

# An Intelligent Robust Fuzzy Temperature Control of a Thermoelectric Cooler

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## Abstract

Thermoelectric coolers (TEC) are used in a variety of applications that require extremely stable temperature control such as logical electronic processors, refrigeration equipment, air conditioning and scientific equipment.

In this paper, the system dynamics of the thermoelectric cooler model including the cooling load heat exchanger and heat sink was driven. The dynamic model is presented with varying operating conditions. A proportional controller will be applied for the cold-end temperature control of a thermoelectric cooler. The feedback control system will be examined under variable cooling load and ambient temperature.

An intelligent fuzzy controller was applied for the cold-end temperature control of a thermoelectric cooler. Fuzzy inference parameters which include the rule base and the shapes of the membership functions was tuned via simulation. The fuzzy control system was simulated under variable cooling load to assured its robustness.

**Keywords:** Thermoelectric cooler, Dynamic model, Fuzzy control.

## INTRODUCTION

Thermoelectric cooler has been frequently used in the cooling of electronic devices such as CPU, infrared sensor, ice point reference in thermocouple thermometry and refrigerators [1-3]. Usually the temperature at the cold side of the thermoelectric module where an electronic part such as infrared sensor is mounted needs to be maintained at a constant and stable value under variable hot-side and ambient temperatures. This relies on a good temperature control.

Thermal failures such as mechanical stresses, thermal debonding and thermal fracture become many of the possible breakdowns of electronics components. Mismatch of the thermal coefficient expansion between two different materials, especially at the interface conditions, could result in the separation of interfaces and bonds between different parts in a module at higher temperature [4]

In optical networking systems [5], TEC requirements generally fall into one of two categories. The first category is that of pump lasers, which are used in erbium-doped fiber amplifiers and Raman amplifiers. These devices generate from  $20 \times 10^{-3}$  Watts to  $500 \times 10^{-3}$  Watts of optical power,

dissipate between 2 Watts and 16 Watts of thermal energy, and require temperature regulation from  $\pm 1^\circ\text{C}$  to  $\pm 0.1^\circ\text{C}$ .

The second category is that of transponder lasers (both fixed wavelength and tunable), which are used as laser light sources in network head end racks, wavelength converters, and elsewhere in the network. These devices generate much less power than pump lasers, typically being less than  $20 \times 10^{-3}$  Watts of optical power. Thermal dissipation is generally less than 5 Watts, but required temperature accuracy ranges from  $\pm 0.1^\circ\text{C}$  down to  $\pm 0.01^\circ\text{C}$ , and possibly even tighter. So, accurate temperature control becomes critical.

Many researchers studied the dynamics or transient behavior of thermoelectric element [4-7]. Using small signal linearization method, Ying-Feng Pang [4] theoretically derived a transfer function model to calculate the temperature response of a thermoelectric element. Bywaters and Blum [6] assumed that the temperature distribution in the thermoelectric elements is linear and solved the governing equations. These studies [4-7] are basically for the analysis of a pair of thermoelectric elements. A linear feedback system is designed for the cold-end temperature control of a thermoelectric cooler using the average linear dynamic model of the thermoelectric cooler and optimal controller structure [8-12]. Also the parameters of the dynamic model are experimentally identified at various operating conditions. TECs are made of a semiconductor couples array. When a DC voltage is supplied, one surface of TEC absorbs heat making the surface cold and the opposite sides gets heated. The minimum temperature that can be obtained through the cold surface depends upon particular factors including ambient temperature, voltage supplied, cooling mechanism [13]. Therefore, in [14], an effective thermal conductivity model for TECs have been proposed with voltage control mechanism. In this study, a standard mathematical model for multi-stage TECs has been developed with thermal resistances from both sides and performance parameters dependent on voltage. Developed system has been evaluated with simulated load and hardware realization. Short term temperature stability of 70mK is achieved and peak power drawn by Peltier element is measured nearly 4W [15]. In the next section, the thermo electrical cooler dynamic model was developed.

