International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 1 (2018) pp. 612-616

Mathematical Model for Frost Heaving Normal Force Calculation

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Abstract

The construction of pipelines on the heaving soils is associated with the risks of unacceptable deformations under the influence of frost heaving forces. The combination of flooded soils or boggy ground with a negative temperature of the pumped product is especially dangerous. In this case, the pipeline is a source of cold and involves liquid water to cryogenic migration. Liquid water crosses the freezing front, falls into a zone with a low potential and crystallizes. Ice conglomerations grow and become a reason of strong bends of pipelines in short sections, which often leads to emergency depressurization. Therefore, the forecasting of the stress-strain state of pipelines and engineering protections on the heaving soils remains priority task.

The influence of heaving soils on the pipelines is carried out by the tangential and normal frost heaving forces. The tangential force does not exceed the strength limit of adfreezing and acts on pile peir or vertical sections of pipelines. The normal force is massively more tangential force. It is the main reason of predominate influence of normal force to reliability of pipelines. A mathematical model of the stress-strain state of the heaving soils is required to calculate the efficiency of engineering protection of a complex geometric shape

The authors showed that the model of linear elastic material describes the stress-strain state of the heaving soils under shallow foundation with sufficient accuracy. For confirmation the results of numerous experimental studies which became the basis for design standards [6,7] were used. Linear relationship between the load on the foundation and its vertical displacement and the excess of the frost heaving force over the bearing capacity (determined according to Coulomb's wedge theory) are additional confirmation of the possibility of using the model of linear elastic material. At the end of the article, the authors showed a way to simulate the force interaction of the soils and underground pipelines according to the model of linear elastic material.

Keywords: frost heaving, thermodynamic driving forces and fluxes, heat and mass transfer in soil, groundwater level, cross effects, kinetic coefficients

INTRODUCTION

Frost heaving is the process accompanied by an increase in the volume of soil as a result of the inflow of water to the freezing front and the formation of ice schliere. The danger of the phenomenon for pipelines is due to considerable forces, which are associated with an increase in the volume. Therefore, the simulation of the force interaction between pipelines and heaving soils is an priority task in the designing of pipelines.

The forces of frost heaving of soils are divided into two types:

1. Tangents force acting along the lateral surface. These forces do not exceed the forces of cohesion of the soil with the lateral surface and line in the range 100-200 kPa [1]. These forces act mainly on pile foundations and vertical sections of pipelines.

2. Normal forces of frost heave acting perpendicular to the surface of the pipeline. They are considered more dangerous than tangent forces, because exceed them by dozens of times. They pose a particular danger for underground pipelines with a negative temperature of the pumped product, because they act along the low half-perimeter and lead to strong bends in short sections. Shall depth of the pipeline leads to the influence of seasonal frost heaving. The smallness of the deformations leads to impossibility to diagnose unacceptable deformations under a deep snow cover.

Thus, mathematical modeling of the tangential forces of frost heaving does not present a significant difficulties. Simulation of the normal forces of frost heaving is associated with the determination of the stress-strain state of the soil and requires the choice of the special mathematical model.

OBJECT AND PURPOSE

In this article the authors solved the following problems:

- choice of a mathematical model for describing the existing regularities of the normal forces of frost heaving;

- validation of the mathematical model.

ESTIMATION OF THE CROSS EFFECTS

The author investigated the possibility of using two models of frost heaving soils deformations for calculating the normal force: linear-elastic and elastic-plastic.

The theoretical study of the interaction force between soils and pipelines was carried out by P.Yu. Mikhailov [2], A.I. Gorkovenko [3], I.A. Ivanov [4], A.B. Ajbinder [5]. The founder of the calculation theory of the normal frost heaving forces acting on the pipeline is A.B. Aibinder. He developed the hypothesis that the maximum force can not exceed the passive pressure or the bearing capacity of the soil at this depth in accordance with the Coulomb's wedge theory. However, the hypothesis has not found experimental confirmation and authors didn't use an elastic-plastic model for calculating the normal frost heaving force.

An important contribution to the study of the heaving soil properties of soils did the staff of the laboratory for studying the properties of frozen soils of PNIIS Gosstroy USSR under the leadership of V.O. Orlov in 1985. They executed a series of experiments and developed recommendations for prevention the deformations and forces of frost heaving, which are reflected in the design standard VSN 29-85 [6]. VSN 29-85 is based on the generalized results of long-term experimental studies of the frost heaving soils and its effect on foundations. The investigation represent huge value not only at designing of the foundation, but also for scientific researches. The authors validated frost heaving forces using VSN 29-85 without resource-consuming industrial studies.

