

# Effect of Channel Estimation Errors on the Performance of a M-MRC Receiver over TWDP Fading Channels

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## Abstract

To meet the demand high-speed data requirement multi-antenna (diversity) systems are well-accepted technique. These types of communication receivers use estimators to optimize the performance. However, an ideal estimator is challenging to obtain which leads degradation of the expected performance of the diversity system. This paper analyzes the outage and average bit error rate (ABER) performance of an arbitrary branch maximal ratio combining (MRC) receiver over two wave diffuse power (TWDP) fading channel with errors in the estimator. The analysis of the obtained results shows that the estimation error is a significant component of the design aspect and can't be neglected.

**Keywords:** Estimation error, ABER, MRC receiver, Coherent modulation, TWDP fading.

## INTRODUCTION

To mitigate the adverse result of the fading, diversity technique is popularly employed in the wireless receiver design. Among the primary diversity combining techniques, maximal ratio combining (MRC) deliver optimal performance. However, the performance is highly depended on the channel estimators. The optimum result is theoretically established considering perfect phase and an envelope estimation of the channel [1]. However, in practice, both phase and the envelope estimation errors are typical, and so the optimal receiver design is a somewhat challenging task [2]. In this paper the performance degradation of the MRC receiver by phase and envelop estimation error has been analyzed, in terms of outage probability and ABER. From the analysis, it can be seen that the receiver performance is affected severely by the envelope estimation error than the phase estimation error. Two wave diffused power (TWDP)

distributed fading model is a new model proposed in the year 2002 [3]. It consists of two dominant paths along with the diffusely propagating wave components and satisfactorily agrees with the practically obtained data. As a particular case the other well-accepted fading models Rayleigh, Rician, One wave fading models can be derived from this. In a propagation scenario like narrow band receiver operation, wideband signals, directional antennas, etc. TWDP may be observed [4]. The closed form expression of this fading model is not available, however; a sufficiently good approximate closed-form expression has been given in [4] for different values of 'K' and 'Δ'. The performance analysis of various diversity receivers over TWDP fading channels with perfect channel estimation is available in the literature [5]-[10]. The performance analysis is done for binary shift keying (BPSK) pre detection MRC receiver over TWDP fading in [5]. In [6] following a moment generating function the average bit error rate of the MRC receiver over TWDP fading channel has been carried out. The capacity analysis of arbitrary branches of selection combining (SC) receiver has been introduced in [7]. The symbol error rate of an SC receiver has been presented in [8] for M-ary phase shift keying and quadrature amplitude modulation (QAM) over TWDP fading channel. The performance of arbitrary branches of SC receiver has been studied in [9]. In [10] the outage probability and average bit error rate of arbitrary branches of an MRC receiver over TWDP fading channel have been presented.

After careful inspection of the literature related to the TWDP fading channels with diversity receivers, we found that the estimation error has not been considered anywhere. This paper presents the performance of an arbitrary branch MRC receiver over TWDP fading channels, considering both phase and envelope estimation error.

## CHANNELS AND SYSTEM MODEL

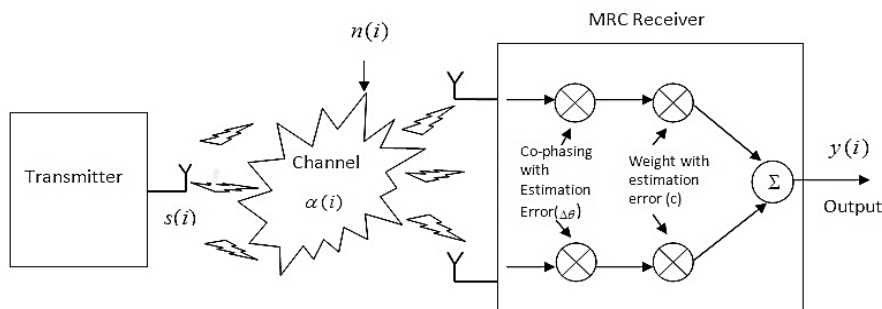


Figure 1: System Model

The system considered to receive multi-path signals has an M-MRC system with arbitrary nos of antennas to improve the signal quality as demonstrated in Fig. 1. The channel in this model is assumed to be slow and frequency non-selective with TWDP distribution. The received signal over 'i<sup>th</sup>' bit duration can be given as,