## SYSTEM DYNAMIC OF A THERMOELECTRIC COOLER

A thermoelectric cooler consists of a thermoelectric module, a heat sink connected to the hot side, and cooling load heat

exchanger connected to the cold side. The thermoelectric module comprises many pairs of p-n type thermoelectric material connected in series and clamped and soldered with two base plates (e.g. aluminum oxide) as shown in Figure 1. The heat load  $Q_L$  is absorbed at the cooling load heat exchanger, conducted to the hot-end plate, and then pumped to the hot side of the thermoelectric module.

Assuming that the temperature distribution inside the cold-end plate and the cooling load heat exchanger are uniform. Energy balance to the cold-end plate and the cooling load heat exchanger as a whole lead to,

$$(M_L C_L + M_c C_c) \frac{dT_L}{dt} = Q_L - Q_k - I \alpha_{pn} T_L \quad (1)$$

where  $Q_k$  is the heat conduction at the cold-end boundary of the thermoelectric module which is expressed as

$$Q_k = -k A \left. \frac{\partial T(x,t)}{\partial x} \right|_{x=0} \quad (2)$$

where  $k$  is the mean thermal conductivity of the p-n material;  $A$  is the total cross sectional area of the thermoelectric material;  $T(x, t)$  is the temperature distribution of the thermoelectric module.

Energy balance to the thermoelectric material will lead to the relation

$$C\gamma \frac{\partial T(x,t)}{\partial x} = k \frac{\partial^2 T(x,t)}{\partial x^2} - \frac{\tau}{A} I \frac{\partial T(x,t)}{\partial x} + \frac{\rho}{A^2} I^2 \quad (3)$$

where  $\tau$  is the Thomson coefficient defined as  $T \alpha_{pn} / dT$ ;  $\rho$  is the mean electrical resistance of the thermoelectric material;  $C$  is the mean specific heat of the thermoelectric material;  $\gamma$  is the mean density of the thermoelectric material. The second term on the right hand side of equation (3) represents the heat resulting from Thomson effect [6].

Similarly, energy balance to the heat sink and the hot side plate as a whole lead to

$$(M_F C_F + M_H C_H) \frac{dT_H}{dt} = I \alpha_{pn} T_H + Q_o - h A_F (T_H - T_a) \quad (4)$$

where  $Q_o$  is the heat conduction at the hot side boundary of the thermoelectric module which can be expressed as

$$Q_o = -k A \left. \frac{\partial T}{\partial x} \right|_{x=L} \quad (5)$$

Eqs. (1), (3) and (4) are the governing equations for the dynamic behavior of a thermoelectric cooler. The model is highly nonlinear due to the temperature dependence of physical properties, the resistive heat and the Peltier effect. Linearization using small signal analysis is necessary since the controller design will be based on linear system theory.

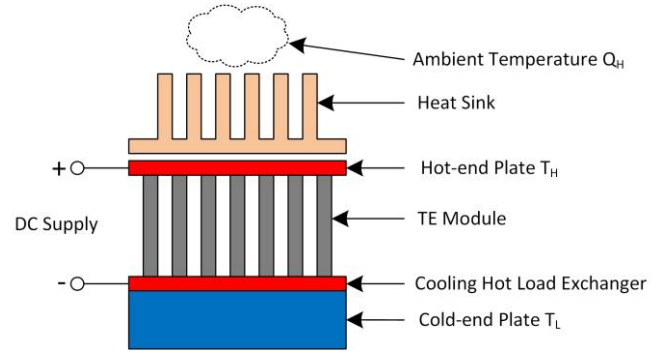


Figure 1: schematic diagram of thermoelectric.

Let all the variables of the thermoelectric cooler be the summation of a steady state value and a perturbed quantity, i.e

$$\begin{aligned} T(x,t) &= T(x) + \tilde{T}(x,t); & T_L(t) &= T_L + \tilde{T}_L(t); \\ T_H(t) &= T_H + \tilde{T}_H(t); & T_a(t) &= T_a + \tilde{T}_a(t); \\ Q_L(t) &= Q_L + \tilde{Q}(t); & I(t) &= I + \tilde{I}(t) \end{aligned} \quad (6)$$

and use the approximate relation of the Seebeck coefficient according to Taylor's series expansion:

$$\alpha_{pn} = \alpha_L + \frac{\tau}{T_L} \tilde{T}_L = \alpha_H + \frac{\tau}{T_H} \tilde{T}_H \quad (7)$$

where  $\alpha_L = \alpha_{pn}(T_L)$ ;  $\alpha_H = \alpha_{pn}(T_H)$ .