Design standards VSN 29-85 allows to assess and prevent deformations and frost heaving forces acting on the shallow foundation rural buildings with a depth of freezing not more than 1.7 m. The impossibility of using this document for the calculation of pipelines is due to the difference of the temperature fields and the conditions of the wetting of the base under the buildings and pipelines. However, the authors of this article used this document to validate the mathematical model of the stress-strain state of the heaving soil under using the example of the foundation of the building, and, then, showed the way to apply this model for the calculation of pipelines.

Design standards VSN 29-85 shows a linear relationship between the vertical heaving and the load on the foundation. It is the main reason of choice of the mathematical model of soils stress-strain state - linear-elastic material. In addition, the reliability of the linear-elastic material model is confirmed by the values of frost heaving normal pressure, which are significantly higher than the bearing capacity according to the Coulomb's wedge theory [7].

Authors performed calculations of the load on the foundation, which could balance the forces of frost heaving and compared it with the analogous calculation in accordance with the method of VSN 29-85, that allowed to proof the possibility of using the linear-elastic material model for the heaving soils. The calculation scheme for linear-elastic material model is shown in Fig. 1 and corresponds to a band of continuous foundation with sand crush-rock pad.

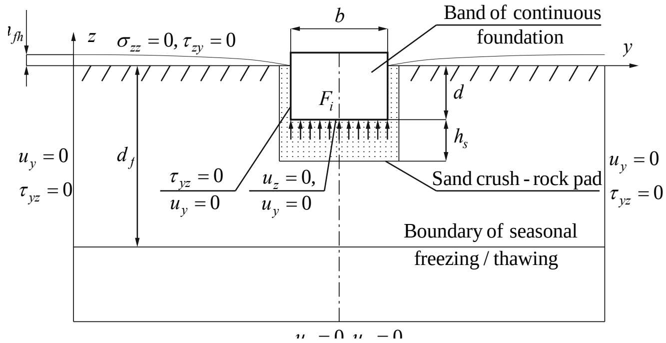

Figure 1: Calculation scheme, for validation of linear-elastic material model (u_y, u_z – displacements along the axis y and z, m; τ_{yz} – tangential stresses along the z axis in a plane perpendicular to the y axis, Pa; τ_{zy} – tangential stresses along the y axis in a plane perpendicular to the z axis, Pa; σ_{zz} – normal stresses along the axis z, Pa; h_{fh} - absolute value of not constrained vertically frost heaving, m; F_i – linear load, which balance the frost heave, N/m)

The choice of the continuous foundation is due to the boundaries of the applicability of design standards VSN 29-85 and the closest similarity continuous foundation to the pipeline (large length and small width). On the free surface, the boundary conditions correspond to zero tangential and normal stresses ($\sigma_{zz} = 0$, $\tau_{zy} = 0$). On the lateral boundaries the conditions correspond to zero tangential stresses and zero displacements along the normal of the soil boundary ($\tau_{yz} = 0$, $u_y = 0$), that corresponds to not constrained vertically frost heaving. On the lower boundary of the ground and the lower surface of the foundation preconditioned zero displacements, which corresponds to the stationary state of the soil massif and the foundation ($u_y = 0$, $u_z = 0$). On the lateral boundaries of the foundation boundary preconditioned zero normal displacements and zero tangential stresses, which corresponds to the protection against the freezing of the foundation with a heaving soil ($\tau_{yz} = 0$, $u_y = 0$).

The deformation modulus and Poisson's ratio were constants along the thickness of the soil, that will be justified after the results of the calculations. The relative frost heave degree ε_{fh} was constant along the thickness of the seasonally frozen layer:

$$\varepsilon_{fh} = \frac{h_{fh}}{d_f} \tag{1}$$

However, directly the value ε_{fh} can not be used in calculations. For the first, we translated it into a relative volumetric deformation, because the soil has volumetric increase in the case of frost heaving. The calculation scheme in Fig. 1 means that the ground has plan strain state away from the foundation. Then the equations of the generalized Hooke's law with $\varepsilon_{xx} = 0$, $\varepsilon_{yy} = 0$, $\sigma_{zz} = 0$:

$$\begin{cases} \varepsilon_{fh} = \frac{1}{E_{\text{soil}}}\left(-\nu_{soil}(\sigma_{xx} + \sigma_{yy})\right) + \frac{\varepsilon_V}{3} \\ 0 = \frac{1}{E_{\text{soil}}}(\sigma_{xx} - \nu_{soil}\sigma_{yy}) + \frac{\varepsilon_V}{3} \\ 0 = \frac{1}{E_{\text{soil}}}(\sigma_{yy} - \nu_{soil}\sigma_{xx}) + \frac{\varepsilon_V}{3} \end{cases} \quad (2)$$

where ε_V – relative volumetric deformation of frost heaving, u.f.; ε_{xx}, ε_{yy}, ε_{zz} – fractional extension along axis *x*, *y*, *z*, u.f.; σ_{xx}, σ_{yy}, σ_{zz} – normal stress along axis *x*, *y*, *z*, Па; E_{soil} – subgrade stiffness modulus, Pa; ν_{soil} – Poisson ratio of soil, u.f.