$$y(i) = \alpha(i)s(i) + n(i) \quad (1)$$

where,  $s(i)$  is the transmitted symbol in 'i<sup>th</sup>' interval, with energy  $E_s = E[|s(i)|^2]$  and noise vector  $n(i) = [n_1(i), n_2(i), \dots, n_n(i)]^T$ , is the complex Gaussian noise having zero mean and two sided power spectral density  $2N_0$ . In the considered model,  $\alpha(i) = [\alpha_1(i), \alpha_2(i), \dots, \alpha_n(i)]^T$  is the channel coefficient vector for 'M' branches and its elements are TWDP distributed. The envelop pdf of a TWDP distributed receive signal is given by [4],

$$f_\alpha(\alpha) = \frac{\alpha}{\sigma^2} \exp\left(\frac{-\alpha^2}{2\sigma^2} - k\right) \sum_{i=1}^L a_i D\left(\frac{\alpha}{\sigma}; k, C_i\right) \quad (2)$$

where,  $C_i = \Delta \cos \frac{\pi(i-1)}{2L-1}$ ,

$$D(x; k, C) = \frac{1}{2} \exp(Ck) I_0\left(x\sqrt{2k(1-C)}\right) + \frac{1}{2} \exp(-Ck) I_0\left(x\sqrt{2k(1+C)}\right)$$

$$K = \frac{\text{Specular power}}{\text{Diffused power}}$$

$$\Delta = [\text{Convenient parameter}] = \frac{\text{Peak Specular Power}}{\text{Average Specular Power}} - 1 \quad \text{and}$$

$$L \text{ is the order } \geq \frac{1}{2} k \Delta.$$

Considering  $\hat{\alpha}(i) = [\hat{\alpha}_1(i), \dots, \hat{\alpha}_L(i)]^T$  as the estimated channel vector at the receiver side, the channel estimation error at each branch can be considered as,

$e_l(i) = \hat{\alpha}_l(i) - \alpha_l(i)$ . As given in [11], a versatile model for channel estimation error of any arbitrary linear channel estimation can be represented as,

$$\alpha_{f,l}(i) = \rho_l \hat{\alpha}_{f,l}(i) + z_{f,l}(i) \quad (3)$$

where,  $l = 1, 2, 3, \dots, L$ , 'f' is the diffused component and the errors  $\{z_{f,l}(i)\}_{l=1}^L$  are the iid equivalent estimation error terms with zero mean and variance  $\sigma_z^2$ . In (3)  $\rho_l$  is denoted as the normalized estimation correlation coefficient between  $\alpha_l(i)$  and  $\hat{\alpha}_l(i)$ , and can be written as  $\rho_l = |\rho_l| e^{j\Delta\theta_l}$ , where  $\Delta\theta_l$  is the phase offset of  $\rho_l$ . In case of ICE  $|\rho_l| < 1$  or  $\Delta\theta_l \neq 0$  or both may arise, which degrades the receiver performance [11].

### A. Effective Output SNR of MRC receiver with ICE

Considering ICE to detect the transmitted symbol 's(i)' at the MRC receiver we have to take the help of complex decision variable (DV), which is given as,  $\tilde{D} = \sum_{l=1}^M \hat{\alpha}_l(i) r_l(i)$  [12].

Applying the half plane decision method [13] the complex DV  $\tilde{D}$  will be rotated with a plane angle  $\beta$  to obtain a new DV as,

$$D(\beta) = \Re(\tilde{D} e^{-j\beta}) \quad (4)$$

where,  $\beta = \pm \left(\frac{\pi}{2} - \frac{\pi}{N}\right)$  and 'N' is the constellation size.

However,  $\beta = 0$  for BPSK (M=2). So considering the half plane decision method for a DV D, the effective output SNR of a MRC receiver is given as [11],

$$\gamma_{ICE}^{MRC} = B(\beta) \sum_{l=1}^M \hat{\gamma}_l \quad (5)$$

where, B is a function of  $\beta$  and given by,

$$B(\beta) = \frac{|\rho_l|^2 \cos^2(\Delta\theta_l - \beta)}{\left[(1 - |\rho_l|^2) \bar{\gamma}_l + 1\right]}, \hat{\gamma}_l = |\hat{\alpha}_l|^2 / N_0.$$

**OUTPUT SNR PDF OF TWDP WITH ESTIMATION ERROR**

MRC is a technique where the received signals are co-phased and weighted as per their SNR value to optimize the quality of the output signal and expression of the output SNR is given as [10],

$$\gamma_{MRC} = \sum_{l=1}^M \gamma_l = \frac{E_b}{N_0} (\alpha_1^2 + \alpha_2^2 + \dots + \alpha_M^2) \tag{6}$$