Substituting Eqs. (6) and (7) into Eqs. (1), (3) and (4), ignoring the high order terms and eliminating the steady state terms, we obtain for constant properties  $\tau, k, \rho, C, \gamma$ ,

$$k \frac{\partial^2 \tilde{T}}{\partial x^2} - \frac{\tau I}{A} \frac{\partial \tilde{T}}{\partial x} + \left[ \frac{2\rho I}{A^2} - \frac{\tau(T_H - T_L)}{AL} \right] \tilde{T} = C\gamma \frac{\partial \tilde{T}}{\partial t} \quad (8)$$

$$\tilde{Q}_L - (\alpha_L + \tau) I \tilde{T}_L - \alpha_L T_L \tilde{I} + k A \left. \frac{\partial \tilde{T}}{\partial x} \right|_{x=0} = (M_L C_L + M_c C_c) \frac{d\tilde{T}_L}{dt} \quad (9)$$

$$\begin{aligned} & (\alpha_H + \tau) I \tilde{T}_H + \alpha_H T_H \tilde{I} - k A \left. \frac{\partial \tilde{T}}{\partial x} \right|_{x=L} - h A_F (\tilde{T}_H - \tilde{T}_a) \\ & = (M_F C_F + M_H C_H) \frac{d\tilde{T}_H}{dt} \end{aligned} \quad (10)$$

Eq. (8) is obtained from an assumption that the steady state temperature distribution in the thermoelectric material is

$$\text{linear, i.e. } \frac{dT(x)}{dx} \approx (T_H - T_L) / L.$$

Solving Eqs. (8)-(10) by Laplace transform we obtain the transfer function of the perturbed cold-end temperature:

$$\tilde{T}_L(s) = G_I(s)\tilde{I}(s) + G_Q(s)\tilde{Q}_L(s) + G_a(s)\tilde{T}_a(s) \quad (11)$$

where

$$G_I(s) = \frac{N(s)}{sD(s)} \quad (12)$$

$$G_Q(s) = \frac{E_H \sinh(qL) + Akq \cosh(qL)}{D(s)} \quad (13)$$

$$G_a(s) = \frac{AA_F h k q}{D(s)} \quad (14)$$

Where

$$N(s) = \left\{ Akq [\alpha_L T_L \cosh(qL) - \alpha_H T_H] + \alpha_L T_L E_H \sinh(qL) \right\} s + \frac{Akq\beta}{C\gamma} [E_H (1 - \cosh(pL)) - Akp \sinh(pL)] \quad (15)$$

$$D(s) = Akq E_L \cosh(pL) + E_H E_L \sinh(pL) + Akq E_H \cosh(pL) + A^2 k^2 p q \sinh(pL) \quad (16)$$

$$p(s) = \frac{\frac{\tau I}{A} + \sqrt{\frac{\tau^2 I^2}{A^2} + 4kC\gamma s}}{2k}; \quad q(s) = \frac{\frac{\tau I}{A} - \sqrt{\frac{\tau^2 I^2}{A^2} + 4kC\gamma s}}{2k} \quad (17)$$

