International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 18 (2018) pp. 13660-13664

A New Efficient Approach to Solve Multi-Objective Transportation Problem in the Fuzzy Environment (Product approach)

M. Afwat A.E.[1], A.A.M. Salama[2], N. Farouk[3,*]

[1]*Professor of Math, Dept. of Physics and Maths, Faculty of engineering, Zagazig University, Egypt.*

[2]*Doctor of Math, Dept. of Physics and Maths, Faculty of engineering, Zagazig University, Egypt.*

[3]*Assistant Lecturer of Engineering Math, Faculty of engineering, Zagazig University, Egypt.*

(Corresponding Author)*

Abstract

In this paper, we present a new method to solve multi-objective transportation problem called (product approach). We use fuzzy programming to convert the objectives which have different units to membership value then aggregate them by product. Our approach is an easy and fast method to find solutions close to the optimum solution. Finally, a numerical example is introduced to illustrate the method.

AMS subject classifications: 90B06, 90C05, 90C70

Keywords

Multi-objective transportation problem, fuzzy membership function, Vogel's approximation method.

INTRODUCTION

Transportation problem (TP) is a special category of linear programming, which is linked up with day-to-day activities in our actual life and mainly deals with logistics.. It aids in resolving problems of distribution and transportation of resources from one position to some other. The commodities are transported from a lot of sources (e.g., factory) to a set of addresses (e.g., warehouse) to conform a certain demand. In our recent situations (TP) not deal with a single objective but nowadays contain a number of conflicting and incommensurable objective functions which can be called multi-objective transportation problem (MOTP). The aim of solving the (MOTP) is to minimize several penalties such as (transportation cost, delivery time… etc), between sources and destinations to find basic feasible or optimum solution to the problem. Many classical methods are used to solve the problem such as north west corner method, minimum cost method and Vogel's approximation method. The transportation problem was firstly studied by Hitchcock in 1947 [1]. Efficient solutions using an application of fuzzy linear programming to (MOTP) were introduced by Bit et al [4]. Solving (MOTP) using a trust-region globalization strategy was presented by Yousria et al [7]. Kundu et al [9] modeled a multi-objective, multi-item solid transportation problem with fuzzy coefficients for the objectives and constraints, then worked by two different methods. Yeola et al [11] introduced a method to solve (MOTP) using a fuzzy programming technique. In this paper, we present a new proposed approach to solve (MOTP) in fuzzy environment to get a basic feasible solution. Our new approach is an easy and uncomplicated method. We illustrate the new approach with a numerical example and compare our results with the other methods.

MATHEMATICAL MODEL

(a) Notations

Before introducing (MOTP) mathematical model. We will clarify some notations.

Let the following:

1. $S_1, S_2, ..., S_m$ indicate (m) sources.
2. $D_1, D_2, ...D_n$ indicate (n) destinations.
3. a_i (i=1,…., m) denote the available units at each m sources.
4. b_j (j=1,…., n) denote the demand units for each n destinations.
5. x_{ij} is the number of units that will be transported from S_i to D_j .
6. C_{ij} stand for any objective(penalty) such as [transportation cost, transportation time, … etc] per unit from S_i to D_j .

(b) Model

The mathematical model of (MOTP) can be expressed as follows:

$$Min \quad Z^k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^k x_{ij}^k \qquad k=1,2,....,p$$

Subject to

$$\sum_{i=1}^{m} x_{ij} = a_i \qquad for\ i = 1,2,....,m$$

$$\sum_{j=1}^{n} x_{ij} = b_j \qquad for\ j = 1,2,....,n$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \qquad (rim\ condition)$$

$$x_{ij} \geq 0$$

Where superscript (k) represents the number of objective functions.

FUZZY PRELIMINARIES

In this paper, we will fuzzify our penalties (objectives) such as (transportation cost, delivery time… etc), to convert them from crisp region to fuzzy region (solution space) to minimize a set of (p) objectives, the membership function used for that is defined as follows:

$$\mu_k(x_{ij}^k) = \begin{cases} 0 & x_{ij}^k \geq U_k \\ \dfrac{U_k - x_{ij}^k}{U_k - L_k} & L_k \leq x_{ij}^k \leq U_k \\ 1 & x_{ij}^k \leq L_k \end{cases} \qquad (1)$$

Note that:

- L_k is the smallest crisp value of x_{ij}^k and U_k is the largest crisp value of x_{ij}^k.
- If $U_k = L_k$ then $\mu_k(x_{ij}^k) = 1$ for all values of k

METHODOLOGY

Step 1: Calculate the membership values using equation (1) for each penalty table.