System of equations (2) allow deriving the expression for the revative volume deformation of the soil:

$$\varepsilon_V = \frac{3h_{fh}(1-\nu_{soil})}{d_f(1+\nu_{soil})} \quad (3)$$

Expression (3) was used to specify consant frost heaving in the soil layer from the surface to the seasonal freezing / thawing boundary. Calculation of the load F_i used the integration over the width of the base *b*.

$$F_i = \frac{1}{b}\int_{-b/2}^{b/2} \sigma_{zz}\, dy \quad (4)$$

Calculation of the load F_i, which balance the forces of frost heaving, according to the design standards VSN 29-85 takes into account the temperature of the winter months, the temperature of the room and the structure of the foundation insulation, the speed of frost heaving, the depth of the foundation and the thickness of the sand cruch-rock pad, hydrogeological conditions, the lie of the ground. For geological conditions with groundwater entering on the surface, which is typical for Western Siberia, and for the base construction in Fig. 1, the following formulas were used [6]:

$$v_f = \frac{h_f}{30t_d}; \quad (5)$$

$$h_f = h_{fh}\left(1 - \frac{d+h_s}{d_f}\right); \quad (6)$$

$$d_f = K_h d_{fn}; \quad (7)$$

$$d_{fn} = d_0\sqrt{\sum_{i=1}^{m}|T_{neg}|}; \quad (8)$$

$$t_d = t_0\left[1 - \left(\frac{d+h_{\text{п}}}{d_f}\right)^2\right]; \quad (9)$$

$$T_d = T_n\left(1 - \frac{d+h_s}{d_f}\right); \quad (10)$$

$$T_n = \min(|T_{n1}|, |T_{n2}|); \quad (11)$$

$$T_{n1} = \frac{2T_{min}t_d}{t_0}\left(1 - \frac{t_d}{2t_0}\right); \quad (12)$$

$$T_{n2} = \frac{T_{min}}{2}; \quad (13)$$

$$\sigma_s = 2 \cdot 10^8 v_f exp(0{,}525T_d); \quad (14)$$

$$F_i = \frac{2k_a(d_f - d - h_s)\sigma_s}{\beta}, \quad (15)$$

where v_{fh} - average speed of frost heaving, m/day; h_f - absolute value of frost heaving under the foundation with sand crush-rock pad, m; d – foundation depth, м; h_s – thickness sand crush-rock pad, m; d_f – depth of soil freezing, м; d_{fn} – standard depth of soil freezing under the foundation, m; K_h - coefficient, depending on the thermal regime of the building and the structure of the foundation (Table 1); d_0 – coefficient, that equals 0,23 for clay loam and clay, 0,28 for sand and sand clay); $\sum_{i=1}^{m}|T_{neg}|$ – the sum of the modules of monthly average negative temperatures, ºC; t_0 – duration of the period with negative temperature, months; t_d – time of soil freezing, months; T_d – calculating soil temperature under the foundation, ºC; T_{min} – minimal monthly average temperature, ºC; b – width of continuous foundation, m; k_a – coefficient that depends on the area of the foundation and the depth of soil freezing (Fig. 3 in [6]); β – coefficient, depending on the relative width of the foundation (Table 2); σ_s – the resistance to displacement of frozen soil relative to the foundation, kPa F_i – load on the foundation, excluding deformations of frost heaving, kN.

Table 1: Coefficient K_h [1]

Foundation without a cellar with floors:	Temperature in the room under the foundations, ºC				
	0	5	10	15	≥20
on the soil	0.9	0.8	0.7	0.6	0.5
on the soil with floor batten	1.0	0.9	0.8	0.7	0.6
with thermal insulation	1.0	1.0	0.9	0.8	0.7

Table 2: Coefficient β for continuous foundation [6]

h_s/b	0.00	0.25	0.50	0.75	1.00	1.25	1.50
β	1.00	0.98	0.96	0.94	0.92	0.88	0.84
h_s/b	1.75	2.00	2.25	2.50	2.75	3.00	
β	0.80	0.76	0.72	0.68	0.64	0.60	

The purpose of the validation is to confirm linear relationship between the load on the foundation and its absolute vertical displacement using linear elastic material and design standards VSN 29-85. In addition, the value of the subgrade stiffness modulus of frozen soil E_{soil} was calculated and its value was compared with the possible interval.