$$E_L(s) = (M_L C_L + M_C C_C) s + (\tau + \alpha_L) I \quad (18)$$

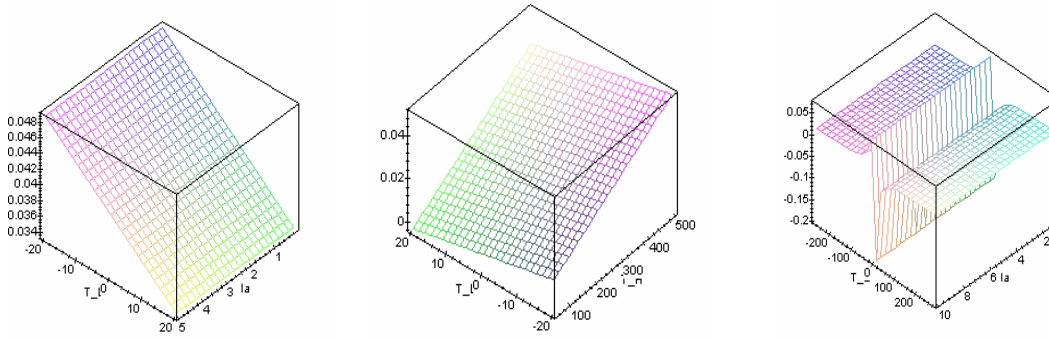
$$E_H(s) = (M_F C_F + M_H C_H) s + h A_F - (\tau + \alpha_H) I \quad (19)$$

$$\beta = \frac{2\rho I}{A^2} - \frac{\tau(T_H - T_L)}{AL} \quad (20)$$

Equation (11) indicates that the cold-end temperature of the thermoelectric cooler  $T_L$  is affected by the variation of the applied current  $I$ , the cooling load  $Q_L$  and the ambient temperature  $T_a$ . The transfer functions  $G_I(s)$ ,  $G_Q(s)$  and  $G_a(s)$  are accounting for the system dynamic behavior caused by current, cooling load, and ambient temperature variations, respectively.

The gain and zero of the dynamic model would vary with the operating points as shown in Fig. 3. The gain  $K$  increases with

decreasing cold end temperature  $T_L$  and increasing hot end temperature  $T_H$ . The zero is approximately constant irrespective of the variations of current and is inverse proportional with variations of cold end temperature. In addition the poles  $p_1$ ,  $p_2$  are however approximately constant conjugate complex irrespective of the variations of current and temperature.



**Figure 3-a:** Variation of gain of the dynamic model \$G\_1(s)\$ **Figure 3-b:** Variation of zero of the dynamic model \$G\_1(s)\$

### FUZZY TEMPERATURE CONTROL OF A THERMO ELECTRIC COOLER

#### Controller design

There are a number of different ways to implement the fuzzy inference engine. Among the very first such proposed techniques is that due to Mamdani [16], which describes the inference engine in terms of a fuzzy relation matrix and uses the compositional rule of inference to arrive at the output fuzzy set for a given input fuzzy set. The output fuzzy set is subsequently defuzzified to arrive at a crisp control action. Other techniques include sum-product and threshold inferencing. A review of ranging these is given by Kosko [17]. The inference methodology we employ here is discussed in [18].

Let the input variables be \$x\_p\$ for \$1 \le p \le P\$. The \$i^{th}\$ membership function in the fuzzifier corresponding to the \$p^{th}\$ input is \$\{\mu\_p^i | 1 \le i \le N\_p\}\$. We denote the single output by \$f\$, with corresponding defuzzification membership functions \$\{v^k | 1 \le k \le K\}\$.

Generalization of inference and adaptation techniques to more than one output is straightforward. In the following analysis, for purposes of simplicity, we consider the case \$P=2\$ without loss of generality. Defining \$N\_p=N\$ for \$p=1\$, and \$N\_p=M\$ for \$p=2\$, for a given output membership function \$v^k\$, the rules are of the form:

If \$x\_1\$ is \$\mu\_1^i\$ and \$x\_2\$ is \$\mu\_2^j\$ OR If \$x\_1\$ is \$\mu\_1^l\$ and \$x\_2\$ is \$\mu\_2^m\$ OR .....

Then, \$f\$ is \$v^k\$

Define a set

$$s_k = \{l, m | \mu_1^l \text{ and } \mu_2^m \text{ are antecedents of a rule with consequent } v^k\} \quad (21)$$

The familiar operations to arrive at the output are as follows:

1. Perform a pair wise fuzzy intersection \$T\$, on each of the membership values of \$x\_1\$ and \$x\_2\$ in \$\mu\_1^l\$ and \$\mu\_2^m\$ for every rule with consequent \$v^k\$, forming activation values \$\zeta\$:

$$\zeta_{lm}^k = T_{l,m \in S_k} (\mu_1^l(x_1) - \mu_2^m(x_2)) \quad (22)$$

Let us assume that the \$(T\$-norm) operator \$T\$ itself is parameterized by \$\alpha\$, i.e., \$T=T(\alpha)\$.