Where, index ‘l’ is used to represent the l<sup>th</sup> receiving antenna and we assume the average input SNR  $\bar{\gamma}_l = \bar{\gamma}/l$ . Considering equal estimation error in all branches of MRC receiver, PDF of SNR for l<sup>th</sup> branch can be given as [10],

$$f_{\hat{\gamma}_l}(\hat{\gamma}_l) = \frac{\eta}{2B} \sum_{i=1}^L \sum_{j=0}^1 a_i e^{-\{\eta B^{-1} \hat{\gamma} + p_{2i-j}\}} I_0 \left( 2\sqrt{p_{2i-j} \eta B^{-1} \hat{\gamma}} \right) \tag{7}$$

where, ‘B’ is the estimation error of the receiver and  $\eta = \frac{K+1}{\bar{\gamma}}$ ,  $p_{2i-j} = K \left[ 1 + (-1)^j \Delta C \cos \frac{\pi(i-1)}{2L-1} \right]$  and  $\hat{\gamma}$  is the actual received signal SNR. As per the (6) the output SNR of the M-MRC receiver can be evaluated by adding all the SNR of individual branches. Following a characteristic function based approach, this output SNR can be derived as shown bellow. The characteristic function of the (7) can be derived as,

$$\phi_{\gamma}(j\omega) = \frac{\eta}{2B} \sum_{i=1}^L \sum_{j=0}^1 a_i e^{-p_{2i-j}} \frac{1}{(\eta B^{-1} + j\omega)} e^{\frac{4 p_{2i-j} \eta B^{-1}}{4(\eta B^{-1} + j\omega)}} \tag{8}$$

Since we have considered the independent branches, therefore the joint characteristic function of input SNRs can be obtained by taking the product of each characteristic function as,

$$\phi_{\gamma_1, \dots, \gamma_M}(j\omega_1, \dots, j\omega_M) = \prod_{t=1}^M \left\{ \frac{\eta}{2B} \sum_{i=1}^L \sum_{j=0}^1 a_i e^{-p_{2i-j_t}} \frac{1}{(\eta B^{-1} + j\omega_t)} e^{\frac{p_{2i-j_t} \eta B^{-1}}{(\eta B^{-1} + j\omega_t)}} \right\} \tag{9}$$

Replacing  $\omega_1 = \omega_2 = \dots = \omega$  in (9) and then writing the expression in the series form expression one exponential term in infinite series the characteristic function of output SNR can be expressed as,

$$\begin{aligned} \phi_{\gamma_1, \dots, \gamma_M}(j\omega_1, \dots, j\omega_M) = \\ \left( \frac{\eta}{2B} \right)^M \sum_{i_1=1}^L \dots \sum_{i_M=1}^L \sum_{j_1=0}^1 \dots \sum_{j_M=0}^1 \sum_{s=0}^{\infty} \frac{\left\{ \prod_{t=1}^M a_{i_t} \right\} e^{-\sum_{t=1}^M p_{2i_t-j_t}} \left( \sum_{t=1}^M p_{2i_t-j_t} \eta B^{-1} \right)^s}{s! (\eta B^{-1} + j\omega)^{M+s}} \end{aligned} \tag{10}$$

The PDF of the output SNR of the M-MRC receiver can be obtained, taking the inverse Fourier transform of the (10) using [[16],3.382.7]as,

$$f_{\hat{\gamma}_{mrc}}(\hat{\gamma}_{mrc}) = \left( \frac{\eta}{2B} \right)^M \sum_{i_1=1}^L \dots \sum_{i_M=1}^L \sum_{j_1=0}^1 \dots \sum_{j_M=0}^1 \frac{\left\{ \prod_{t=1}^M a_{i_t} \right\}}{\Gamma(M)} \times e^{-\sum_{t=1}^M p_{2i_t-j_t}} \hat{\gamma}^{M-1} e^{-\eta B^{-1} \hat{\gamma}} \sum_{s=0}^{\infty} \frac{\left( \hat{\gamma} \sum_{t=1}^M p_{2i_t-j_t} \eta B^{-1} \right)^s}{s! (M)_s} \tag{11}$$