Step 2: Construct a new table in which each cell is the product of all membership values of the corresponding cells in the three penalties tables.

Step 3: Calculate the difference (Δ) between the highest membership value and the next highest membership value for each row and column.

Step 4: Select the maximum (Δ) and search for the highest membership value in that row or column. When finding that cell we allocate it with the minimum value of a_i or b_j .

Step 5: When making an allocation in a cell and either a row or column or both are satisfied we cancel that row or Column or both and excluded from the next calculations.

Step 6: After elimination of the satisfied row or column or both, we calculate again (Δ) for the remaining rows and columns and making allocation for the maximum membership value cell.

Step 7: Repeat the previous steps until or columns and rows are satisfied.

Step 8: from allocation we get the values of x_{ij} then substitute in the objective functions

<u>Notes:</u>

- If there is a tie in the maximum (Δ) we search for the maximum membership value in the corresponding row or column
- If there is a further tie in the maximum (Δ) and maximum membership value. We select the cell with the highest a_i .

NUMERICAL EXAMPLE

Let us consider the following numerical multi-objective Transportation example presented by Aneja and Nair [2]; Ringuest and Rinks [3] to illustrate our approach. The problem has the following characteristics.

Table 1: Penalty 1 (P_1)

From \ To		Destination					Supply
		D_1	D_2	D_3	D_4	D_5	a_i
Source	S_1	9	12	9	6	9	5
	S_2	7	3	7	7	5	4
	S_3	6	5	9	11	3	2
	S_4	6	8	11	2	2	9
Demand b_j		4	4	6	2	4	20

Table 2: Penalty 2 (P_2)

From \ To		D_1	D_2	D_3	D_4	D_5	Supply a_i
Source	S_1	2	9	8	1	4	5
	S_2	1	9	9	5	2	4
	S_3	8	1	8	4	5	2
	S_4	2	8	6	9	8	9
Demand b_j		4	4	6	2	4	20

Table 3: Penalty 3 (P_3)

From \ To		D_1	D_2	D_3	D_4	D_5	Supply a_i
Source	S_1	2	4	6	3	6	5
	S_2	4	8	4	9	2	4
	S_3	5	3	5	3	6	2
	S_4	6	9	6	3	1	9
Demand b_j		4	4	6	2	4	20

According to the first step we evaluate the membership values of the first penalty (p_1) table, where $U_K = 12$ and $L_K = 2$ then using the equation (1) we get the following table.

Table 4: Membership values for Penalty 1 (P_1)

From \ To		D_1	D_2	D_3	D_4	D_5	Supply a_i
Source	S_1	0.3	0	0.3	0.6	0.3	5
	S_2	0.5	0.9	0.5	0.5	0.7	4
	S_3	0.6	0.7	0.3	0.1	0.9	2
	S_4	0.6	0.4	0.1	1	1	9
Demand b_j		4	4	6	2	4	20

By similarity we do that for (P_2) and (P_3) we get:

Table 5: Membership values for Penalty 2 (P_2)

From \ To		D_1	D_2	D_3	D_4	D_5	Supply a_i
Source	S_1	0.88	0	0.12	1	0.62	5
	S_2	1	0	0	0.5	0.88	4
	S_3	0.12	1	0.12	0.62	0.5	2
	S_4	0.88	0.12	0.38	0	0.12	9
Demand b_j		4	4	6	2	4	20

Table 6: Membership values for Penalty 3 (P_3)

From \ To		D_1	D_2	D_3	D_4	D_5	Supply a_i
Source	S_1	0.88	0.62	0.38	0.75	0.38	5
	S_2	0.62	0.12	0.62	0	0.88	4
	S_3	0.5	0.75	0.5	0.75	0.38	2
	S_4	0.38	0	0.38	0.75	1	9
Demand b_j		4	4	6	2	4	20

Now from the second step we calculate the product of all membership values for the 3 penalties we get the following table.