2. Collect activation values for like output membership functions and perform a fuzzy union \$T^+\$, where \$T^+ = T^+(\beta)\$

$$w_k = T_{l,m \in S_k} (\zeta_{lm}^k) \quad (23)$$

3. These values are defuzzified to generate the output estimated value, \$f(x\_1; x\_2)\$, by computing the centroid of the composite membership function \$\mu\$:

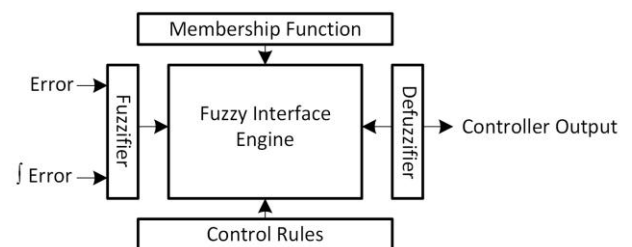
$$\mu = \sum_{k=1}^K w_k v^k \quad (24)$$

$$y(x_1, x_2) = \frac{\sum_{k=1}^K w_k C_k A_k}{\sum_{k=1}^K w_k A_k} \quad (25)$$

Where

$$A_k = \int v_k(x) dx ; C_k = \frac{\int x v^k(x) dx}{\int v^k(x) dx} \quad (26)$$

\$A\_k\$ and \$c\_k\$ are, respectively, the area and centroid of the consequent membership function \$v^k\$. All of the stages of the fuzzy inference system are affected by the choice of certain parameters.



**Figure 4:** The basic fuzzy logic controller

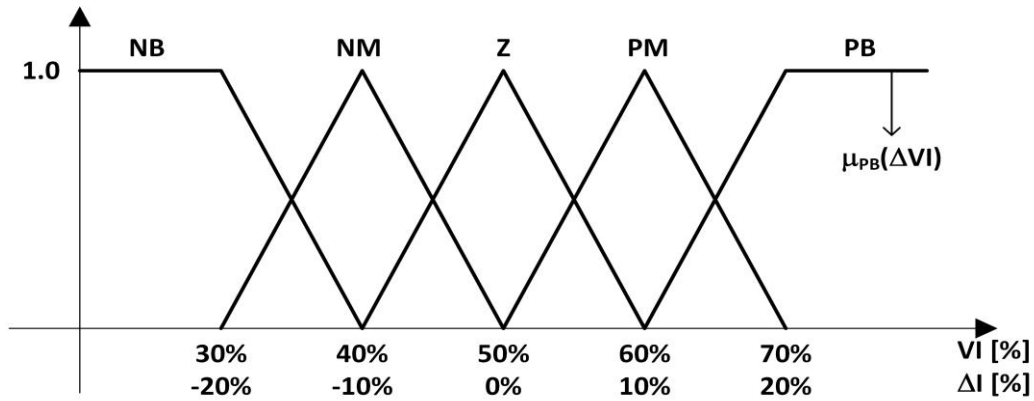


Figure 5: Membership function of error and change in error

The fuzzifier in Fig. 4 maps the input onto the possibility domain. The inference engine is the system “decision maker” and determines how the system interprets the fuzzy linguistics. Its parameters are those of the aggregation operators, which provide interpretation of connectives “AND” and “Or”. While the defuzzification stage maps fuzzy consequents into crisp output values.

Both the fuzzification and defuzzification stages require choices of cardinality, position and shape of membership functions. The defuzzification operation itself can be parameterized, and the inference engine requires choices to be made among numerous fuzzy aggregation operators, which could be parameterized.

All of these parameters can be adaptively adjusted by monitoring a certain target performance measure in a supervised learning environment. Over the years numerous techniques for adaptation of fuzzy membership functions, rule bases, and aggregation operators have been proposed.