Expressing the above equation in terms of Hypergeometric function the closed form equation of the PDF of the output SNR of the M-MRC receiver with estimation error can be obtained as,

$$f_{\hat{\gamma}}(\hat{\gamma}) = \left(\frac{\eta}{2B}\right)^M \sum_{i_1=1}^L \dots \sum_{i_M=1}^L \sum_{j_1=0}^1 \dots \sum_{j_M=0}^1 \frac{\left\{ \prod_{t=1}^M a_{i_t} \right\} e^{-\sum_{t=1}^M p_{2i_t-j_t}} \hat{\gamma}^{M-1} e^{-\eta B^{-1} \hat{\gamma}}}{\Gamma(M)} {}_0F_1\left(; M; \hat{\gamma} \sum_{t=1}^M p_{2i_t-j_t} \eta B^{-1}\right) \quad (12)$$

where,  ${}_0F_1 = (; a; z)$  is the confluent hypergeometric function and  $\Gamma(\cdot)$  is the gamma function.

#### IV.ABER expression for MRC receiver with ICE

In this section we have derived the ABER expression using (12) and the general formula of bit error rate given in [14],

$$P_e = \int_0^{\infty} P(e/\hat{\gamma}) f_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} \quad (13)$$

Where,  $P(e/\hat{\gamma})$  is a conditional BER. For coherent binary modulation scheme  $P(e/\hat{\gamma})$  is given as [15],

$$P(e/\gamma) = Q(\sqrt{2a\hat{\gamma}}) \quad (14)$$

where, 'a=1 for BPSK and 'a=0.5' for BFSK. Substituting (12) and (14) in (13) and solving the integral using [[16],6.455(1)], the final ABER expression can be written as,

$$P_{e,coh} = \left(\frac{\eta}{2B}\right)^M \sum_{i_1=1}^L \dots \sum_{i_M=1}^L \sum_{j_1=0}^1 \dots \sum_{j_M=0}^1 \sum_{s=0}^{\infty} \frac{\left\{ \prod_{t=1}^M a_{i_t} \right\} a e^{-\sum_{t=1}^M p_{2i_t-j_t}} \left( \sum_{t=1}^M p_{2i_t-j_t} \eta B^{-1} \right)^s}{\Gamma(M) s!(M)_s 2\sqrt{\pi}} \frac{\sqrt{\frac{b}{2}} \Gamma(M + s + \frac{1}{2})}{(M + s) \left(\frac{b}{2} + \eta B^{-1}\right)^{M+s+\frac{1}{2}}} {}_2F_1\left(1, M + s + \frac{1}{2}; M + s + 1; \frac{\eta B^{-1}}{\frac{b}{2} + \eta B^{-1}}\right) \quad (15)$$

Simplifying eq.(15) the expression can be rewrite as,

$$P_{e,coh} = \frac{B^{-M} \eta^M}{2\sqrt{\pi}} \sum_{i_1=1}^L \dots \sum_{i_M=1}^L \sum_{j_1=0}^1 \dots \sum_{j_M=0}^1 \sum_{s=0}^{\infty} \frac{\left\{ \prod_{t=1}^M a_{i_t} \right\} a e^{-\sum_{t=1}^M p_{2i_t-j_t}} \left( 2 \sum_{t=1}^M p_{2i_t-j_t} \eta B^{-1} \right)^s}{\Gamma(M) s!(M)_s} \frac{\sqrt{b} \Gamma(M + s + \frac{1}{2})}{(M + s) (b + 2\eta B^{-1})^{M+s+\frac{1}{2}}} {}_2F_1\left(1, M + s + \frac{1}{2}; M + s + 1; \frac{2\eta B^{-1}}{b + 2\eta B^{-1}}\right) \quad (16)$$

### OUTAGE PROBABILITY ANALYSIS FOR M-MRC RECEIVER WITH ICE

When the instantaneous error probability exceeds a specified value, or the output SNR,  $\hat{\gamma}$  falls below a particular threshold value  $\gamma_{th}$  [10] then it is known as outage probability. The outage probability can be derived from the equation, as given in [10],

$$P_{out} = \int_0^{\gamma_{th}} f_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} \quad (17)$$

where,  $\gamma_{th}$  is the threshold SNR value. Now, substituting (12) in (17) and solving the equation using [[16], 3.381.1] the final expression can be expressed for M-MRC system with channel estimation error as,

$$P_{out} = \left(\frac{\eta}{2B}\right)^M \sum_{i_1=1}^L \dots \sum_{i_M=1}^L \sum_{j_1=0}^1 \dots \sum_{j_M=0}^1 \sum_{s=0}^{\infty} \left\{ \prod_{t=1}^M a_{i_t} \right\} e^{-\sum_{t=1}^M p_{2i_t-j_t}} \left( \sum_{t=1}^M p_{2i_t-j_t} \eta B^{-1} \right)^s (\eta B^{-1})^{-(s+M)} g((s+M), \gamma_{th} \eta B^{-1}) \quad (18)$$

where  $g(\cdot)$  is the lower order incomplete Gamma function.