Table 7: The product of membership values

From \ To		D_1	D_2	D_3	D_4	D_5	Supply a_i
Source	S_1	0.23	0	0.01	0.45	0.07	5
	S_2	0.31	0	0	0	0.54	4
	S_3	0.04	0.52	0.02	0.05	0.17	2
	S_4	0.2	0	0.01	0	0.12	9
Demand b_j		4	4	6	2	4	20

International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 18 (2018) pp. 13660-13664

Now applying the remaining steps in our proposed approach we get:

Table 8: Proposed product approach

From \ To	D_1	D_2	D_3	D_4	D_5	Supply a_i	Δ					
Source S_1	[3] 0.23	0	0.01	[2] 0.45	0.07	5/3/0	0.22	0.22	0.16	0.22←	-	-
Source S_2	0.31	0	0	0	[4] 0.54	4/0	0.23	0.23	0.23	-	-	-
Source S_3	0.04	[2] 0.52	0.02	0.05	0.17	2/0	0.35	-	-	-	-	-
Source S_4	[1] 0.2	[2] 0	[6] 0.01	0	0.12	9/8/2/0	0.08	0.08	0.08	0.19	0.19	0.01
Demand b_j	4/1/0	4/2/0	6/0	2/0	4/0							
Δ	0.08	0.52↑	0.01	0.4	0.37							
	0.08	0	0	0.45↑	0.42							
	0.08	0	0	-	0.42↑							
	0.03	0	0	-	-							
	0.2↑	0	0.01	-	-							
	-	1	0.01↑	-	-							

The set of solution is

$$\{x_{11}=3, x_{14}=2, x_{25}=4, x_{32}=2, x_{41}=1, x_{42}=2, x_{43}=6\}.$$

Now calculate the corresponding objective functions we get

$$Z^1(x)=157,\ Z^2(x)=72 \text{ and } Z^3(x)=86$$

Table 9: Comparison between different approaches

The Name of approach	$Z^1(x)$	$Z^2(x)$	$Z^3(x)$
The fuzzy approach [5]	112	106	80
Interactive approach [6]	127	104	76
Trust Region Approach [7]	144	104	73
Proposed parallel method [11]	157	72	86
Our proposed (product) approach	157	72	86
Ideal Solution	102	72	64

CONCLUSION

In this paper, we propose a new approach name it with (product approach) to solve (MOTP). We use fuzzy programming to convert different penalty units (cost, time,.. etc) to membership value, then aggregate them by product. The main features of our approach can be summarized as follows:

1. It can solve (MOTP) with higher dimensions.
2. It is an easy and uncomplicated method
3. As compared to other approaches, we see its results rather close to the optimum solution.

REFERENCES

[1] Hitchcock, F.L., 1941," The Distribution of a Product from Several Sources to Numerous Localities," Journal of Mathematics and Physics, 20 (1-4), pp. 224-230.

[2] Aneja, Y.P. and Nair, K.P.K., 1979," Bicriteria Transportation problem,"Management Science, 25 (1), pp.73-78.

[3] Ringuest, J.L. and Rinks, D.B., 1987," Interactive solutions for the linear multi-objective transportation problem," European Journal of Operational Research, pp. 96--106.

[4] Bit, A.K., Biswal, M.P. and Alam, S.S., 1992," Fuzzy programming approach to multicriteria decision making transportation problem," Fuzzy Sets and Systems, 50 (2), pp.135-141.

[5] El-Wahed, W.F.A., 2001," A multi-objective transportation problem under fuzziness," Fuzzy Sets and Systems, 117 (1), pp.27-33.

[6] Abd El-Wahed, W.F. and Lee, S.M, 2006," Interactive fuzzy goal programming for multi-objective transportation problems," Omega, 34 (2), pp.158-166.

[7] Abo-elnaga, Y., El-sobky, B. and Zahed, H., 2012," Trust Region Algorithm for Multi-objective Transportation, Assignment, and Transshipment Problems," Life Science Journal, 9(3), pp. 1765-1772.

[8] Zaki, S.A., Mousa, A.A.A., Geneedi, H.M. and Elmekawy, A.Y., 2012" Efficient Multiobjective Genetic Algorithm for Solving Transportation, Assignment, and Transshipment Problems,". Applied Mathematics, 03 (01), pp. 92-99.

[9] Kundu, P., Kar, S. and Maiti, M., 2013," Multi-objective multi-item solid transportation problem in fuzzy environment," Applied Mathematical Modelling, 37 (4), pp.2028-2038.

[10] Jha, P.J. 2014 Operations Research. Mcgraw Hill Publishing Co Ltd.

[11] M.C, Y. and V.A, J. ,2016," Solving Multi-Objective Transportation Problem Using Fuzzy Programming Technique-Parallel Method," International Journal of Recent Scientific Research, 7 (1), pp.8455-8457.