The inference engine is the system “decision maker” and determines how the system interprets the fuzzy linguistics. Its parameters are those of the aggregation operators, which provide interpretation of connectives “AND” and “Or”. The inference engine surfaces of the rules bases and its membership function is shown in Fig. 6. It is thus seen that both the fuzzification and defuzzification stages require choices of cardinality, position and shape of membership functions. The defuzzification operation itself can be parameterized, and the inference engine requires choices to be made among numerous fuzzy aggregation operators, which could be parameterized. In the next section, the simulation results of the proposed controller was illustrated.

## RESULTS AND DISCUSSION

The controller design for temperature control of a thermoelectric cooler is not very simple due to the nonlinear dynamic behavior of the thermoelectric module. The effect of dynamic behavior of the thermal masses of the heat sink and the cooling load heat exchanger imposes another problem on the control system design.

By adopting the dynamic model for the thermoelectric cooler. The controller gain is tuned by computer simulation for feedback system at critical damping condition and without overshoot. In Fig. 7 (a and b), the simulation results illustrate that  $\tilde{T}_L$  can be maintained at the fixed value within 0.01°C irrespective of the variation of the cooling load and the ambient conditions. The resistance to the variation of plant dynamics is quit important for a controller design. The corresponding fuzzy controller signal is shown in Fig. 7.c. While Fig.7.d shows the crossponding change of  $\tilde{T}_H$  for the same input.