### NUMERICAL RESULTS AND ANALYSIS:

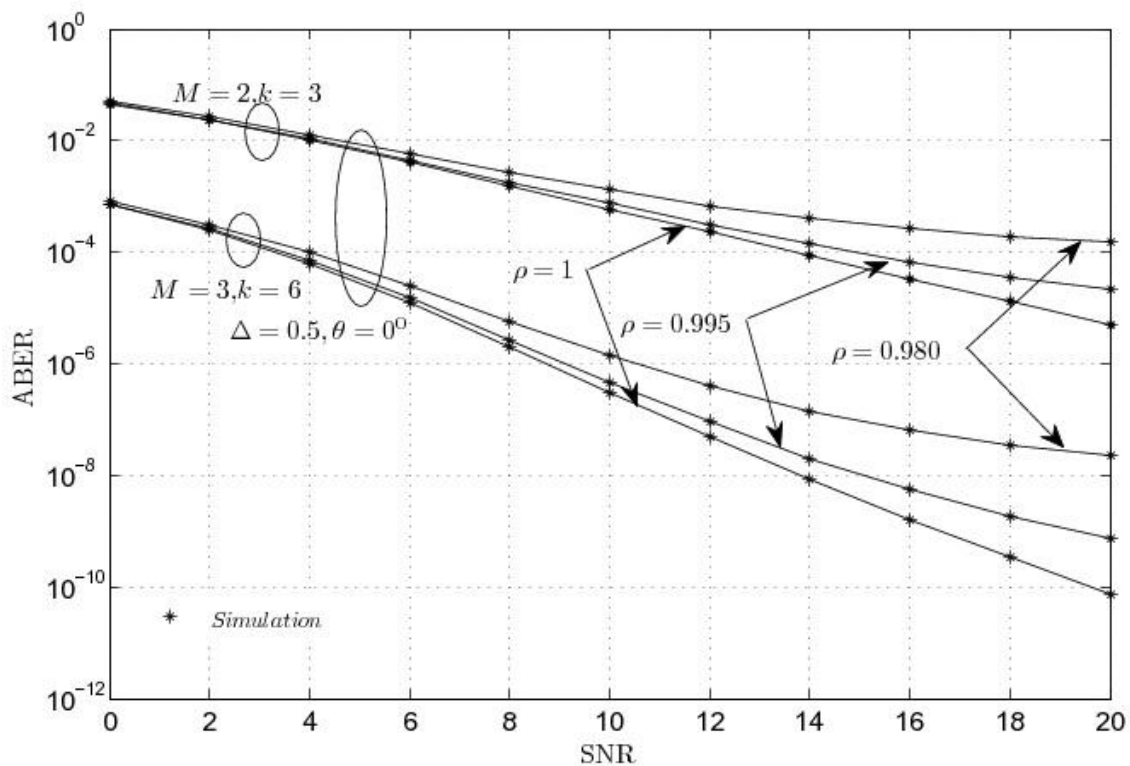
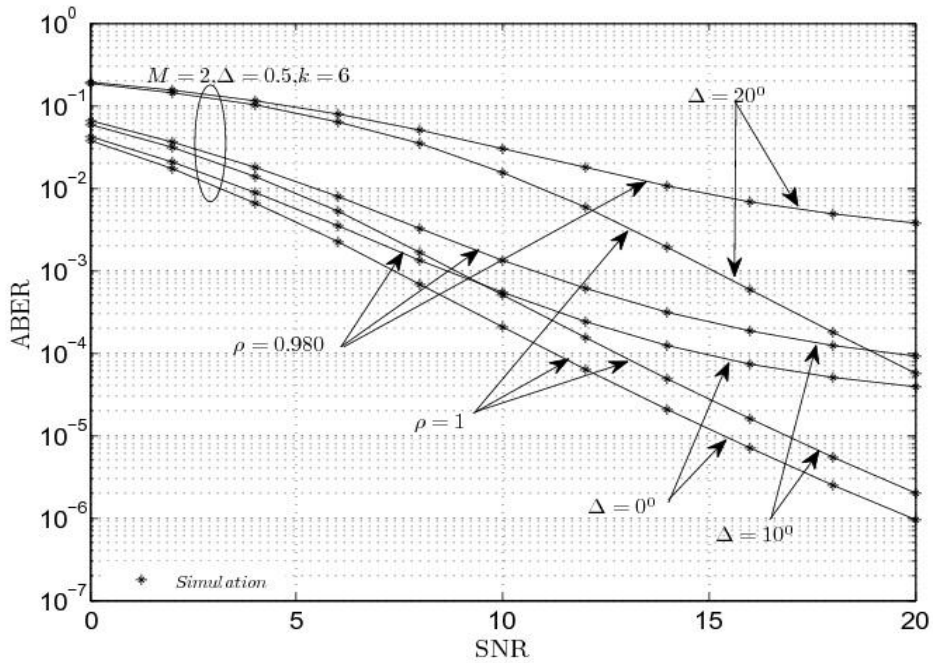
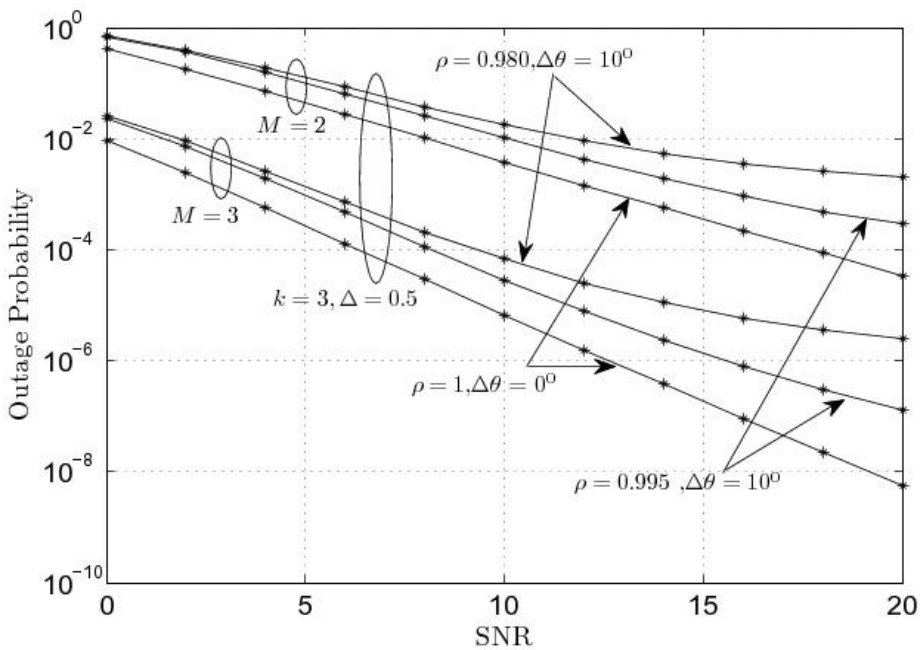