Application field of VSN 29-85 is the heaving soils of the temperate climate with the depth of seasonal freezing not more than 1.7 m. The climatic characteristics of the Tyumen meteorological station (Table 3) were chosen for calculations, because it is closest station in terms of climate and geology to significant industrial regions of the north of Western Siberia, where more than 100,000 km of pipelines are located.

Table 3: Monthly average temperature

XI	XII	I	II	III	year
-7.9	-13.7	-17.4	-16.1	-7.7	+0.9

According to the expression (8) and table 3, the standard depth of freezing within the territory of the Tyumen meteorological station can vary from 1.82 m on clay loam and clay to 2.22 m on sand clay and sand. Thermal insulating properties of the snow cover reduces the depth of freezing. Authors took depth of freezing far from the foundation equal to $d_f = 1{,}7$ m, that corresponds to the upper limit of the applicability of the design standards VSN 29-85.

The geometry of the calculation scheme corresponds to a shallow continuous foundation: b=1 m, d=0.5 m, h_s=0.3 m. The parameters d and h_s provide a freezing depth of the heaving soil under the foundation not more 0.9 m, that corresponds to the limits of applicability of VSN 29-85 in the choice of the coefficient k_a. The depth of seasonal freezing is the same both under and away the foundation, that corresponds to a coefficient K_h=0.93 on clay loam and clay and K_h=0.77 on sand clay and sand. The resulting values K_h correspond to the possible values in Table 1. Thus, the calculation scheme in Figure 1 fully corresponds to the limits of applicability of VSN 29-85.

Table 4 shows the results of calculating of the load F_i according to design standards VSN 29-85 and linear elastic material model for different values of absolute value of frost heaving h_{fh}.

Table 4: The results of calculating of the frost heaving force using the design standards VSN 29-85 and linear elastic material model

h_{fh}	0,01	0,04	0,14
Design standards VSN 29-85			
t_0	5	5	5
d_f	1,7	1,7	1,7
t_d	3,89	3,89	3,89
h_f	0,005	0,021	0,074
v_f	5,0e-5	1,8e-4	6,3e-4
T_{n1}	16,55	16,55	16,55
T_{n2}	8,70	8,70	8,70
T_n	8,70	8,70	8,70
T_d	4,61	4,61	4,61
σ_z	102	407	1425
K_a	0,14	0,14	0,14
β	0,98	0,98	0,98
F_i	26	105	366
Linear elastic material model			
ν_{soil}	0,35	0,35	0,35
ε_{fh}	0,0059	0,0235	0,0824
ε_V	0,0028	0,0113	0,0397
F_i	26	105	366
E_{soil}	10,36	10,36	10,36

Notes: Poisson's ratio ν_{soil} =0.35 to correspond to any type of soil (sand, sand clay, clay loam, clay) in the melted state [3] and the frozen state [8].

DISCUSSION

The results of calculations according to the linear elastic material model, given in Table 4, show the linear dependence between the load on the foundation F_i and value of absovertical lute displacement of frost heaving h_{fh}, which coincide with the design standard VSN 29-85. At the same time, calculated subgrade stiffness modulus is 10.36 MPa and corresponds to the possible interval for thawed and frozen soils [3, 8]. The last confirms the correctness of the previously accepted assumption of the constancy of the subgrade stiffness modulus in the different depth. Together, the results show the possibility of using linear elastic material model to calculate the frost heaving forces acting on the foundations.

To use the linear elastic material model for calculation of frost heaving forces acting on underground pipelines, it is necessary to take into account the differences in the configuration of temperature fields and moisture conditions under buildings and pipelines (i.e. to eliminate the limitations of design standard VSN 29-85). For this purpose, the author suggests to use the method described in [9-12]

CONCLUSIONS

The authors of the article showed that the linear elastic material model correctly describes the dependence between the vertical displacement of the foundation and the load on the foundation.

The obtained values of the subgrade stiffness modulus correspond to the possible values, which is an additional confirmation of the possibility to use the linear elastic material model.

The results obtained for the shallow foundations can be extended to underground pipelines. But it is necessary to into account the peculiarities of heat and mass transfer and ice formation using mathematical models described in [9-11].

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