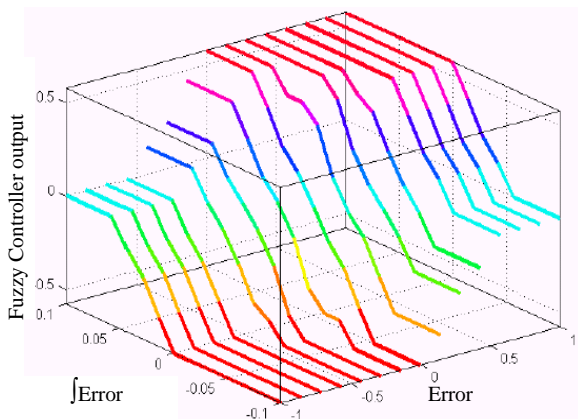
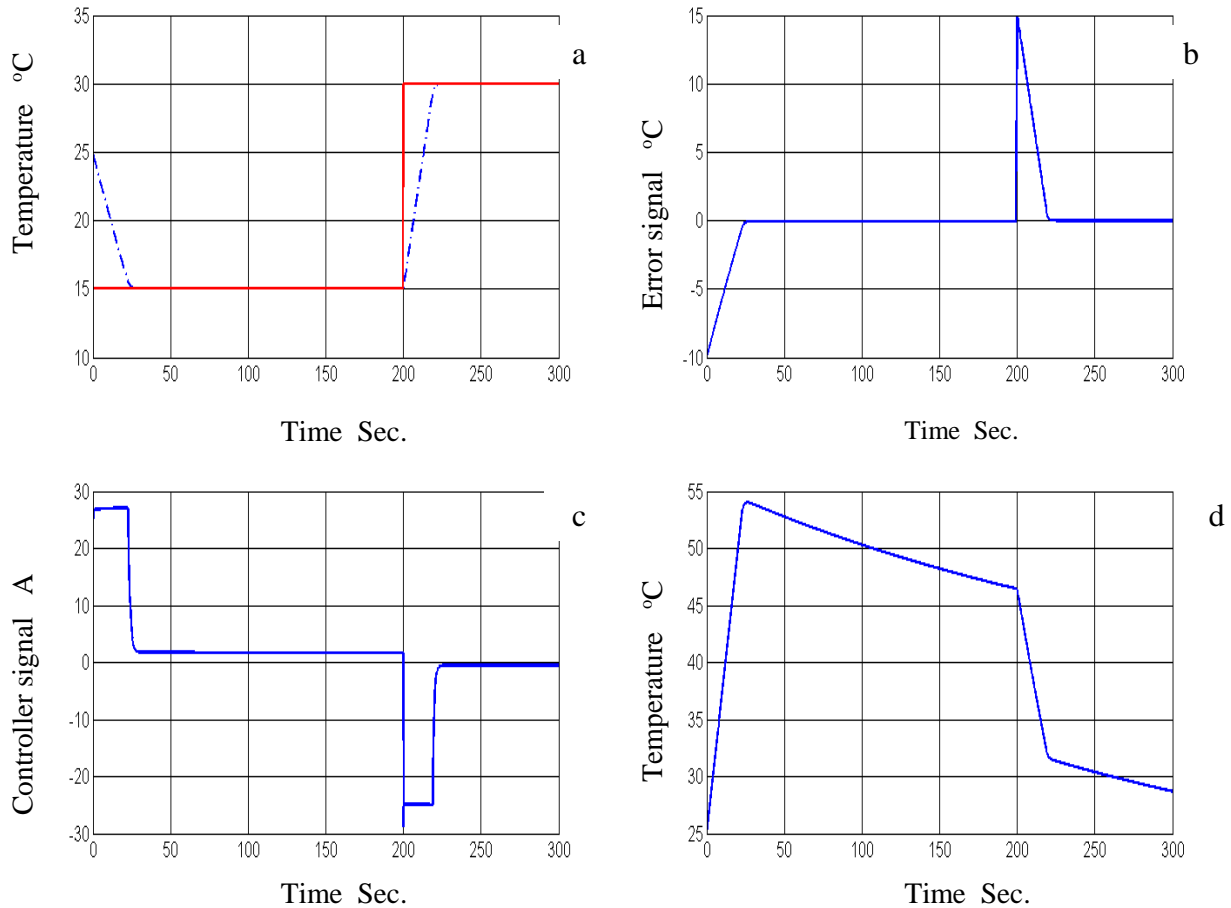
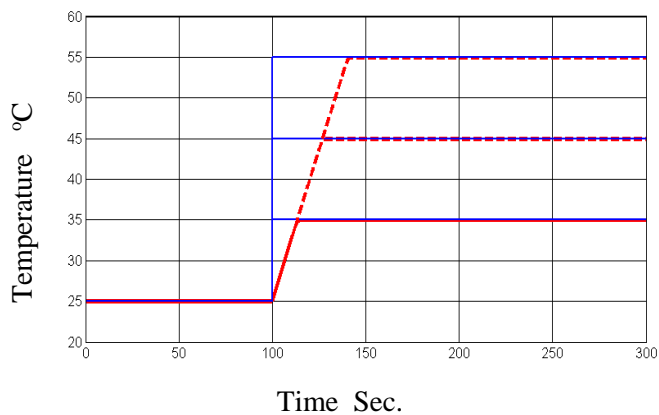


Figure 6: Surfaces of the rules bases and its membership function.



**Figure 7:** a. Time response of cold end temperature  $\tilde{T}_L$  for a step setting at  $Q_L = 10$  W, b. the error signal for the step input change, ( $^{\circ}\text{C}$ )., c. The proposed controller signal, (A) and d. The corresponding change of  $\tilde{T}_H$  for the same input, ( $^{\circ}\text{C}$ ).

To check the proposed controller robustness, a different levels of  $\tilde{T}_L$  (35, 45 and 55 $^{\circ}\text{C}$ ) were applied at Fig.8 and the results assured the robustness of the controller.



**Figure 8:** The time response for different level of  $\tilde{T}_L$ , ( $^{\circ}\text{C}$ ).

## CONCLUSION

In the present study, we derived a dynamic model for the whole thermoelectric cooler.

We use a fuzzy logic controller as a design controller for the cold end temperature.

In simulation, the TEC model is considered perfect without considering the environmental factor that have effect on the system such as internal heating, heat transfer from hot region into the system. This can affect the tuning coefficient for Fuzzy control algorithm.

Furthermore, computer simulation results of the controller show a robustness against thermal parameter uncertainties of the model, such as differences of the low temperature values of the TECs. Future work will deal with extended model design using more than one elements, building an array consisting of multiple TECs.

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