^{n} \big((i,j) - \beta cl(G)\big)'\right) \in \mathcal{J}$. That is $(1 - \vee_{i=1}^{n} G_i') \in \mathcal{J}$. That is $\wedge_{i=1}^{n} G_i \in \mathcal{J}$, which gives a contradiction to the fact that G_i are the elements of fuzzy $\mathcal{J}$-filter bases ζ. Therefore $\wedge_{G\in\zeta}(i,j) - \beta cl(G) \neq \bar{0}$ for every fuzzy $\mathcal{J}$-filter bases ζ.

Definition 4.5. A function f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called fuzzy (i,j)-β-continuous(resp. fuzzy (i,j)-$M\beta$-continuous) if the inverse image of every both τ_1- open and τ_2- open fuzzy set in Y(resp. (i,j)-β-open fuzzy set) is (i,j)-β-open fuzzy set in X.

Definition 4.6. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be fuzzy (i,j)-open if $u \in (\tau_1 \cap \tau_2)$ then $f(u) \in (\sigma_1 \cap \sigma_2)$*

Definition 4.7. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called fuzzy (i,j)-$M\beta$ open if u is a fuzzy (i,j)-β open set in X then $f(u)$ is a fuzzy (i,j)-β open set in Y.*

Theorem 4.8. *If a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is fuzzy (i,j)-$M\beta$-continuous and u is a fuzzy (i,j)-$\mathcal{J}\beta$-compact relative to X, then so is $f(u)$.*

Proof Let $\{B_i : i \in I\}$ be a (i,j)-β-open fuzzy covering for f(u). As f is fuzzy (i, j)-Mβ-continuous, we have $\{f^{-1}(B_i) : i \in I\}$ is a (i,j)-β open fuzzy covering of u. Since u is fuzzy (i, j)-$\mathcal{J}$β-compact relative to X; there exists a finite subfamily $\{f^{-1}(B_i) : i = 1,2,\ldots\ldots n\}$ such that $(u - \vee_{i=1}^{n} f^{-1}(B_i)) \in \mathcal{J}$. That is $u - f^{-1}(\vee_{i=1}^{n} B_i) \in \mathcal{J}$. Now $\mathrm{f}\big(u - f^{-1}(\vee_{i=1}^{n} B_i)\big) \in \mathrm{f}(\mathcal{J})$. Then f(u) - $\mathrm{f}\big(f^{-1}(\vee_{i=1}^{n} B_i)\big) \in \mathrm{f}(\mathcal{J})$. That is $(f(u) - \vee_{i=1}^{n} B_i) \in \mathrm{f}(\mathcal{J})$. Therefore f(u) is fuzzy (i,j)- f($\mathcal{J}$)β-compact relative to Y.

Theorem 4.9. *Let $f : X \to Y$ be a fuzzy (i,j)-$M\beta$-open bijective function and Y is fuzzy (i,j)-f($\mathcal{J}$)β-compact then X is fuzzy (i,j)* -$\mathcal{J}$β-compact.

Proof Let $\{B_i : i \in I\}$ be a collection (i,j)- β open fuzzy sets covering of X, then {f(B_i) : $i \in I$} is a (i,j) -β-open fuzzy set covering of Y. As Y is fuzzy f($\mathcal{J}$)β-compact, there exists a finite subset $F \subseteq I$ such that $(1_Y - \vee_{i\in F} f(B_i)) \in$ f($\mathcal{J}$). Since f is a bijective function, $f^{-1}(1_Y - \vee_{i\in F} f(B_i)) \in \mathcal{J}$ and f^{-1}(f(B_i)) = B_i. Therefore $(1_X - \vee_{i\in F} B_i) \in \mathcal{J}$.

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