Figure 2: ABER vs SNR with Envelope estimation error



**Figure 3:** ABER vs SNR with Phase estimation error



**Figure 4:** Outage vs SNR with Phase and Envelop estimation error

The result obtained in the previous sections has been numerically evaluated and plotted to minutely observe the behavior of the system along with the estimation error. In Fig. 2-4 ABER and outage probability have been found with envelope ( $\rho$ ) and phase ( $\theta$ ) estimation error for different diversity order with varying channel condition. In Fig.2 effects of envelope estimation error have been shown on coherent ABER for various diversity order. It has been noted that for an error of 2% in the estimation to maintain a bit error

rate of  $10^{-4}$  approximately extra 8 dB power is needed with dual diversity. Same may also be observed in higher order diversity branch. Similarly, in Fig.3 the ABER vs. SNR is shown for various phase errors in coherent BPSK modulations. Fig.4 shows the outage probability analysis with both phase and envelope error. From this analysis, we observed that the expected benefit of diversity might not be achievable if the estimator uses are not perfect.

## CONCLUSION

In this paper the analysis of an M- MRC receiver with estimation error has been presented in terms of ABER and outage probability over TWDP fading channels. An output SNR PDF expression of an M-MRC receiver with ICE has been calculated followed by a CF based approach. From the calculated SNR PDF the ABER and outage probability expression with ICE has been obtained and plotted. From the study it can be concluded that, since an M-MRC receiver requires the estimators to optimize the performance; so if the estimates are not perfect, then if the SNR is degraded considerably, the practical application of M-MRC is inefficient. However, increasing the nos of antennas at the receiver side the receiver performance can be increased.

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